For a particle in a box

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$E_1 = \frac{\pi^2\hbar^2}{2ma^2}, \quad E_2 = \frac{4\pi^2\hbar^2}{2ma^2}, \quad E_3 = \frac{9\pi^2\hbar^2}{2ma^2}.$$

$$\therefore \langle E\rangle = \frac{1}{6}E_1 + \frac{1}{2}\cdot 4E_1 + \frac{1}{3}9E_1$$

$$= \left[\frac{1}{6} + 2 + 3\right]\frac{\pi^2\hbar^2}{2ma^2}$$

$$= \frac{31}{6}\cdot\frac{\pi^2\hbar^2}{2ma^2}.$$

Problem Zettili

A particle of mass m, which freely moves inside an infinite potential well of length a, has the following initial wave fn at $t=0$.

$$\Psi(x,0) = \frac{A}{\sqrt{a}}\sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}}\sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}}\sin\left(\frac{5\pi x}{a}\right)$$

where A is a real constant

(a) Find A so that $\Psi(x,0)$ is normalized.

(b) If measurements of energy are carried out what are the values that will be found & what are the corresponding probabilities?

(c) Find the wave fn $\Psi(x,t)$ at any later time t.

(d) Calculate average energy

$$\rightarrow \psi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}.$$

$$\Psi(x,0) = \frac{A}{\sqrt{2}}\underbrace{\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)}_{\psi_1} + \frac{\sqrt{3}}{\sqrt{5}\sqrt{2}}\underbrace{\sqrt{\frac{2}{a}}\sin\left(\frac{3\pi x}{a}\right)}_{\psi_3} + \frac{1}{\sqrt{5}\sqrt{2}}\underbrace{\sqrt{\frac{2}{a}}\sin\left(\frac{5\pi x}{a}\right)}_{\psi_5}$$

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$$\Psi(x,0) = \frac{A}{\sqrt{2}}\psi_1(x) + \sqrt{\frac{3}{10}}\,\psi_3(x) + \frac{1}{\sqrt{10}}\psi_5(x).$$

ⓐ <u>Normalization</u>

$$\int \Psi^*(x,0)\,\Psi(x,0)\,dx = 1.$$

$$\Psi^*(x,0) = \frac{A}{\sqrt{2}}\psi_1^*(x) + \sqrt{\frac{3}{10}}\,\psi_3^*(x) + \frac{1}{\sqrt{10}}\psi_5^*(x)$$

$$\Psi^*(x,0)\,\Psi(x,0) = \frac{A}{2}\psi_1^*(x)\psi_1(x) + \frac{3}{10}\psi_3^*(x)\psi_3(x) + \frac{1}{10}\psi_5^*(x)\psi_5(x)$$

$$\int \Rightarrow \frac{A}{2} + \frac{3}{10} + \frac{1}{10} = 1$$

$$\Rightarrow A = \sqrt{\frac{6}{5}}.$$

$$\therefore \Psi(x,0) = \sqrt{\frac{3}{5}}\,\psi_1(x) + \sqrt{\frac{3}{10}}\,\psi_3(x) + \frac{1}{\sqrt{10}}\psi_5(x).$$

ⓑ <u>Energies</u> :- $E_1 = \frac{\pi^2\hbar^2}{2ma^2}$, $E_3 = \frac{9\pi^2\hbar^2}{2ma^2}$, $E_5 = \frac{25\pi^2\hbar^2}{2ma^2}$.

probability of measuring $E_1 = |c_1|^2 = \frac{3}{5}$

" " " $E_3 = |c_3|^2 = \frac{3}{10}$.

" " " $E_5 = |c_5|^2 = \frac{1}{10}$.

ⓒ. $\Psi(x,t) = \sqrt{\frac{3}{5}}\,\psi_1(x)\,e^{-iE_1t/\hbar} + \sqrt{\frac{3}{10}}\,\psi_3(x)\,e^{-iE_3t/\hbar} + \frac{1}{\sqrt{10}}\psi_5(x)\,e^{-iE_5t/\hbar}$.

ⓓ Average energy $\langle H\rangle = \sum_{n=1,3,5} |c_n|^2 E_n$

$$= |c_1|^2E_1 + |c_3|^2E_3 + |c_5|^2E_5$$

$$\langle H\rangle = \frac{3}{5}E_1 + \frac{3}{10}E_3 + \frac{1}{10}E_5$$

$$= \frac{3}{5}E_1 + \frac{3}{10}9E_1 + \frac{1}{10}25E_1$$

$$= \frac{58}{10}E_1 = \frac{29}{5}\cdot\frac{\pi^2\hbar^2}{2ma^2}$$

$$\boxed{\langle H\rangle = \frac{29\pi^2\hbar^2}{10ma^2}}$$

Griffith Prob 2.5

A particle in the infinite square well has as its initial wavefunction an even mixture of the first two stationary states

$$\Psi(x,0) = A[\psi_1(x) + \psi_2(x)].$$

(a) Normalize $\Psi(x,0)$

(b) Find $\Psi(x,t)$

(c) If you measured the energy of this particle what values might you get & what is the probability of getting each of them? Find the expectation value of H. How does it compare with E_1 & E_2?

$$\rightarrow \Psi^*(x,0) = A[\psi_1^*(x) + \psi_2^*(x)].$$

$$\int_0^a \Psi^*(x,0)\Psi(x,0)\,dx = 1$$

$$\Rightarrow \int_0^a A[\psi_1^*(x)+\psi_2^*(x)]\,A[\psi_1(x)+\psi_2(x)]\,dx = 1$$

or, $\int_0^{a} A^2 \left[\underbrace{\psi_1(x)^* \psi_1(x)}_{1} + \underbrace{\psi_2(x)^* \psi_2(x)}_{1} + \underbrace{\psi_1(x)^* \psi_2(x)}_{0} + \underbrace{\psi_2^*(x) \psi_1(x)}_{0} \right] = 1$

$$\int_0^a \psi_m^*(x)\, \psi_n(x)\, dx = \delta_{mn}$$

$\therefore \int_0^a (A^2 + A^2)\, dx = 1$

$\Rightarrow 2A^2 . a = 1$

$\Rightarrow A^2 = \frac{1}{2a} \Rightarrow A = \frac{1}{\sqrt{2a}}$.

(b) $\Psi(x,t) = \frac{1}{\sqrt{2}} \left[\psi_1(x)\, e^{-\frac{iE_1 t}{\hbar}} + \psi_2(x)\, e^{-iE_2 t/\hbar} \right]$.

(c) Probability of getting $E_1 = |c_1|^2 = \frac{1}{2}$

$E_2 = |c_2|^2 = \frac{1}{2}$.

$$\langle H \rangle = \sum_{n=1,2} |c_n|^2 E_n = |c_1|^2 E_1 + |c_2|^2 E_2$$

$$= \frac{1}{2} E_1 + \frac{1}{2} E_2$$

$$= \frac{1}{2} E_1 + \frac{1}{2} . 4E_1 = \frac{1}{2} E_1 + 2E_1$$

$$= \frac{5}{2} . \frac{\pi^2 \hbar^2}{2ma^2}$$

$$= \frac{5}{4} \frac{\pi^2 \hbar^2}{ma^2}$$