Wave particle duality

A moving body behaves ~~like a~~ in certain ways as though it has a wave nature.

A photon of light of frequency ν has the momentum

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

Since $\lambda\nu = c$

$$\Rightarrow \boxed{\lambda = \frac{h}{p}}$$ is called photon wavelength

This is a general expression and can be applied to any material body.

The momentum of a particle of mass m and velocity v is $\boxed{p = mv}$

and hence the equivalent wavelength (λ)

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

If p is greater $\Rightarrow$ shorter is the wavelength.

For Relativistic Case:

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Q. Find the de Broglie wavelength of (a) 46 g golf ball with a velocity of 30 m/s and
(b) an electron with velocity of 10^7 m/s.

Soln: (a) as $v << c \Rightarrow m = m_0$

& $\lambda = 4.8 \times 10^{-34}$ m.

as λ is very small in comparison to its dimension hence its ~~λ~~ λ is not detectable.

$\Rightarrow$

(b) Again $v << c$

$m = m_0 = 9.1 \times 10^{-31}$ kg

$\Rightarrow \lambda = 7.3 \times 10^{-11}$ m.

as the dimension of electron is comparable with its λ in this ~~by~~ form — The radius of H atom is 5.3×10^{-11} m.

$\Rightarrow$ it is detectable.

$\Rightarrow$ The wave character of moving e^- is the key to understand atomic structure and behaviour.

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Find the K.E of a proton whose de Broglie wavelength is $1.000\ fm = 1.000 \times 10^{-15}\ m$. Which is roughly the proton diameter.

A relativistic calculation is needed unless pc for the proton is much smaller than the proton rest mass $E_0 = 0.938\ GeV$.

$$\Rightarrow \quad pc = (mv)c = \frac{hc}{\lambda} = \frac{(4.136 \times 10^{-15}\ eV s)(2.998 \times 10^{8}\ m/s)}{1.00 \times 10^{-15}\ m}$$

$$= 1.24 \times 10^{9}\ eV = 1.2410\ GeV$$

Since $pc > E_0$, the relativistic calculation is required.

The total energy

$$E = \sqrt{E_0^2 + p^2c^2} = \sqrt{(0.938\ GeV)^2 + (1.2340\ GeV)^2}$$

$$= 1.555\ GeV.$$

The corresponding K.E is

$$K.E = E - E_0 = (1.555 - 0.938)\ GeV$$

$$= 617\ MeV.$$

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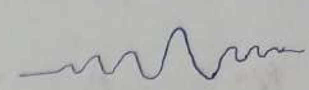
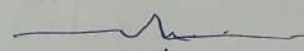
Need of Uncertainity principle

Wave Particle ~~duality~~ duality leads the wave Nature of Particle ⇒ • (Particle Classically) ≡ 

Wave group called wave packet

The particle is inside the equivalent wave packet anywhere

As the probability density $|\psi|^2$ is very high at the centre and hence it is likely to be here only. Nevertheless, we may still find the particle anywhere that $|\psi|^2$ (Region) is not actually zero.

Now Narrow the wave group will leads to more preciously a particle position can be specified. 

$\overleftrightarrow{\Delta x}$ small

But here the wavelength is not well defined as not enough wave to measure ~~define~~ wavelength λ accurately and as $\lambda = \frac{h}{p}$

⇒ p i.e the momentum is not a precise Quantity

⇒ If we make a ~~measurements~~ series of measurements then we will find a broad range of values.

On the otherhand a wide wavegroup,  in the picture λ is clearly defined and hence momentum "p" is precise Quantity and a series of measurements will give a narrow range of values. But where is the particle located ?

The width of the group is now too great for us to be able to say exactly where ~~is the~~ it is at a given time ⇒ Thus we have the

Uncertainity Principle

Heisenberg's Uncertainty Principle

The Product of the Uncertainties in determining the position and momentum of the particle Can never be smaller than the number of the order $\frac{\hbar}{2}$

$$\therefore \quad \boxed{\Delta x \, \Delta P \geq \frac{\hbar}{2}}$$

Similarly if ΔE is the uncertainty in the energy and Δt is the " " " " time

Then

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

Similarly $\Delta J \, \Delta \theta \geq \hbar/2$

$\Delta J \rightarrow$ uncertainity in total angular momentum of test

and $\Delta \theta \rightarrow$ uncertainity in angular position.

Q: A measurement estblishes the position of a proton with an accuracy of $\pm 1.00 \times 10^{-11}$ m. Find the uncertainity in the proton's position 1.00 s later. Assume $v \ll c$

Ans: at $t=0$, $\Delta x_0 = 1.00 \times 10^{-11}$ m

$$\Rightarrow \Delta P \geq \frac{\hbar}{2\Delta x_0}$$

$$\text{or } \Delta(mv) \geq \hbar/2\Delta x_0 \quad \text{as } v \ll c$$

$$\Delta(mv) = m_0 \Delta v$$

$$\Delta v \geq \frac{\hbar}{2\Delta x_0 m_0}$$

So after time t the new uncertainity in x will be

$$\Delta x = t\Delta v \geq \frac{\hbar t}{2 m_0 \Delta x_0}$$

$$\Delta x \geq \frac{1.054 \times 10^{-34} \times (1.00 \text{ sec})}{2(1.672 \times 10^{-27}) \times (1.00 \times 10^{-11}) \text{ m}}$$

$$\boxed{\Delta x \geq 3.15 \times 10^{3} \text{ m}}$$

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Application of Uncertainty Principle

A typical atomic nucleus is about 5.0×10^{-15}m in radius. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of a nucleus.

Soln: Letting $\Delta x = 5.0\times10^{-15}$ m

Now, $\Delta x \Delta p \geq \frac{\hbar}{2} \Rightarrow \Delta p \geq \frac{\hbar}{2\Delta x} \geq \frac{1.054\times10^{-34}}{2\times5.0\times10^{-15}}$

$\Rightarrow \Delta p \geq 1.1\times10^{-20}$ kg m/sec

If this is the uncertainty in the nucleus e's momentum, the momentum p itself must be at least comparable in magnitude. An e^- with such a momentum has a kinetic energy many times greater than its rest mass energy m_0c^2. As we know $K.E = p.c$ here to a sufficient degree of accuracy.

$\therefore K.E = pc \geq 1.1\times10^{-20}$ kg m/sec $\times 3\times10^{8}$ m/sec

$K.E \geq 3.3\times10^{-12}J \geq \frac{3.3\times10^{-12}J}{1.6\times10^{-19}} \geq 20$ MeV

If it is to be inside the nucleus.

Experiments show that even the electron associated with unstable atoms never have more than a fraction of this energy, and we conclude that nuclei do not contain electrons.

Q.2 A hydrogen atom is 5.3×10^{-11} m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom.

Soln: here $\Delta x = 5.3 \times 10^{-11}$ m

$$\Rightarrow \Delta p \geq \frac{\hbar}{2\Delta x} = 9.9 \times 10^{-25} \text{ kgm/sec.}$$

An electron whose momentum is of the order of magnitude behaves like a classical particle, and its K.E is

$$K.E = \frac{p^2}{2m} \geq \frac{(9.9 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} \geq 5.4 \times 10^{-19} J$$

$= 3.4$ ev.

The K.E of an electron in the lowest energy level of a H-atom is actually 13.6 ev.

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Q. An "excited atom" gives up its excess energy by emitting a photon of characteristic frequency, ~~as described~~. The average period that elapses between the excitation of an atom and the time it radiates is 1.0×10^{-8} sec. Find the inherent uncertainty in the frequency of the photon.

Soln:- Given: $\Delta t = 1.0 \times 10^{-8}$ sec.

$\Rightarrow$ The corresponding uncertainty in energy is

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2} \Rightarrow \Delta E \geq \frac{\hbar}{2\Delta t} = \frac{1.054 \times 10^{-34}\ J\,sec}{2(1.0 \times 10^{-8}\ sec)}$$

$$\Rightarrow \Delta E \geq 5.3 \times 10^{-27}\ J.$$

Now the corresponding uncertainty in frequency is

as $\Delta E = h \Delta \nu$

$$\Rightarrow \Delta \nu = \frac{\Delta E}{h} \geq \frac{5.3 \times 10^{-27}\ J}{6.63 \times 10^{-34}\ J\,sec} \geq 8.0 \times 10^{6}\ Hz.$$

This is the irreducible limit to the accuracy with which we can determine the frequency of the radiation emitted by an atom. As a result, the radiation ~~of~~ from a group of excited atoms does not appear with the precise frequency ν.

Calculate the de-Broglie wavelength associated with the electrons, which are accelerated by a voltage of 50 KV.

Soln: given $V = 50$ KV.

We know $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$ as $E = \frac{p^2}{2m}$

$$= \frac{h}{\sqrt{2meV}}$$

$\Rightarrow eV = 1.6 \times 10^{-19} \times 50 \times 10^{3}\ V = 8.0 \times 10^{-15}\ J$

$$\Rightarrow \lambda = \frac{6.63 \times 10^{-34}\ J\,sec}{\sqrt{2 \times 8.0 \times 10^{-15}\ J \times 9.1 \times 10^{-31}\ kg}}$$

$\Rightarrow \boxed{\lambda = 5.48 \times 10^{-12}\ m}$ Ans.