Magnetic field

The space around a current carrying conductor or magnet is defined as site of magnetic field.
The magnetic field around a moving charge exists in addition to the electrostatic field.
Magnetic field like the electric field is a vector field and its magnitude and direction at any point are specified by a vector $\vec{B}$ called as magnetic induction or magnetic flux density.
It is represented by lines called as lines of induction. It is related to its lines as :

(1) Tangent drawn at any pt to a line of induction gives the direction of $\vec{B}$ at that pt.

(2) No. of lines per unit area gives an idea about the magnitude of $\vec{B}$.

Magnetic flux :- (ϕ_B)

Magnetic flux across a surface is defined in the same way as the electric flux for the electric field.

$d\phi_B$ across an area dA

$$d\phi_B = B \cdot dA$$

$$d\phi_E = E \cdot dA = E \cdot ds$$

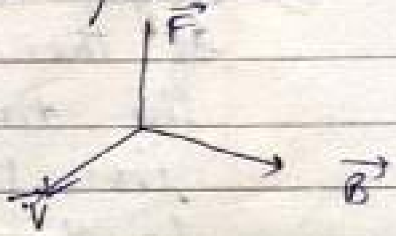
$$\phi_B = \int d\phi_B$$

$$\phi_B = B \cdot A$$

If $\vec{B}$ is uniform and normal to the area $\vec{A}$, then flux is BA

Force acting on Isolated moving charge

If the velocity of the moving charge is $\perp$ to the magnetic field, then the force acting on charge is observed to be $\perp$ to both $\vec{V}$ and $\vec{B}$ and this force is given as



$$F = q_0 VB$$
$$\vec{F} = q_0(\vec{V} \times \vec{B})$$

Note

1. Force is $\perp$ to the plane formed by $\vec{V}$ and $\vec{B}$
2. F vanishes as velocity tends to 0. or when $B = 0$
3. $\vec{V}$ and $\vec{B}$ are parallel or anti parallel in case of electric field, the force acting on a test charge is defined as
$$\vec{F} = q_0 \vec{E}$$
Here, the only characteristic direction is that of $\vec{F}$ whereas in case of magnetic field there are 2 characteristics dir^n i.e. $\vec{V}$ and F
4. Unit of $\vec{B}$ = Tesla or Weber/m^2 (SI unit)
$$1T = 10^4 G$$

Rules used to determine the direction of force

1. Fleming's Left Hd Rule

Fore finger, central finger and thumb of left hd are stretched in such a way that they are mutually ⊥ to each other.

Central finger → i or velocity of charges

Fore finger → $\vec{B}$

Thumb → $\vec{F}$

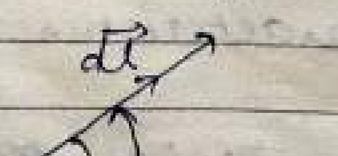
2. Right Hd Palm Rule

If the thumb of the right hand points in the dir^n of velocity and the fingers point in the dir^n of $\vec{B}$, then palm points in the dir^n of $\vec{F}$.

3. Right Hd screw Rule

$\vec{dl} \times \vec{r}$

Look at the plane containing the vectors $\vec{dl}$ and $\vec{r}$. Imagine moving from the first vector ($\vec{dl}$) towards the second vector ($\vec{r}$). If the movement is anticlockwise, the resultant is towards the reader. If clockwise, Rstt is away from the reader.



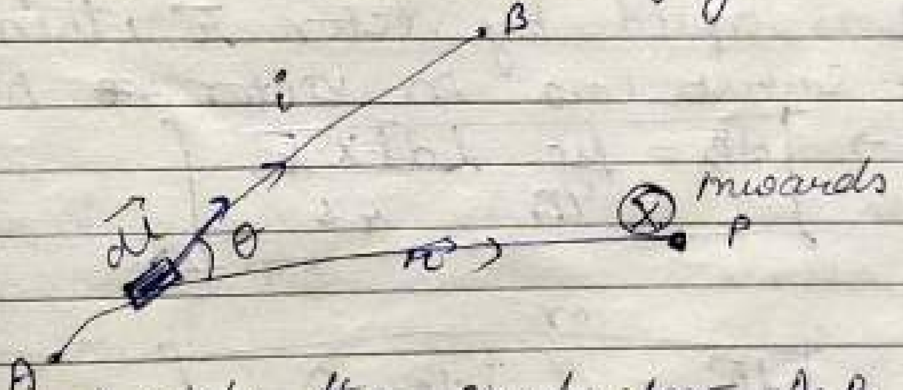
4. Rght Hd Thumb Rule
Curl the palm of right hand along the circular wire with the fingers pointing in the dirn of current, the right hand thumb gives the dirn of magnetic field.

Notes: Symbols of $\vec{B}$

• or ⊙ ⇒ field lines is coming towards reader

× or ⊗ → $\vec{B}$ going away from reader.

BIOT & SAVART LAW
It gives the procedure for calculating the magnetic field at any pt in space around a current carrying conductor



Divide the conductor AB into infinitesimal current element of length $\vec{dl}$

$\vec{dB}$ at pt P due to $\vec{dl}$ depends on 1. Directly proportional to current i in conductor

2. Inversely proportional to square of distance of pt P from the element

3. directly proportional to dl sinθ where θ is angle b/w current carrying element and vector $\vec{r}$.

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$$dB \propto \frac{i\,dl \sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2}$$

μ_0 = permeability of free space $= 4\pi \times 10^{-7}$ W/Am
$Wb\,A^{-1}m^{-1}$

dir^n of $\vec{B}$ at $\vec{P}$ is given by Right Hd Thumb Rule.

Grasp the conductor in the palm of your right hd so that the thumb pts in the dir^n of current and fingers curl gives the direction of magnetic field. So the lines of force are away from the reader.

<u>Note</u> 1. If the current element $d\vec{l}$ is represented by a vector $d\vec{l}$ & the pt P loc^n is rptd by vector $\vec{r}$, then Biot Savart law in vector notation is →

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,d\vec{l} \times \vec{r}}{r^3}}$$

or

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,d\vec{l} \times \hat{r}}{r^2}}$$

Total magnetic field at P due to entire conductor is $B = \int dB$

$$B = \int_L \frac{\mu_0}{4\pi} \frac{i\,dl \times r}{r^3}$$

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$$B = \frac{\mu_0}{4\pi}\int_L \frac{I\,dl \times r}{r^3} \quad \Big/ \quad \underset{\text{line current}}{i\,dl}$$

2. In case of surface currents, the current distribution is $K\,ds$

K = Surface current density

$= \frac{\text{current}}{\text{distance}}$

$$B = \frac{\mu_0}{4\pi}\int_S \frac{K \times r}{r^3}\,ds$$

3 Volume current $J\,dV$

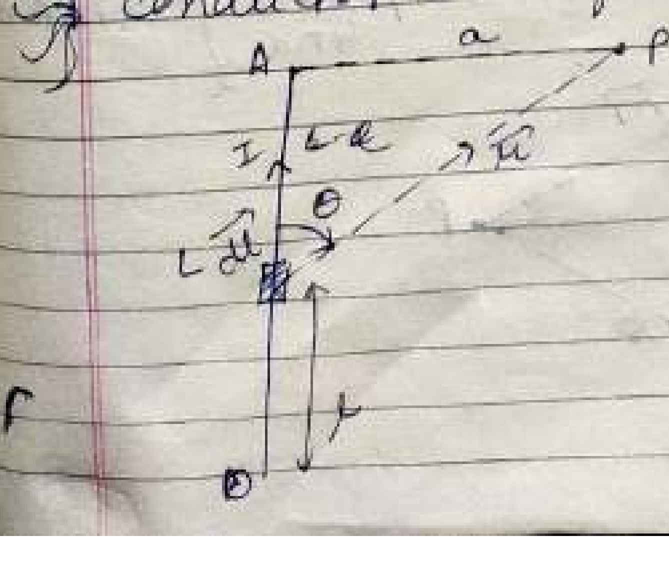
J = Volume current density $= \frac{\text{current}}{\text{area}}$

$J\,dV = \frac{i}{r^2}\,r^3 = ir \;(i\,dl)$

$$B = \frac{\mu_0}{4\pi}\int_V \frac{J \times r}{r^3}\,dV$$

B due to a straight current carrying conductor/wire using B-S-Law

Consider a straight current carrying conductor OA of length L



Magnetic field at P due to $\vec{dl}$:

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \times r}{r^3} = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}$$

Total magnetic field at P due to wire OA:

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin\theta}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl\, a}{r^3}$$

$r = a \operatorname{cosec}\theta$ $\qquad\qquad \sin\theta = a/R$

$$\tan\theta = \frac{a}{L - l}$$

$L - l = a \cot\theta$

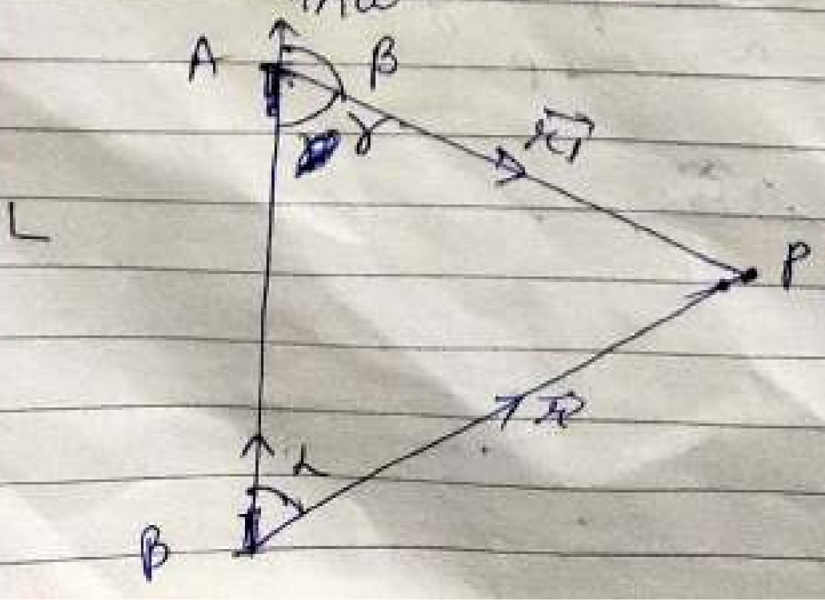
$l = L - a\cot\theta$

$dl = a \operatorname{cosec}^2\theta \, d\theta$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{a \operatorname{cosec}^2\theta \sin\theta}{a^2 \operatorname{cosec}^2\theta} d\theta$$

$$B = \frac{\mu_0 I}{4\pi a} \int \sin\theta \, d\theta = \frac{\mu_0 I}{4\pi a} \int_{\alpha}^{\beta} \sin\theta \, d\theta$$

~~$B = \frac{-\mu_0 I}{4\pi a} \cos\theta + C$~~ $\quad = \frac{\mu_0 I}{4\pi a} \int_{\alpha}^{180-\gamma} \sin\theta \, d\theta$



$$B = -\frac{\mu_0 I}{4\pi a}\left(\cos\theta\right)_{\alpha}^{180-\gamma}$$

$$= -\frac{\mu_0 I}{4\pi a}\left(\cos(180-\gamma) - \cos\alpha\right)$$

$$= -\frac{\mu_0 I}{4\pi a}\left(-\cos\gamma - \cos\alpha\right)$$

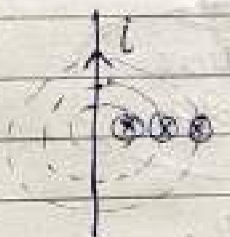
$$\boxed{B = \frac{\mu_0 I}{4\pi a}\left(\cos\gamma + \cos\alpha\right)}$$

For ∞ly long wire , $\alpha, \gamma = 0$

$$B = \frac{\mu_0 I}{4\pi a}(1+1)$$

$$\boxed{B = \frac{\mu_0 I}{2\pi a}}$$ magnitude

Dirn of $\vec{B}$



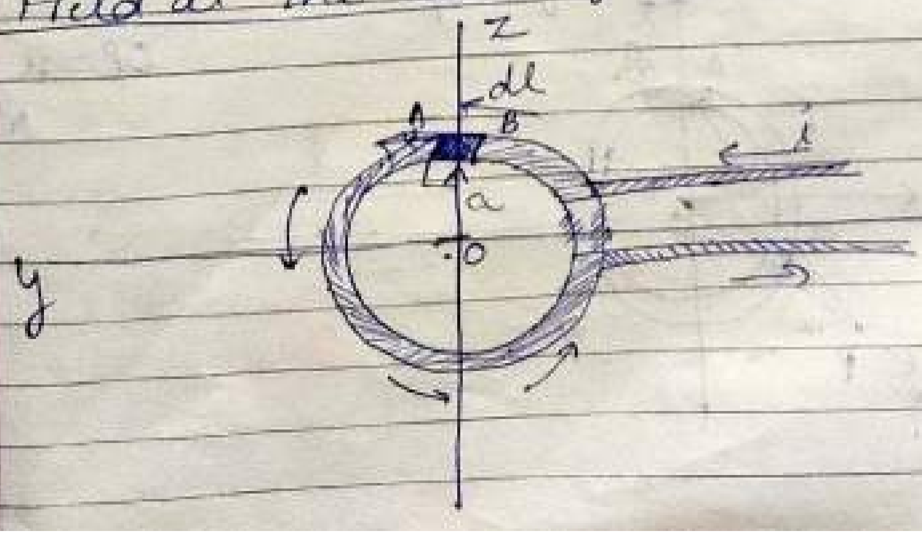
Away from the reader.

B due to circular loop or current loop

Consider a single coil of rad. a carrying a current i

(a) Field at the centre of the coil

loop is YZ plane



Magnetic field at O due to AB

$$dB = \frac{\mu_0}{4\pi} \frac{i\, dl \times a}{a^3}$$

$$= \frac{\mu_0}{4\pi} \frac{i\, dl\, a \sin 90^\circ}{a^3}$$

$$dB = \frac{\mu_0}{4\pi} \frac{i\, dl}{a^2}$$

Total magnetic field at O due to entire loop

$$B = \int dB$$

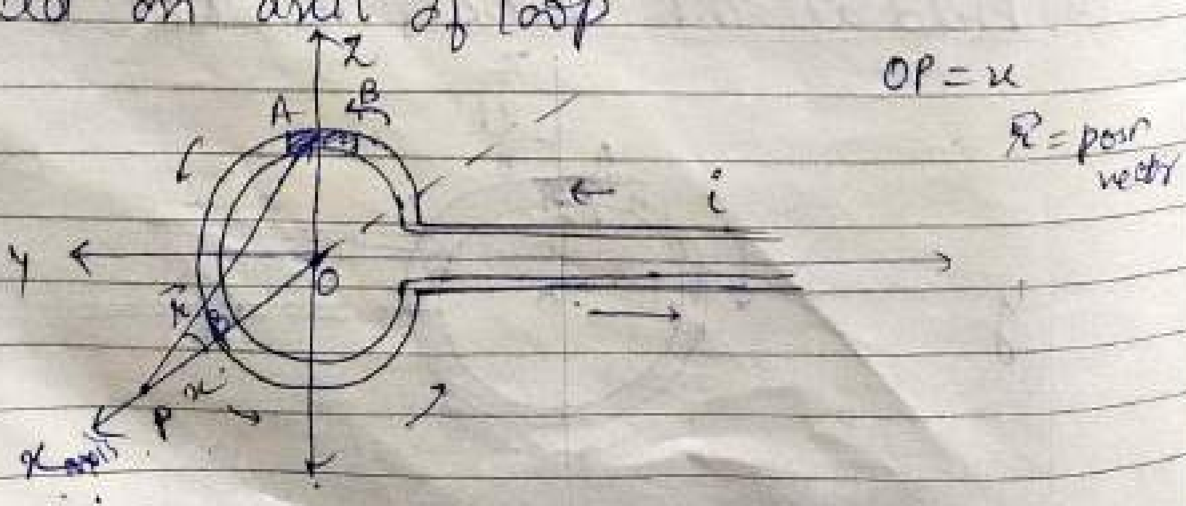
$$B = \frac{\mu_0 i}{4\pi a^2} \int dl$$

$$= \frac{\mu_0 i}{4\pi a^2} 2\pi a$$

$$B = \frac{\mu_0 i}{2a} \quad \text{magnitude}$$

dir^n : curl the fingers of the right hd along dir^n of i, dir^n of thumb gives the dir^n of B to x-axis

(b) Field on axis of loop



$\vec{dl}$ is in yz plane

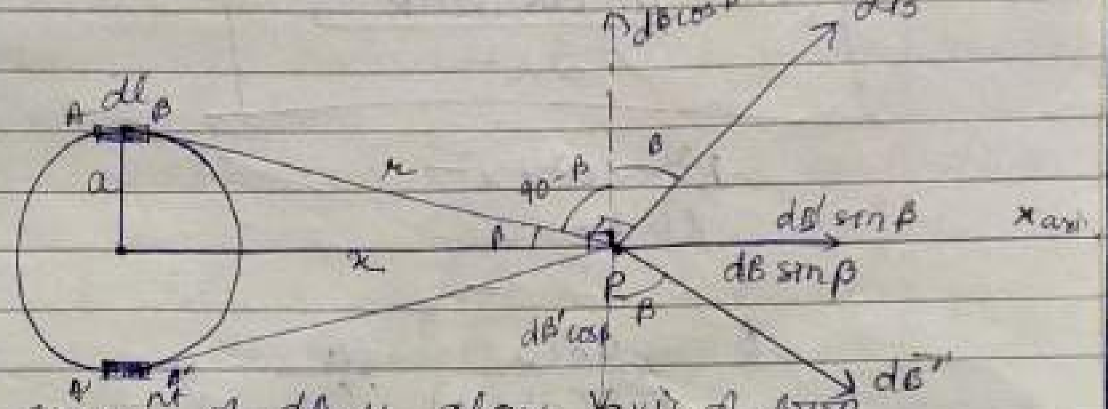
$\vec{r}$ is in xz plane

$\vec{B}$ at P due to AB

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, \vec{dl} \times \vec{r}}{r^3} = \frac{\mu_0 i}{4\pi} \frac{dl \sin\theta}{r^2}$$

$\theta = 90° \rightarrow$ diff planes

$$d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{dl}{r^2} \rightarrow xy \text{ plane dir}^n$$



one comp^{nt} of dB is along axis of loop

second is $\perp$ axis of loop

Comp^{nt} of $\vec{B}$ along axis of loop is

$$dB \sin\beta = \frac{\mu_0}{4\pi} \frac{i\,dl}{r^2} \sin\beta$$

Component of magnetic field along the axis $\perp$ to OX axis

$$dB \cos\beta = \frac{\mu_0}{4\pi} \frac{i\,dl}{r^2} \cos\beta$$

Consider element A'B' of same element A'B' of same length dl on other side of loop

dB' is $\perp$ to both r and A'B'

$\because$ element AB & A'B' are same

Hence $dB = dB'$.

comp. $\perp$ axis A/P

$dB \cos\beta = dB' \cos\beta$ (mag.) are same

dir^n is opposite

$\therefore$ No contribution from comp^n of $\vec{B}$ along $\perp$ to axis of loop

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Resultant $\vec{B}$ at P due to entire loop

$$B = \int dB \sin\beta$$

$$B = \frac{\mu_0 i}{4\pi} \int \frac{dl \sin\beta}{r^2}$$

$$= \frac{\mu_0 i}{4\pi} \frac{\sin\beta}{r^2} \int dl = \frac{\mu_0 i}{\cancel{4\pi}_2} \frac{\sin\beta}{r^2} 2\cancel{\pi} a$$

$$\boxed{B = \frac{\mu_0 i a \sin\beta}{2r^2}}$$

$$r^2 = a^2 + x^2$$

$$\sin\beta = \frac{a}{r}$$

$$B = \frac{\mu_0 i a}{2(a^2+x^2)} \frac{a}{(a^2+x^2)^{1/2}}$$

$$\boxed{B = \frac{\mu_0 i a^2}{2(a^2+x^2)^{3/2}}}$$ (Mag.)

Note

If $x >> a$

$$B = \frac{\mu_0}{2} \frac{i a^2}{x^3}$$

* and → with 2π

$$B = \frac{\mu_0}{4\pi} \frac{2\pi a^2 i}{x^3}$$

$$\boxed{B = \frac{\mu_0}{2\pi} \frac{iA}{x^3}}$$ A = area

$m = iA$ = magnetic moment of a current carrying loop

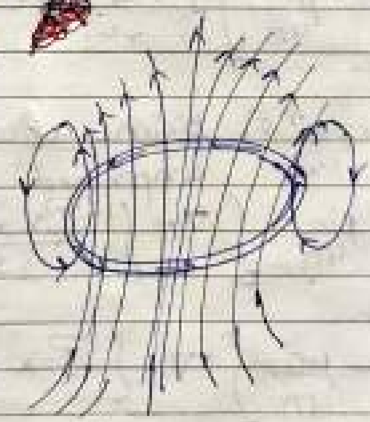
Electric field along axis of dipole

$$E = \frac{1}{2\pi\epsilon_0}\frac{p}{r^3} \quad p \rightarrow \text{dipole moment}$$

$$\boxed{B = \frac{\mu_0}{2\pi}\frac{m}{r^3}}$$

1. At far off point, a circular behaves as a dipole of moment m where m = product of current flowing & the area of coil. ~~M = pa~~
2. Any planar current loop is equivalent to a magnetic dipole whose dipole moment has a magnitude equal to the product of current through the loop & area of loop. The dirⁿ of m is normal to the plane of the loop

Direction

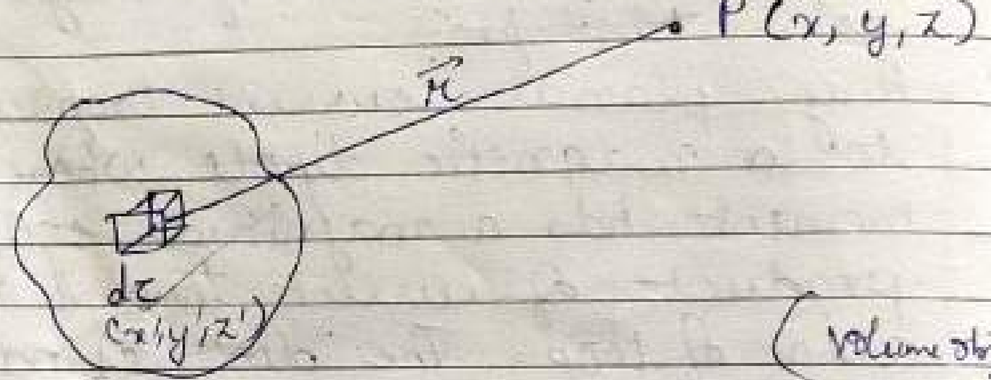


Steady current – a continuous flow of charge that has been going on forever without change and without charge piling up anyway is called as steady current

Steady current $\Rightarrow$ constant magnetic field
(continuous of charges) - $\frac{dB}{dt} = 0$
magnetostatics

Continuity eqn $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} = 0$ (steady currents)

Divergence of B or Free Magnetic poles donot exist



(Volume object)

$\vec{B}$ at P due to object

$$B = \frac{\mu_0}{4\pi} \int_\tau \frac{\vec{J} \times \hat{r}}{r^2} d\tau$$

$\vec{J}$ = Func^n of prime coordinates
$\vec{B}$ " " unprimed "

$$\nabla \cdot \vec{B} = \frac{\mu_0}{4\pi} \int_\tau \nabla \cdot \left\{ \frac{\vec{J} \times \vec{r}}{r^3} \right\} d\tau$$

$$= \frac{\mu_0}{4\pi} \int_\tau \nabla \cdot \left\{ \vec{J} \times \frac{\vec{r}}{r^3} \right] d\tau$$

Vector Identity $\nabla \cdot (\vec{P} \times \vec{Q}) = \vec{Q} \cdot [\nabla \times \vec{P}] - \vec{P} \cdot [\nabla \times \vec{Q}]$

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$$\nabla \cdot \vec{B} = \cancel{\frac{\mu_0}{4\pi}} \ldots$$

$$= \frac{\mu_0}{4\pi} \int_{\tau} \left\{ \underbrace{\frac{\vec{r}}{r^3} \cdot (\nabla \times \vec{J})}_{\text{unprimed}} - \underbrace{\vec{J} \cdot \left(\nabla \times \frac{\vec{r}}{r^3}\right)}_{\text{primed}} \right\} d\tau$$

$=0$

$$\nabla \cdot \vec{B} = -\frac{\mu_0}{4\pi} \int_{\tau} \vec{J} \cdot \left[\nabla \times \frac{\vec{r}}{r^3}\right] d\tau$$

$$\vec{r} = (x-x')\hat{i} + (y-y')\hat{j} + (z-z')\hat{k}$$

$\nabla \times \vec{r}$

$$\nabla \times \frac{\vec{A}}{f} = \frac{1}{f}(\nabla \times \vec{A}) + \frac{1}{f^2} \vec{A} \times (\nabla f)$$

$$\nabla \times \frac{\vec{r}}{r^3} = \frac{1}{r^3}(\nabla \times \vec{r}) + \frac{1}{r^6} \vec{r} \times (\nabla r^3)$$

$$\nabla \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x-x' & y-y' & z-z' \end{vmatrix} = 0$$

$$\nabla r^3 = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)^{3/2}$$

$$= \hat{i}\, \frac{3}{\cancel{2}} \underbrace{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}}_{\downarrow} \cancel{2}(x-x')$$

$+\hat{j}$ $\downarrow$ $(y-y')$

$+\hat{k}$ $\downarrow$ $(z-z')$

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$$\nabla r^3 = 3\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{1/2}$$
$$\left[(x-x')\hat{i}+(y-y')\hat{j}+(z-z')\hat{k}\right]$$

$$\nabla r^3 = 3r\vec{r}$$
$$\nabla \times \frac{\vec{r}}{r^3} = 0 - \frac{1}{r^6}\,\vec{r}\times(3r\vec{r})$$
$$= -\frac{3r}{r^6}(\vec{r}\times\vec{r}) = 0$$

$$\nabla\cdot\vec{B} = -\frac{\mu_0}{4\pi}\int_\tau \vec{J}\cdot\left(\nabla\times\frac{\vec{r}}{r^3}\right)d\tau$$

$$\boxed{\nabla\cdot\vec{B} = 0}$$

1. The above eqn physically means the magnetic ~~d~~ field doesnot have any sources similar to the electric charges for the electric field.
2. Mathematically, it is equivalent of the stmt that free magnetic poles donot exist

Note: Comparing
1. This eqn to the analogous eqn in electrostatics i.e ~~$\nabla\cdot B$~~ $\nabla\cdot D = \rho$, we conclude that there is no magnetic analog to the electric charge.
2. There are no magnetic charges from which the magnetic lines can emerge
3. Magnetic flux, $\phi_B = \oint B\cdot ds$
Gauss divergence theorem
$$\int_V \underbrace{\nabla\cdot B}_{\to 0}\, dV$$

$$\phi_B = \oint B \cdot ds = 0$$

Magnetic flux over any closed surface is 0.

4. $\boxed{\oint_S \vec{B} \cdot d\vec{s} = 0}$ Gauss law of magnetic field or magnetostatics (Integral Form)

Ampere's circuital law

Line integral of magnetic field around a closed curve is μ_0 times the current (c) passing through the ~~loop~~ path of integration

$$\oint B \cdot dl = \mu_0 i$$

Proof: $\text{curl } B = \mu_0 \vec{J}$ we know

i is the current flowing along a closed surface S.

$$\oint_S \text{curl } B \, ds = \mu_0 \oint J \, ds$$

converted to line integral using stoke's theorem

$$\int_S (\nabla \times A) \cdot ds = \oint_P A \cdot dl$$ Stoke's Theorem

Integral of curl over a region (surface) is equal to the value of the func^n at ~~at~~ the boundary (perimeter).

$$\oint B \cdot dl = \mu_0 \underbrace{\oint J ds}_{i}$$

$$\boxed{\oint B \cdot dl = \mu_0 i}$$

Illustration of Ampere Circuital law

Consider a straight current carrying wire, the magnetic field at a dist r is given as $B = \frac{\mu_0 i}{2\pi r}$

$$\oint B \cdot dl = \oint \frac{\mu_0 i}{2\pi r} dl$$

$$= \frac{\mu_0 i}{2\pi r} 2\pi r = \mu_0 i$$

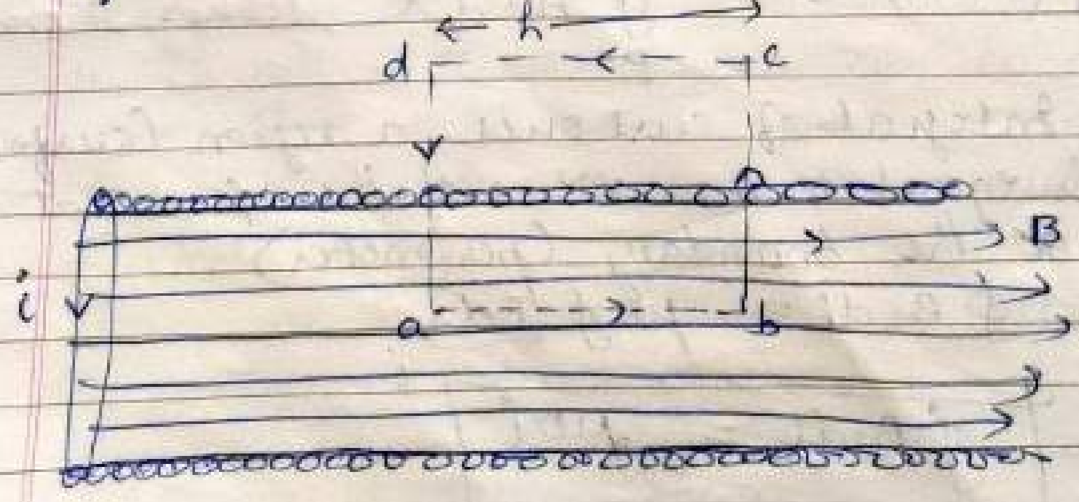
Note: Ampere circuital law is a convenient method of calculating $\vec{B}$ compared to the direct method based on the Biot savart's law

Using Ampere's law

Magnetic Field due to a solenoid

Inside

Consider a cylinder whose length is large in comparison to its radius. On closely winding uniform turns of wire on it, are obtained a long ~~solenoid~~ solenoid



L = Total length

N = Total no of turns in L

no. of turns per unit length $= \frac{N}{L}$

I_0 = current in 1 turn

$$\oint B\cdot dl = \int_a^b B\,dl + \int_b^c B\cdot dl + \int_c^d B\cdot dl + \int_d^a B\cdot dl$$

$\theta = 90^\circ$ $\theta = 90^\circ$

$$= ~~\int_a^b B\cdot dl + \int_c^d B\cdot dl~~$$

→ negligible as it is far off from the solenoid

$$= \int_a^b B\cdot dl + \int_a^b B\,dl\cos\theta$$

$$= B\int_a^b dl = Bh$$

$\oint B\cdot dl = \mu_0 i$ → current enclosed in path of integration

No. of turns in the path of integration (i.e. length h) $= \frac{N}{L}h$

i enclosed path $= \frac{N}{L} h \times I_0$

$$\oint B\cdot dl = \frac{\mu_0 N h I_0}{L}$$

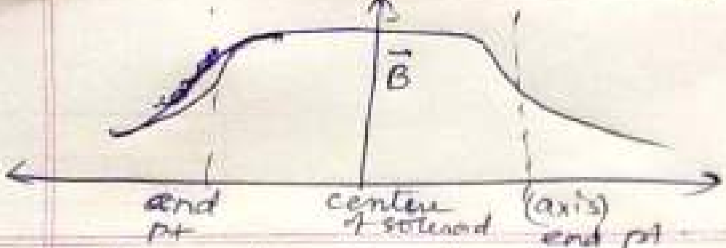
$$Bh = \frac{\mu_0 h I_0 N}{L}$$

$$\boxed{B = \frac{\mu_0 N I_0}{L}} = \mu_0 n I_0 \quad \text{where } n = \frac{N}{L}$$

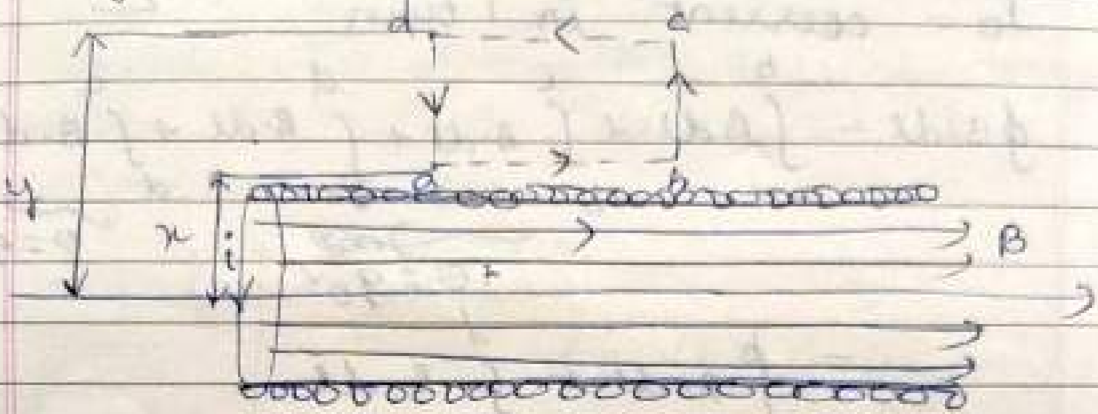
Note:

If the point P lies at one end of the solenoid

$$B = \frac{\mu_0 n I_0}{2}$$



Magnetic Field due to a solenoid outside it



I_0 = current in 1 turn

$$\oint B\cdot dl = \int_a^b B\cdot dl + \underbrace{\int_b^c B\cdot dl}_{0} + \int_c^d B\cdot dl + \underbrace{\int_d^a B\cdot dl}_{0}$$

$\theta = 90^\circ$

$$= \int_a^b B\cdot dl + \int_c^d B\cdot dl$$

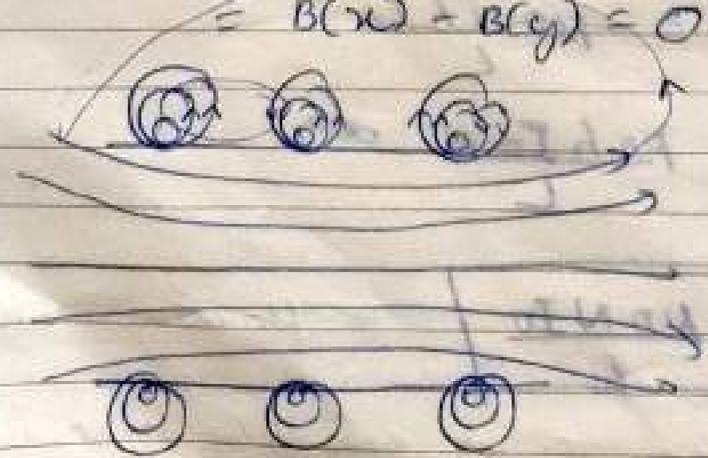
$$= \int_a^b B\,dl\cos 0 + \int_c^d B\,dl\cos \pi$$

$$= B\int_a^b dl - B\int_c^d dl$$

cancels

$$= Bh - Bh = 0$$

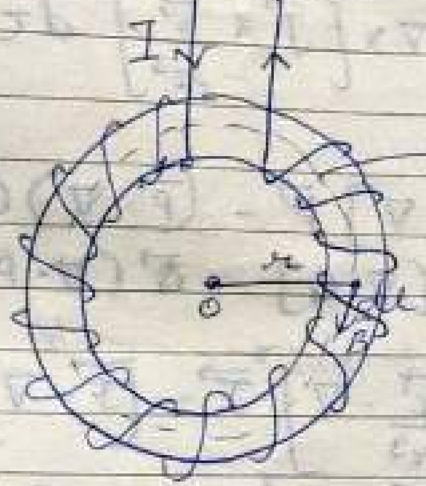
$$= B(x) - B(y) = 0$$



Outside solenoid, $B = 0$

Magnetic field of a Toroid

Toroid can be considered as a solenoid i.e. bent into a circle with the ends joined. Since the axis of the resulting bent solenoid is circular, the lines of B is also circular.



I_0 = current in each turn

N = Total no of turns

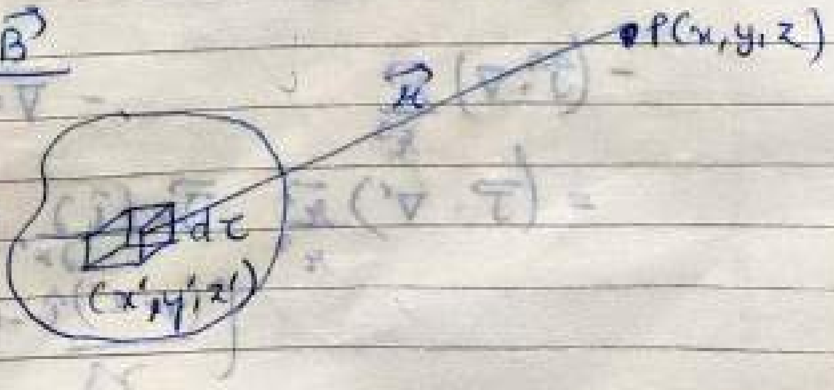
$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos 0 = B\, 2\pi r$

$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$ → current enclosed in the path of integration — Here pt of integration includes all the turns

$B\, 2\pi r = \mu_0 N I_0$

$$\boxed{B = \frac{\mu_0 N I_0}{2\pi r}}$$

Curl of $\vec{B}$



Magnetic field at P due to volume τ

$$\vec{B} = \frac{\mu_0}{4\pi} \int_\tau \frac{\vec{J} \times \hat{r}}{r^2} \, d\tau \quad , \ \vec{J} \text{ is volume current density}$$

$$\nabla \times \vec{B} = \frac{\mu_0}{4\pi} \int_\tau \nabla \times \left[\vec{J} \times \frac{\hat{r}}{r^2} \right] d\tau$$

$$= \frac{\mu_0}{4\pi} \int_\tau \nabla \times \left[\vec{J} \times \frac{\vec{r}}{r^3} \right] d\tau$$

$$\nabla \times (\vec{P} \times \vec{Q}) = (\vec{Q} \cdot \nabla) \vec{P} - (\vec{P} \cdot \nabla) \vec{Q} + \vec{P} (\nabla \cdot \vec{Q}) - \vec{Q} (\nabla \cdot \vec{P})$$

$$\nabla \times \left[\vec{J} \times \frac{\vec{r}}{r^3} \right] = \left[\frac{\vec{r}}{r^3} \cdot \nabla \right] \vec{J} - \left[\vec{J} \cdot \nabla \right] \frac{\vec{r}}{r^3} + \vec{J} \left[\nabla \cdot \frac{\vec{r}}{r^3} \right] - \frac{\vec{r}}{r^3} \left[\vec{\nabla} \cdot \vec{J} \right]$$

$\vec{J}$ is a function of primed coordinates,
$\vec{\nabla}$ is " unprimed coordinates

① & ④ terms $= 0$ due to prime & unprimed coord.

$$\nabla \times \left[\vec{J} \times \frac{\vec{r}}{r^3} \right] = -\left[\vec{J} \cdot \nabla \right] \frac{\vec{r}}{r^3} + J \left[\nabla \cdot \frac{\vec{r}}{r^3} \right]$$

$$(\nabla \times \vec{B}) = \frac{\mu_0}{4\pi} \left[-\int_\tau (\vec{J} \cdot \nabla) \frac{\vec{r}}{r^3} \, d\tau + \int_\tau \vec{J} \left(\nabla \cdot \frac{\vec{r}}{r^3} \right) \right]$$

$$-(\vec{J} \cdot \nabla) \frac{\vec{r}}{r^3} \qquad \because -\vec{\nabla} = \vec{\nabla}'$$

$$= (\vec{J} \cdot \nabla') \frac{\vec{r}}{r^3} \qquad \vec{J} \cdot \left(\hat{i} \frac{\partial}{\partial x'} + \hat{j} \frac{\partial}{\partial y'} + \hat{k} \frac{\partial}{\partial z'} \right)$$

$$\left[\frac{(x - x')\hat{i}}{r^3} + \frac{(y - y')\hat{j}}{r^3} + \frac{(z - z')\hat{k}}{r^3} \right]$$

x component :

$$\left[\vec{J}\cdot\vec{\nabla}'\frac{(x-x')}{r^3}\right]_x$$

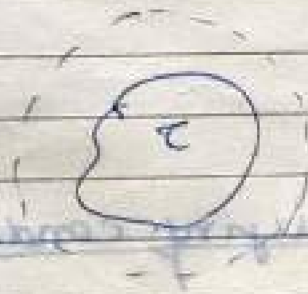
$$\nabla\cdot(f\vec{A}) = f(\nabla\cdot\vec{A}) + \vec{A}\cdot\nabla f$$

$$\left[\vec{J}\cdot\nabla'\left(\frac{x-x'}{r^3}\right)\right] = \nabla\cdot\left(\frac{x-x'}{r^3}\vec{J}\right) - \left(\frac{x-x'}{r^3}\right)\underbrace{(\nabla\cdot\vec{J})}_{\substack{=0 \\ \text{steady current}}}$$

$$\vec{\nabla}\times\vec{B} = \frac{\mu_0}{4\pi}\left[\int_\tau \vec{\nabla}'\cdot\left\{\frac{x-x'}{r^3}\vec{J}\right\}d\tau + \int_\tau \vec{J}\left(\nabla\cdot\frac{\vec{r}}{r^3}\right)d\tau\right]$$

Surface integral

$$\vec{\nabla}\times\vec{B} = \frac{\mu_0}{4\pi}\left[\underbrace{\oint_S \frac{x-x'}{r^3}\,\vec{J}\cdot d\vec{s}}_{x\text{ component}} + \int_\tau \vec{J}\left(\nabla\cdot\frac{\vec{r}}{r^3}\right)d\tau\right]$$



If we increase volume of integral, at --- surface there is no current density

Integral is carried out over the dotted line & surface integral is 0 because there is no $\vec{J}$ on the path of integral

$$\vec{\nabla}\times\vec{B} = \frac{\mu_0}{4\pi}\int_\tau \vec{J}\left(\nabla\cdot\frac{\vec{r}}{r^3}\right)d\tau$$

Gauss law $\Rightarrow$ $\nabla\cdot E = \rho/\epsilon_0$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\text{charge}}{r^2} \hat{r}$$

$$= \frac{1}{4\pi\epsilon_0} \int_{\tau} \frac{\rho \, d\tau}{r^2} \hat{r}$$

$$= \frac{1}{4\pi} \frac{1}{\epsilon_0} \int_{\tau} \frac{\hat{r}}{r^2} \, d\tau$$

$$\nabla \cdot E = \frac{1}{4\pi} \frac{1}{\epsilon_0} \int_{\tau} \left(\nabla \cdot \frac{\hat{r}}{r^2} \right) d\tau$$

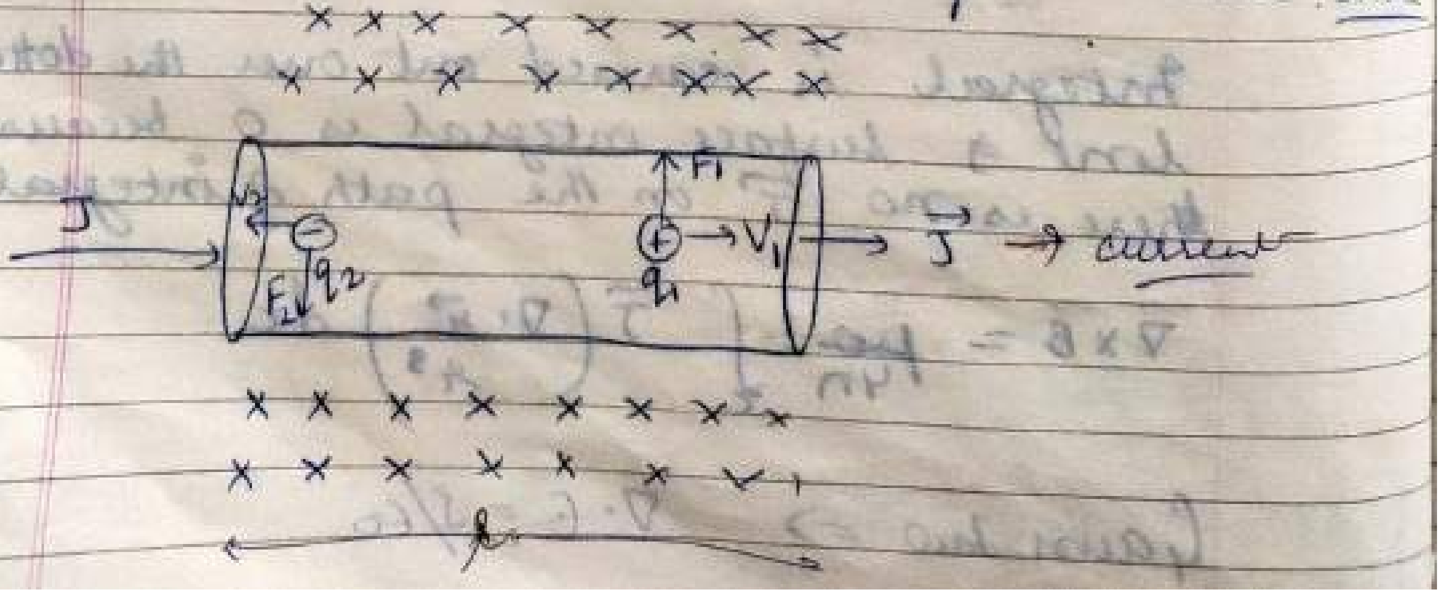
$$\cancel{\frac{1}{\epsilon_0}} = \frac{1}{4\pi} \cancel{\frac{1}{\epsilon_0}} \int_{\tau} \left(\nabla \cdot \frac{\hat{r}}{r^2} \right) d\tau$$

$$1 = \frac{1}{4\pi} \int_{\tau} \left(\nabla \cdot \frac{\vec{r}}{r^3} \right) d\tau \qquad (2)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \underbrace{\frac{1}{4\pi} \int_{\tau} \left(\nabla \cdot \frac{\vec{r}}{r^3} \right) d\tau}_{1}$$

✓ $\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$ Ampere's Law in Curl Form

<u>Force on a current carrying conductor or wire</u>



λ = length of wire l

B is $\perp$ to the plane of wire.

If q_1 is moving with a drift velocity v_1, then force acting on it is

$$\vec{F_1} = q_1(\vec{V_1} \times \vec{B})$$

If q_2 is moving with a drift velocity V_2, then force

$$\vec{F_2} = q_2(\vec{V_2} \times \vec{B})$$

Direction is opposite to $\vec{F_1}$

$$\vec{F_2} = -q_2(\vec{V_2} \times \vec{B})$$ (-) forcing -ve charge to go along $\vec{J}$

or to make $\vec{F_2} \parallel$ to $\vec{F_1}$

Let n_1 be the no. of positive charges per unit volume

n_2 = " (-ve) "

Al = volume

Total force acting on wire

$$\vec{F} = n_1 Al \vec{F_1} + n_2 Al \vec{F_2}$$

$$= n_1 Al q_1(\vec{V_1} \times \vec{B}) - n_2 Al q_2(\vec{V_2} \times \vec{B})$$

$$\vec{F} = \sum_{i=1}^{2} (Al)\, n_i q_i (\vec{V_i} \times \vec{B})$$

$$= Al \sum_{i=1}^{2} (n_i q_i \vec{V_i} \times \vec{B})$$ $J = n_i q_i \frac{x}{time}$

$$\boxed{\vec{F} = V(\vec{J} \times \vec{B})}$$

$i(\vec{l} \times \vec{B})$

$V(\vec{J} \times \vec{B})$

$\vec{J} = i/A$

$= V\vec{J} \times \vec{B}$

$= \frac{Vi}{A} \times \vec{B} = \vec{l}\, i \times \vec{B} = i(\vec{l} \times \vec{B})$

$$\boxed{\vec{F} = I(\vec{l} \times \vec{B})}$$

$$\boxed{F = i(\vec{l} \times \vec{B})}$$

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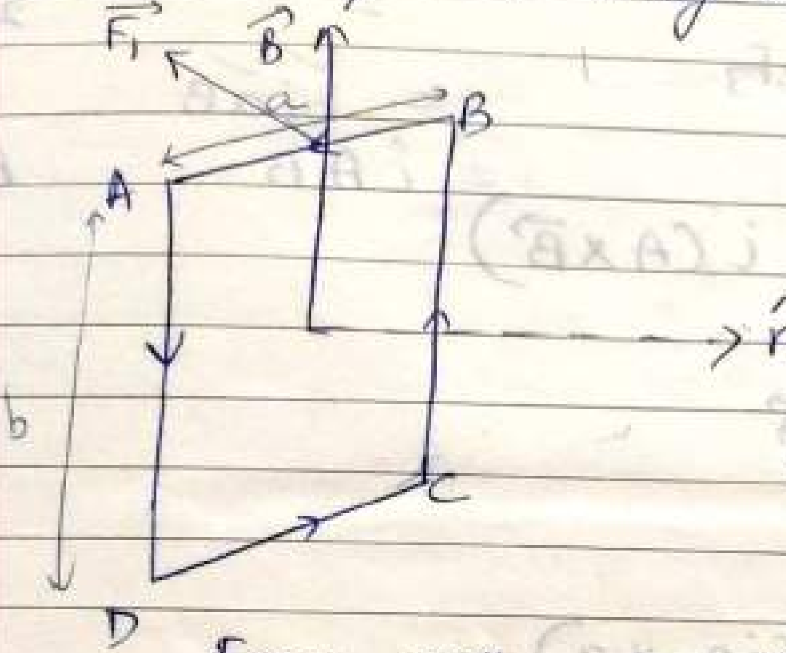
Force and Torque on a Current loop in a uniform magnetic loop.



$\vec{B}$ is taken along the plane of rectangular loop $\hat{n}$ direction of the surface of loop,

$\theta = 90°$, b/w $\vec{B}$ & $\hat{n}$

Force acting on BA is

$F_1 = i(a \times B)$

↳ away from the reader

$F_1 = iaB$ ① $(\theta = 90°)$

side BA ⊥ $\vec{B}$

Force acting on side AD

$F_2 = i(b \times \vec{B})$

$= ibB \sin \pi = 0$ ②

Force acting on DC

$\vec{F_3} = i(a \times B) = iab$ ③

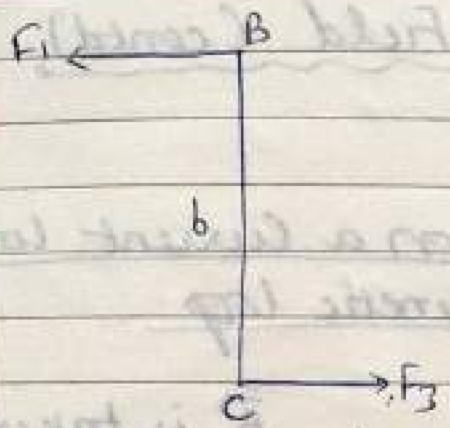
towards the reader

Force acting on side CD is

$\vec{F_4} = i(b \times B) = 0$ ④

Net force acting on loop $= 0$ (having opposite dirn)

$\vec{\tau} = \vec{r} \times \vec{F}$



τ ~~towards~~ at B + τ at C

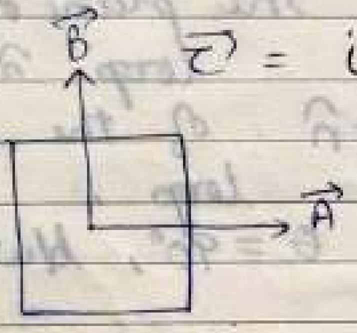
$= r \times F_1 + r \times F_3$

$= \frac{b}{2} F_1 \sin 90 + \frac{b}{2} F_3 \sin 90$

$= \frac{b}{2}(iaB) + \frac{b}{2}(iaB)$

$= biaB$

$= iAB$ A = area



$\vec{\tau} = i(A \times \vec{B})$

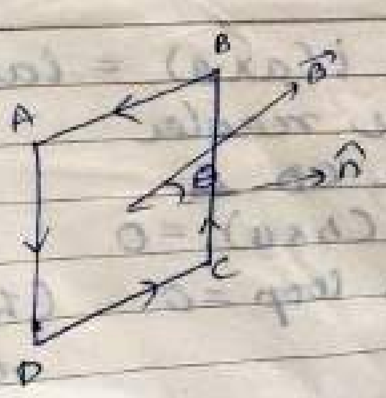
Note:

$\vec{\tau} = (iA \times B)$

$$\boxed{\vec{\tau} = \vec{m} \times \vec{B}}$$

When a current carrying loop is kept in a uniform magnetic field, it will experience a torque given by above eqn ie $\vec{\tau} = \vec{m} \times \vec{B}$

Potential Energy of a Current Loop



θ is the angle b/w $\vec{B}$ and $\hat{n}$ in zero potential energy state or stable state

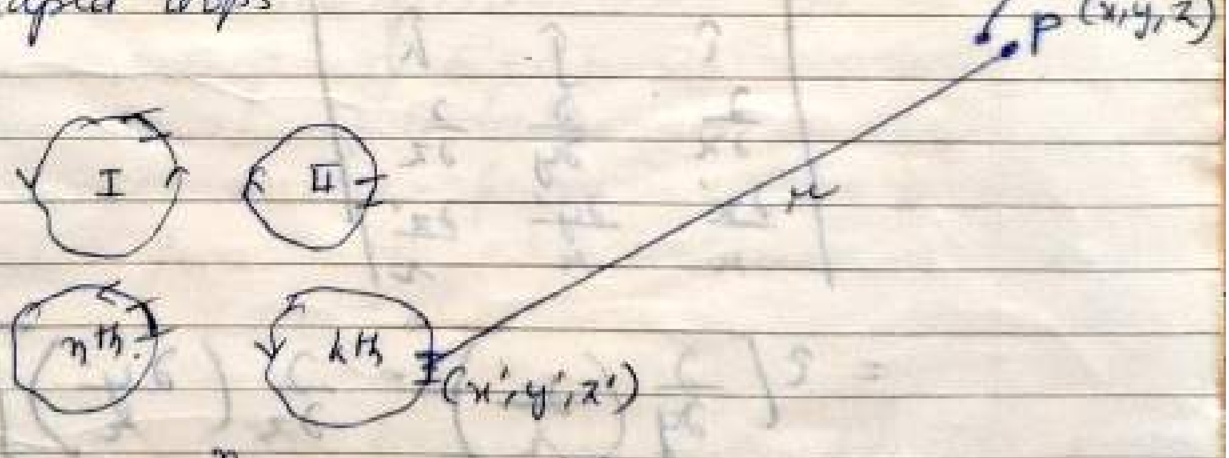
Work is done against the force due to $\vec{B}$ to change angle b/w $\vec{B}$ & $\hat{n}$ to 90°

$$\cancel{V=}\ U = -\int_0^{\pi/2} \tau \, d\theta = -\int_0^{\pi/2} \frac{iA}{n} B \sin\theta \, d\theta$$

$$U = -mB\cos\theta$$
$$U = -(\vec{m}\cdot\vec{B})$$

Magnetic Vector Potential ($\vec{A}$)

Starting from the BIOT-savart's law, we find that magnetic field can be expressed in terms of simpler functions. This function is called as magnetic vector potential. Consider n current loops producing the magnetic fields. Let us take arbitrary shaped loops



$$\vec{B} = \frac{\mu_0}{4\pi} \sum_{k=1}^{n} i_k \oint_k \frac{\vec{dl} \times \hat{r}}{r^2}$$

$$\vec{dl} = dx'\,\hat{i} + dy'\,\hat{j} + dz'\,\hat{k}$$
$$\vec{r} = (x-x')\,\hat{i} + (y-y')\,\hat{j} + (z-z')\,\hat{k}$$

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$$\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2}$$

$$\frac{dl \times \hat{r}}{r^2} = \frac{\vec{dl} \times \hat{r}}{r^2} = -\vec{dl} \times \vec{\nabla}\left(\frac{1}{r}\right)$$

$$= \vec{\nabla}\left(\frac{1}{r}\right) \times \vec{dl}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x}\left(\frac{1}{r}\right) & \frac{\partial}{\partial y}\left(\frac{1}{r}\right) & \frac{\partial}{\partial z}\left(\frac{1}{r}\right) \\ dx' & dy' & dz' \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}\left(\frac{1}{r}\right) dz' - \frac{\partial}{\partial z}\left(\frac{1}{r}\right) dy'\right]$$

$$\hat{i}\left[\frac{\partial}{\partial y}\left(\frac{dz'}{r}\right) - \frac{\partial}{\partial z}\left(\frac{dy'}{r}\right)\right]$$

$$-\hat{j} \qquad +\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial x'}{r} & \frac{\partial y'}{r} & \frac{\partial z'}{r} \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}\left(\frac{\partial z'}{r}\right) - \frac{\partial}{\partial z}\left(\frac{\partial y'}{r}\right)\right]$$

$$\nabla \times \frac{\vec{dl}}{r} = \text{curl}\left(\frac{\vec{dl}}{r}\right)$$

$$\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\left[\frac{\partial x'}{r}\hat{i} + \frac{\partial y'}{r}\hat{j} + \frac{\partial z'}{r}\hat{k}\right]$$

$$\frac{d\vec{l} \times \hat{r}}{r^2} = \text{curl}\left(\frac{d\vec{l}}{r}\right)$$

$$\vec{B} = \frac{\mu_0}{4\pi} \sum_{k=1}^{n} i_k \oint_k \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi} \sum_{k=1}^{n} i_k \oint_k \nabla \times \left(\frac{d\vec{l}}{r}\right)$$

$$\vec{B} = \nabla \times \underbrace{\left[\frac{\mu_0}{4\pi} \sum_{k=1}^{n} i_k \oint_k \frac{d\vec{l}}{r}\right]}_{\vec{A}}$$

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

Magnetic Scalar Potential

In electrostatics, the electric field E is a conservative field.

i.e. why $\oint \vec{E} \cdot dl = 0 \rightarrow$ path independ-

This nature of E helps us to define a single value func^n which is a scalar func^n i.e. Electrostatic potential V

$$V = -\int_{\infty} E \cdot dl$$

It is natural to think of a similar scalar func^n for magnetic fields which are very useful. If such a func^n exists, we call it as magnetic scalar potential which can be written as

$$V_m = -\int_{\infty} B \cdot dl$$

But from Ampere's Circuital Law

$$\oint B \cdot dl = \mu_0 i \neq 0$$

It is 0 only when $i = 0$ i.e. path of integration includes no current. This tells the line integral of the magnetic field is not path independent.
Hence, they are non-conservative force

Due to this feature of magnetic field, the magnetic scalar potⁿ is not a single valued funcⁿ where it is being defined. Hence, it is very difficult to express magnetic scalar potential in a simpler form

Q Consider an e^- moving in a circle of radius 5×10^{-11} m around a nucleus with constant speed 2.18×10^8 m/s. Find value of magnetic field at centre & find magnetic dipole moment.

soln

$$B = \frac{\mu_0 i}{2a}$$

$$\text{speed } V = \frac{2\pi a}{t}$$

$$t = \frac{2\pi \times 5 \times 10^{-11}}{2.18 \times 10^8} = 1.44 \times 10^{-16} \text{ sec}$$

charge per sec $i = \frac{q_e}{t} = \frac{1.6 \times 10^{-19}}{1.44 \times 10^{-16}} = 1.11 \times 10^{-3}$ amp

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$$B = \frac{4\pi \times 10^{-7} \times 1.11 \times 10^{-3}}{2 \times 5 \times 10^{-11}}$$

$B = 13.941$ Tesla

$m = iA = 1.11 \times 10^{-3} \times \pi \, (5 \times 10^{-11})^2$

$= 87 \times 10^{-19}$ Amp m^2