12 November 2020
09:20

Q. Let $\beta \in S_7$ and $\beta^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7)$.

find β? If $\beta \in S_9$, then also find β.

Sol^n //

Given that $\beta \in S_7$

and $\beta^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7)$ ——— (1)

$S_7 \to$ set of all permutations of the set $\{1, 2, 3, 4, 5, 6, 7\}$

from eq^n (1), $\beta^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7) \to 7$-cycle

$\therefore \Rightarrow |\beta^4| = 7$

$\Rightarrow (\beta^4)^7 = \varepsilon$

$\Rightarrow \beta^{28} = \varepsilon$

$\Rightarrow |\beta|$ divides 28

$\Rightarrow |\beta|$ can be 1 or 2 or 4 or 7 or 14 or 28. ——— (2)

Case-I:

let $|\beta| = 1$ or 2 or 4

In this case, $\beta^4 = \varepsilon$

But from eqn ①, $\beta^4 = (2\,1\,4\,3\,5\,6\,7) \neq \varepsilon$

$\longrightarrow \Leftarrow$

$\therefore |\beta|$ can not be 1 or 2 or 4

~~Case II:~~ If $|\beta| = 14$,

then possible disjoint cycle structure of β are

$(7)(2)$ or $(14)(1)$

These disjoint cycle structure requires 9 symbols and 15 symbols respectively but we have only 7 symbols in S_7

$\therefore |\beta|$ can not be 14.

~~Case III:~~ If $|\beta| = 28$.

Then the possible disjoint cycle structure of β are

$(7)(4)$ or $(28)(1)$

requires 11 symbols

requires 29 symbols

In S_7, there are only seven symbols

$\therefore$ $|\beta|$ cannot be 28.

from eqn ②, $|\beta| = 7$ ——— ③

$\Rightarrow \beta^7 = \varepsilon$

from eqn ①, $\beta^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7)$

$\Rightarrow (\beta^4)^2 = (2\ 1\ 4\ 3\ 5\ 6\ 7)(2\ 1\ 4\ 3\ 5\ 6\ 7)$

$\Rightarrow \beta^8 = (2\ 4\ 5\ 7\ 1\ 3\ 6)$

$\Rightarrow \beta^7\beta = (2\ 4\ 5\ 7\ 1\ 3\ 6)$

$\Rightarrow \beta = (2\ 4\ 5\ 7\ 1\ 3\ 6)$ [from eq③ $\beta^7 = \varepsilon$]

~~II part~~

If $\beta \in S_9$ $\Rightarrow$ $|\beta| = 7$ or 14

then possible disjoint cycle structures are (7) or $(7)(2)$

~~Case-I~~: If $|\beta| = 7$

the possible disjoint cycle structure is (7)

and $\beta = (2\ 4\ 5\ 7\ 1\ 3\ 6)$

Case-II: If $|\beta| = 14$

the possible disjoint cycle structure is (7)(...

-

the possible disjoint cycle structure is (7)(2)

and $\beta = (2\ 4\ 5\ 7\ 1\ 3\ 6)\ (8\ 9)$

Q. let $\beta = (1 2 3)(1 4 5)$. find β^{99} ?

Sol^n :: $\beta = (1 2 3)(1 4 5)$

$\Rightarrow \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 3 & 5 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 5 & 2 \end{bmatrix}$

$= (1\ 4\ 5\ 2\ 3)$ ~~+5~~ = cycle ①

$\Rightarrow |\beta| = 5$

$\Rightarrow \beta^5 = \varepsilon$

$\Rightarrow (\beta^5)^{20} = \varepsilon^{20}$

$\Rightarrow \beta^{100} = \varepsilon$

$\Rightarrow \beta^{99} . \beta = \varepsilon$

$\Rightarrow \beta^{99} \beta \beta^{-1} = \varepsilon \beta^{-1} \Rightarrow \beta^{99} = \beta^{-1}$

$\Rightarrow \beta^{99} = (1\ 4\ 5\ 2\ 3)^{-1}$ [from eq^n ①]

$\Rightarrow \beta^{99} = (3\ 2\ 5\ 4\ 1)$ ✓

$\Rightarrow \beta^{99} = (1\ 3\ 2\ 5\ 4)$ ✓

$\left[\because (a_1\ a_2 \cdots a_n)^{-1} = (a_n\ a_{n-1} \cdots a_2\ a_1)\right]$