Entropy

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Entropy of a perfect Gas

From first law

$$dQ = dU + dW$$

$$= C_v\, dT + P\, dV \quad -(1)$$

$$dQ = T\, dS \quad -(2)$$

$$\Rightarrow T\, dS = C_v\, dT + P\, dV$$

$$\boxed{dS = C_v \frac{dT}{T} + \frac{P\, dV}{T}} \quad -(3)$$

(a)Entropy in terms of volume and temperature

$$S_2 - S_1 = \int_1^2 dS = \int_{T_1}^{T_2} C_v \frac{dT}{T} + \int_{V_1}^{V_2} \frac{P\,dV}{T} \quad \text{— (4)}$$

$$\left\{ \begin{array}{l} PV = RT \\ \Rightarrow P = \frac{RT}{V} \end{array} \right\}$$

$$S_2 - S_1 = C_v \int_{T_1}^{T_2} \frac{dT}{T} + R \int_{V_1}^{V_2} \frac{dV}{V}$$

$$= C_v \left| \log_e T \right|_{T_1}^{T_2} + R \left| \log_e V \right|_{V_1}^{V_2}$$

$$\Rightarrow \boxed{S_2 - S_1 = C_v \log_e \frac{T_2}{T_1} + (C_p - C_v) \log_e \frac{V_2}{V_1}} \quad \text{— (5)}$$

$$S_2 - S_1 = \frac{1}{M} \left[C_v \log_e \frac{T_2}{T_1} + (C_p - C_v) \log_e \frac{V_2}{V_1} \right]$$

(b) Entropy in terms of pressure and temperature

$$PV = RT \longrightarrow V = \frac{RT}{P}$$

$$PdV + VdP = RdT$$

$$\Rightarrow PdV = RdT - VdP \quad - (6)$$

$$S_2 - S_1 = \int_{T_1}^{T_2} C_v \frac{dT}{T} + \int_{T_1}^{T_2} R \frac{dT}{T} - \int_{P_1}^{P_2} R \frac{dP}{P}$$

$$= C_v \int_{T_1}^{T_2} \frac{dT}{T} + (C_p - C_v) \int_{T_1}^{T_2} \frac{dT}{T} - R \int_{P_1}^{P_2} \frac{dP}{P}$$

$$S_2 - S_1 = C_p \int_{T_1}^{T_2} \frac{dT}{T} - (C_p - C_v) \int_{P_1}^{P_2} \frac{dP}{P}$$

$$= C_p \left| \log_e T \right|_{T_1}^{T_2} - (C_p - C_v) \left| \log_e P \right|_{P_1}^{P_2}$$

$$\boxed{S_2 - S_1 = C_p \log_e \frac{T_2}{T_1} - (C_p - C_v) \log_e \frac{P_2}{P_1}} \quad - 7$$

$$S_2 - S_1 = \frac{1}{M} \left[C_p \log_e \frac{T_2}{T_1} - (C_p - C_v) \log_e \frac{P_2}{P_1} \right]$$

(c) Entropy in terms of pressure and volume

$$PV = RT \longrightarrow T = \frac{PV}{R} \quad -(8)$$

$$\downarrow$$

$$PdV + VdP = RdT$$

$$\Rightarrow dT = \frac{PdV + VdP}{R} \quad -(9)$$

$$dS = C_V \frac{PdV + VdP}{\cancel{R}} \times \frac{\cancel{R}}{PV} + \frac{Pdv}{PV} \times R$$

$$ds = C_v \frac{P\,dV + V\,dP}{PV} + \frac{R\,dV}{V}$$

$$= C_v\left(\frac{dV}{V} + \frac{dP}{P}\right) + (C_p - C_v)\frac{dV}{V}$$

$$= C_v \frac{dP}{P} + C_p \frac{dV}{V}$$

$$S_2 - S_1 = \int_1^2 dS = C_v \int_{P_1}^{P_2} \frac{dP}{P} + C_p \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\boxed{S_2 - S_1 = C_v \log_e \frac{P_2}{P_1} + C_p \log_e \frac{V_2}{V_1}} \quad \text{— (18)}$$

$$s_2 - s_1 = \frac{1}{M}\left[C_v \log_e \frac{P_2}{P_1} + C_p \log_e \frac{V_2}{V_1}\right]$$

Third Law of Thermodynamics

According to Nernst, “ **The heat capacities of all solids tend to zero as the absolute zero of temperature is approached and that the internal energies and entropies of all the substances become equal there, approaching their common value asymptotically tending to zero”**.

In terms of entropy, the theorem may be stated as:

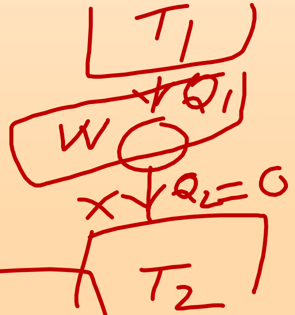
“ **at absolute zero temperature, the entropy tend to zero and the molecules of a substance or a system are in perfect order (well arranged)**”.

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\left(\frac{Q_1}{T_1} - \frac{Q_2}{T_2}\right) \rightarrow \text{+ve quantity}$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$T_2 = 0 \Rightarrow Q_2 = 0$$



$$\boxed{T_2 = 0K}$$


Thankyou