Example: The set $S = \{(1,1,0), (0,1,0), (0,1,1)\}$ generates $\mathbb{R}^3$.

Proof 2

TS $\text{span}\, S = \mathbb{R}^3$

$\Rightarrow \{a_1(1,1,0) + a_2(0,1,0) + a_3(0,1,1) \mid a_1, a_2, a_3 \in \mathbb{R}\}$
$= \mathbb{R}^3$ —— (1)

Clearly $\text{span}(S) \subseteq \mathbb{R}^3$ —— (2)

Let (a,b,c) be any arbitrary element of $\mathbb{R}^3$

TS $(a,b,c) \in \text{span}(S)$

Let $a_1(1,1,0) + a_2(0,1,0) + a_3(0,1,1) = (a,b,c)$

$\Rightarrow (a_1, a_1, 0) + (0, a_2, 0) + (0, a_3, a_3) = (a,b,c)$

$\Rightarrow (a_1, a_1 + a_2 + a_3, a_3) = (a,b,c)$

$$\Rightarrow \left.\begin{array}{r} a_1 = a \\ a_1 + a_2 + a_3 = b \\ a_3 = c \end{array}\right\} \Rightarrow \begin{array}{l} a_1 = a,\ a_2 = b - a - c \\ a_3 = c \end{array}$$

$$\therefore (a,b,c) = a(1,1,0) + (b-a-c)(0,1,0) + c(0,1,1)$$

$$\Rightarrow (a,b,c) \in \text{Span}.\{(1,1,0),\ (0,1,0),\ (0,1,1)\}$$

$$\Rightarrow (a,b,c) \in \text{Span}.(S)$$

$$\therefore \mathbb{R}^3 \subseteq \text{Span}.(S) \quad \text{———} \quad (3)$$

From eqn (2) & (3),

$$\text{Span}(S) = \mathbb{R}^3$$

Q. Show that the matrix $\begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$ is not a lineal combination of the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.

Soln: Let $v = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$, $v_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$,

$v_3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$v_3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

Suppose $v = a_1 v_1 + a_2 v_2 + a_3 v_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} = a_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + a_2 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + a_3 \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 + a_3 & a_2 \\ a_3 & a_1 + a_2 + a_3 \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} a_1 + a_2 + a_3 &= 1 \\ a_2 &= 0 \\ a_3 &= 2 \\ a_1 + a_2 + a_3 &= 2 \end{aligned}\right\} \text{———— } \mathrm{I}$$

equations (i) and (iv) of system I contradicts each other, so there is no solutions of system of equations I

Therefore, v is not a linear combination of v_1, v_2 and v_3.