COMBINED STANDARD DEVIATION

Just as it is possible to compute combined mean of two or more than two groups. Similarly we can also compute combined standard deviation of two or more groups. It is denoted by σ_{12} and is computed as follows.

$$\sigma_{12} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}}$$

where σ_{12} = combined standard deviation

σ_1 = standard deviation of first group

σ_2 = standard deviation of second group

$$d_1 = |\bar{X}_1 - \bar{X}_{12}| ; \quad d_2 = |\bar{X}_2 - \bar{X}_{12}|$$

The above formula can be extended to find out the standard deviation of three or more groups. For ex:-

$$\sigma_{123} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_3\sigma_3^2 + N_1 d_1^2 + N_2 d_2^2 + N_3 d_3^2}{N_1 + N_2 + N_3}}$$

where,

$$d_1 = |\bar{X}_1 - \bar{X}_{123}| ; \quad d_2 = |\bar{X}_2 - \bar{X}_{123}| ; \quad d_3 = |\bar{X}_3 - \bar{X}_{123}|$$

$$\bar{X}_{123} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2 + N_3\bar{X}_3}{N_1 + N_2 + N_3}$$

* The numbers of workers employed, the mean wage (in Rs.) per month, & standard deviation (in Rs) in each section of a factory are given below. Calculate the mean wages and standard deviation of all workers taken together.

Section	No. of workers employed	Mean wage	Standard deviation
A	50	113	6
B	60	120	7
C	90	115	8

Solution:
$$\bar{X}_{123} = \frac{N_1 + \bar{X}_1 + N_2\bar{X}_2 + N_3\bar{X}_3}{N_1 + N_2 + N_3}$$

$$= \frac{(50\times113) + (60\times120) + (90\times115)}{50+60+90}$$

$$= \frac{5650 + 7200 + 10350}{200}$$

$$= \frac{23200}{200} = 116.$$

combined SD of three series.

$$\sigma_{123} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_3\sigma_3^2 + N_1d_1^2 + N_2d_2^2 + N_3d_3^2}{N_1+N_2+N_3}}$$

$$d_1 = |\bar{X}_1 - \bar{X}_{123}| = |113-116| = 3$$

$$d_2 = |\bar{X}_2 - \bar{X}_{123}| = |120-116| = 4$$

$$d_3 = |\bar{X}_3 - \bar{X}_{123}| = |115-116| = 1$$

$$\sigma_{123} = \sqrt{\frac{50(60)^2 + 60(7)^2 + 90(8)^2 + 50(3)^2 + 60(4)^2 + 90(1)^2}{50+60+90}}$$

$$= \sqrt{\frac{1800 + 2940 + 5760 + 450 + 960 + 90}{200}}$$

$$= \sqrt{\frac{12000}{200}} = \sqrt{60} = 7.75$$

- COEFFICIENT OF VARIATION

$$C.V. = \frac{\sigma}{\bar{x}} \times 100$$

Calculation:-

ranges x	m.p. m	f	(m−110)/10 d	fd	fd^2
		10	−6	−60	360
40-60	50	15	−6	−60	240
60-80	70	28	−4	−56	112
80-100	90		−2	0	0
110-120	110	32	0		
120-140	130	20	2	40	80
140-160	150	10	4	40	160
160-180	170	8	6	48	288
180-200	190	5	8	40	320
		N=128		$\Sigma fd = -8$	$\Sigma fd^2 = 1560$

$$C.V. = \frac{\sigma}{\bar{X}} \times 100$$

Mean: $\bar{X} = A + \frac{\Sigma f d}{N} \times i$

$$= 110 - \frac{8}{128} \times 20$$

$$= 110 - 1.25$$

$$= 108.75$$

$$\sigma = \sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N}\right)^2} \times i$$

$$= \sqrt{\frac{1560}{128} - \left(\frac{-8}{128}\right)^2} \times 20$$

$$= \sqrt{12.1875 - 0.0039} \times 20$$

$$= \sqrt{12.1836} \times 20$$

$$= 69.81$$

$$C.V. = \frac{69.81}{108.75} \times 100 = 64.19\%$$