Polarization

Q.1 What kind of polarization has plane electromagnetic wave if the projection of vector E on x and y axes are perpendicular to the propagation direction and are defined by the equation:

a) $E_x = E\cos(\omega t - Kz)$, $E_y = E\sin(\omega t - Kz)$

b) $E_x = E\cos(\omega t - Kz)$, $E_y = E\cos(\omega t - Kz + \frac{\pi}{4})$

c) $E_x = E\cos(\omega t - Kz)$, $E_y = E\cos(\omega t - Kz + \pi)$

Solution a)

$$E_x = E\cos(\omega t - Kz)$$

$$E_y = E\sin(\omega t - Kz)$$

$$\frac{E_x}{E} = \cos(\omega t - Kz) \quad \text{--- (I)}$$

$$\frac{E_y}{E} = \sin(\omega t - Kz) \quad \text{--- (II)}$$

Squaring both then

$$\frac{E_x^2}{E^2} + \frac{E_y^2}{E^2} = 1$$

Page No. ______
Date ______

This is the equation of the circle. This system represent the anticlockwise circular polarized.

b) $E_x = E\cos(\omega t - kz)$.

$E_y = E\cos(\omega t - kz + \frac{\pi}{4})$

$$\frac{E_y}{E} = \frac{1}{\sqrt{2}}\cos(\omega t - kz) - \frac{1}{\sqrt{2}}\sin(\omega t - kz)$$

$$\left(\frac{E_y}{E} - \frac{1}{\sqrt{2}}\frac{E_x}{E}\right)^2 = \left(-\frac{1}{\sqrt{2}}\sin(\omega t - kz)\right)^2$$

$$\sin(\omega t - kz) = \sqrt{1 - \cos^2(\omega t - kz)}$$

$$= \sqrt{1 - \frac{E_x^2}{E^2}}$$

$$\frac{E_y^2}{E^2} + \frac{E_x^2}{E^2} - \sqrt{2}\frac{E_y E_x}{E^2} = \frac{1}{2}$$

This is ellipse we see this ~~Component~~ represent elliptical clockwise polarization when view forward incoming wave

$$E_x + E_y = 2E \cos\left(\omega t - kz + \frac{\pi}{8}\right) \cos\frac{\pi}{8}$$

$$E_x - E_y = 2E \sin\left(\omega t - kz + \frac{\pi}{8}\right) \sin\frac{\pi}{8}$$

$$\left(\frac{E_x + E_y}{2E\cos\frac{\pi}{8}}\right)^2 + \left(\frac{E_x - E_y}{2E\sin\frac{\pi}{8}}\right)^2 = 1$$

c) $E_x = E \cos(\omega t - kz)$

$E_y = E \cos(\omega t - kz + \pi)$

$= -E \cos(\omega t - kz)$

$E_y = -E_x$

plane polarization

Page No. ______
Date ______

Q.2. A manufacture a quartz plate cut parallel to its optic axis and not exceeding 0.50 mm in thickness. Find the maximum thickness of plate allowing plane polarized light with wavelength 589 nm.

a) only rotation of polarization plane

b) again circular polarization

Solution: For quartz $n_e = 1.553$, $n_o = 1.544$ } for $\lambda = 589$ nm.

$$\text{phase diff} = \frac{2\pi}{\lambda}(n_e - n_o)d$$

d = thickness of plate

a) for emergent light to exp only rotation of polarization plane:

$$\delta = (2K+1)\pi \qquad K = 0, 1, 2, 3 \ldots$$

$$d = \frac{(2K+1)\lambda}{2(n_e - n_o)}$$

$$= \frac{(2K+1)\,589}{2 \times .009} = (2K+1)\frac{589}{18}\text{ mm}$$

Max value of $(2K+1) < 0.50$

$$= \frac{0.50 \times 589}{18} = 15.28$$

Page No. ______
Date ______

Then $K=7$, $d = 15 \times \frac{589}{18} = 0.4908\,mm$

b) for circular polarization

$$\delta = \frac{\pi}{2}$$

$$\delta = (4K+1)\frac{\pi}{2}$$

$$d = (4K+1)\frac{\lambda}{4(n_e-n_o)}$$

$$= (4K+1) \times \frac{0.589}{36}$$

$$= \frac{0.50 \times 36}{.589} = 30.56$$

The nearest integer less than this width $4K+1 = 29$

$$K = 7$$

$$d = 0.4749\,mm$$