

Practical:

(iv) (b) Newton Raphson Method

```

In[21]:= NewtonRaphson[x0_, n_, f_] :=
Module[{xk1, xk = N[x0]},
  k = 0;
  Output = {{k, x0, f[x0]}};
  While[k < n,
    fPrimexk = f'[xk];
    If[fPrimexk == 0,
      Print["The derivative of function at ",
        k,
        "th iteration is zero, we can not
        proceed further with the iterative
        scheme"]; Break[]];
    xk1 = xk - f[xk] / fPrimexk;
    xk = xk1;
    k++;
    Output = Append[Output,
      {k, xk, f[xk]}}];];
  Print[
    NumberForm[TableForm[Output,
      TableHeadings ->
        {None, {"k", "xk", "f[xk]"}}, 10]];
  Print["Root after ", n,
    " iterations xk = ",
    NumberForm[xk, 10]];
  Print[
    "Function value at approximated
    root, f[xk] = ",
    NumberForm[f[xk], 10]];];

```

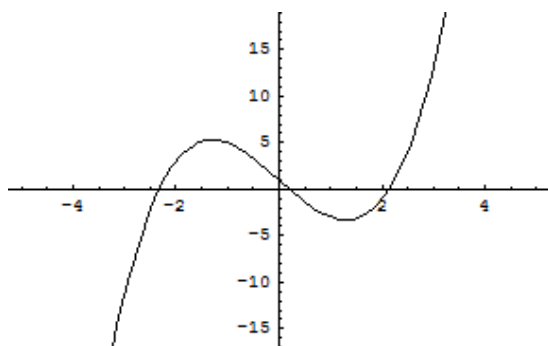
Q1: Perform 5 iterations of the Newton Raphson Method to find out the smallest positive root of the function $f(x) = x^3 - 5x + 1$.

Solution: First let's have an idea about initial approximation by plotting the function.

```

f[x_] := x^3 - 5*x + 1;
Plot[f[x], {x, -5, 5}]

```



Out[38]= - Graphics -

From the graph we see that one of the smallest positive root lies in the interval $[0, 1]$.

So,

lets start with the initial approximation $x_0 = 0.5$.

```
In[40]:= NewtonRaphson[0.5, 5, f]
```

| k | x_k | $f[x_k]$ |
|---|--------------|-------------------------------|
| 0 | 0.5 | -1.375 |
| 1 | 0.1764705882 | 0.1231426827 |
| 2 | 0.2015680743 | 0.0003492763989 |
| 3 | 0.2016396751 | $3.100484314 \times 10^{-9}$ |
| 4 | 0.2016396757 | $1.110223025 \times 10^{-16}$ |
| 5 | 0.2016396757 | $1.110223025 \times 10^{-16}$ |

Root after 5 iterations $x_k = 0.2016396757$

Function value at approximated root, $f[x_k] = 1.110223025 \times 10^{-16}$

Q2: Perform four iterations of the Newton-Raphson Method to obtain approximate value of $\sqrt[3]{17}$ starting with the initial approximation $x_0 = 2$.

Solution: We can take the function $f(x) = x^3 - 17$ having $\sqrt[3]{17}$ as its root. Here initial approximation is given as $x_0 = 2$.

```
In[22]:= f[x_] = x^3 - 17;
NewtonRaphson[2, 4, f]
```

| k | x_k | $f[x_k]$ |
|---|-------------|------------------------------|
| 0 | 2 | -9 |
| 1 | 2.75 | 3.796875 |
| 2 | 2.582644628 | 0.2263772599 |
| 3 | 2.571331512 | 0.0009901837441 |
| 4 | 2.571281592 | $1.922353121 \times 10^{-8}$ |

Root after 4 iterations $x_k = 2.571281592$

Function value at approximated root, $f[x_k] = 1.922353121 \times 10^{-8}$

Next we find out the Actual Value of $\sqrt[3]{17}$ to compare it with the approximated root obtained by Newton Raphson Method

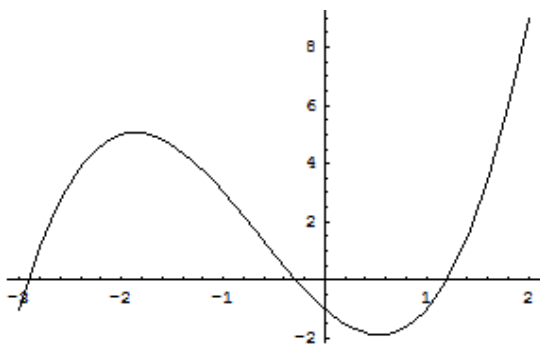
```
In[29]:= actualValue = N[17^(1/3), 20]
```

```
Out[29]= 2.5712815906582353555
```

Q3 Perform four iterations of the Newton-Raphson Method to approximate the root of the function $f(x) = x^3 + 2x^2 - 3x - 1$ near $x = -3$.

Solution: First let's have an idea about initial approximation by plotting the function

```
In[34]:= f[x_] := x^3 + 2 x^2 - 3 x - 1;
Plot[f[x], {x, -3, 2}]
```



```
Out[35]= - Graphics -
```

From the graph we see that one of the roots lies in the interval $[-3, -2]$.

So, let's start with the initial approximation $x_0 = -3$.

```
In[36]:= NewtonRaphson[-3, 4, f]
```

| k | x_k | $f[x_k]$ |
|---|--------------|-------------------------------|
| 0 | -3 | -1 |
| 1 | -2.916666667 | -0.04803240741 |
| 2 | -2.912241416 | -0.0001320975296 |
| 3 | -2.912229179 | $-1.008864103 \times 10^{-9}$ |
| 4 | -2.912229178 | 0. |

Root after 4 iterations $x_k = -2.912229178$

Function value at approximated root, $f[x_k] = 0.$