(Q) what is order of following

(a) (14) — 2 (b) 147 — 3 (c) 14762 — 5 (d) $(124)(357)$ — $\mathrm{lcm}(3,3) = 3$

(e) $(124),(345)$ common (first write in product of disjoint cycle

$$\begin{pmatrix}1&2&4\\2&4&1\end{pmatrix}\begin{pmatrix}3&4&5\\4&5&3\end{pmatrix} = \begin{pmatrix}1&2&3&4&5\\2&4&1&5&3\end{pmatrix}$$
$$= (12453)$$
$$= 5 \text{ (order)}$$

(f) $(124)(3578) = \mathrm{lcm}(3,4) = 12$

(Q) what is order of each of following permutation?

(a) $\begin{bmatrix}1&2&3&4&5&6\\2&1&5&4&6&3\end{bmatrix} = (12)(356)(4)$
$$= (12)(356)$$
$$\mathrm{lcm}(2,3) = 6$$

(b) $\begin{bmatrix}1&2&3&4&5&6&7\\7&6&1&2&3&4&5\end{bmatrix} = (1753)(264)$
$$= \mathrm{lcm}(4,3)$$
$$= 12$$

(Q) Determine whether the following permutation are even or odd

(a) $(135) = (15)(13) = E$

(b) $(1357) = (17)(16)(15)(13) = E$

(c) $(1243)(3521) = \begin{pmatrix}1&2&3&4\\2&4&1&3\end{pmatrix}\begin{pmatrix}3&5&2&1\\5&2&1&3\end{pmatrix}$

(common)

$$= \begin{pmatrix}1&2&3&4&5\\1&2&5&3&4\end{pmatrix}$$

$(2)(1)(354)$
$$= (354)$$
$$= (34)(35) = E$$

(d) $(12)(134)(152) = \begin{pmatrix}1&2\\2&1\end{pmatrix}\begin{pmatrix}1&3&4\\3&4&1\end{pmatrix}\begin{pmatrix}1&5&2\\5&2&1\end{pmatrix} =$

$$\begin{pmatrix}1&2&3&5&4\\5&3&4&1&2\end{pmatrix}$$
$$= (15)(234)$$
$$= (15)(24)(23)$$
$$= \text{odd}$$

let

$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$

find $\alpha^{-1}, \beta\alpha, \alpha\beta$

$\alpha = (12)(45)$

$\alpha^{-1} = (21)(54)$

$= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$

$\beta\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 2 & 3 & 4 & 5 \end{pmatrix}$

$= (26543)$

$\alpha\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 1 & 5 & 3 & 4 \end{pmatrix}$

$= (16453)$

(e) let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{bmatrix}$

$\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$

(a) write α & β as product of disjoint cycles

(b) product of 2 cycles

Sol^n

(a) $\alpha = (12)(45)(67)$

(b) $\alpha = (12)(45)(67)$

(a) $\beta = (2\,3\,8\,4\,7)(56)$

(b) $\beta = (27)(24)(28)(23)(56)$

Thm: The order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles.

Alternating Group (A_n)

A_n = set of all even permutations of S_n

A_n form a group w.r.t to mapping composition

$$O(A_n) = \frac{O(S_n)}{2} = \frac{n!}{2}$$

(Q) A_n is group w.r.t to mapping composition.

(Sol^n) A_n is set of all even permutation of S_n

Now $I \in S_n$ & I is always even permutation

$\Rightarrow I \in A_n$

$\Rightarrow$ A_n is non-empty.

let $f, g \in A_n$ we get f & g are even permutation

$\Rightarrow fg^{-1} \in A_n$ ($g \in A_n \Rightarrow g^{-1}$ is also even)

(product of two even permutation is also even)

$\Rightarrow$ A_n is subgrp of S_n

$\Rightarrow$ A_n is group.

... subgrp of S_n.

Note: (a) Product of two even permutation is even permutation

(b) Product of two odd permutation is even permutation

(c) Product of odd permutation & even permutation is odd permutation.

(d) If f is even $\Rightarrow$ f^{-1} is even

If g is odd $\Rightarrow$ g^{-1} is odd.

(Q) Let $\alpha, \beta \in S_n$. Prove that $\alpha^{-1}\beta^{-1}\alpha\beta$ is even permutation

(Sol^n) Let $f = \alpha^{-1}\beta^{-1}\alpha\beta$ and $\alpha, \beta \in S_n$

case(i) If α & β both even permutation

then α^{-1} & β^{-1} " " "

s.t. $\alpha^{-1}\beta^{-1}\alpha\beta = f$ is even permutation

even even

even

case(ii) If α is even & β is odd permutation

then α^{-1} is even & β^{-1} is odd "

s.t. $f = \alpha^{-1}\beta^{-1}\alpha\beta$ is even permutation

even odd even odd

odd odd

even

case(iii) If α is odd & β is even

then $f = \alpha^{-1}\beta^{-1}\alpha\beta$ is even permutation

odd even odd even

odd odd

even

case (iv) If α & β both odd

then α^{-1} & β^{-1} are odd

$\alpha^{-1}\beta^{-1}\alpha\beta$ is even

odd odd odd odd

even even

even

Construction of A_1

A_1 = set of all even permutation of S_1

$S_1 = \{I\} = A_1$

$A_2 = \{I\}$

$S_2 = \{I, (1\,2)\}$

$A_3 = \{I, (1\,2\,3)(1\,3\,2)\}$

$S_3 = \{I, (1\,2), (2\,3), (1\,3), (1\,2\,3), (1\,3\,2)\}$

$S_4 = \{ \cdot \cdot \}$

$A_4 = \{I, (1\,2\,3), (1\,2\,4), (1\,3\,4), (2\,3\,4), (4\,3\,2),$
$(4\,3\,1), (4\,2\,1), (3\,2\,1)\ (1\,2)(3\,4),$
$(1\,3)(2\,4), (1\,4)(2\,4)\}$