Rate of Return regulation:

if regulatory agencies permit monopoly to charge a price above MC, that is sufficient to earn a fair rate of return on investment.

if rate of return allowed to firms is grater than what it would have been under competitive framework then there will be incentive to use relatively more capital input then the level that minimizes cost.

Formal Model

Production function $q = f(K, L)$

let rate of return on capital is constrained at level [be regulation] at $\bar{s}$.

$\rightarrow$ Profit $= TR - TC$

$= P.q - WL - VK$

$$\boxed{\pi = P.f(K,L) - WL - VK}$$

Lagrangian Problem

$$\mathcal{L} = P.f(K,L) - WL - VK + \lambda(WL + \bar{s}K - P.f(K,L))$$

Case 1 $\lambda = 0$

regulation is ineffective and monopoly behaves like profit maximising firm.

Case 2 $\lambda = 1$

$\mathcal{L} = -VK + \bar{s}K$

$$\boxed{\mathcal{L} = (\bar{s} - V)K}$$

for $\bar{s} > v$

Monopoly has incentive to higher as much capital as he can to maximise profit, since profit is increasing in K.

$$\boxed{\frac{\partial \mathcal{L}}{\partial K} > 0}$$

here, monopoly intends to hire infinite amounts of capital which is an implausible (impossible) result.

Case III

$0 < \lambda < 1$ or $\lambda \in (0,1)$

$$\frac{\partial \mathcal{L}}{\partial L} = Pf_L - w + \lambda[w - Pf_L]$$

$$\frac{\partial \mathcal{L}}{\partial L} = 0$$

$$Pf_L - w = -\lambda(w - Pf_L)$$

$$Pf_L - w = \lambda(Pf_L - w)$$

$$\boxed{\lambda = 1}$$

only thing for $\frac{\partial \mathcal{L}}{\partial L} = 0$

$\therefore \boxed{w = Pf_L}$

$$\frac{\partial \mathcal{L}}{\partial K} = P \cdot f_K - v + \lambda(\bar{s} - Pf_K) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{s}K + wL - P \cdot f(K, L) = 0$$

from (1)st FOC

here, regulated monopoly hires labour up to the point where

$$w = P \cdot f_L$$

from IInd FOC

$$\frac{\partial \mathcal{L}}{\partial K} = 0$$

$$P \cdot f_K - r + \lambda(\bar{S} - P \cdot f_K) = 0$$

$$(P \cdot f_K - \lambda P \cdot f_K) = r - \lambda \bar{S}$$

$$(1-\lambda) P \cdot f_K = r - \lambda \bar{S}$$

$$\boxed{P \cdot f_K = \frac{r - \lambda \bar{S}}{(1-\lambda)}}$$

L	VMP_L	W	
1	100	70	under emp
2	80	70	
3	70	70	
4	50	70	over emp
5			

in Nr add and subtract $r\lambda$

$$P \cdot F_K = \frac{r - \lambda \bar{S} + r\lambda - r\lambda}{1-\lambda}$$

$$P \cdot F_K = \frac{r(1-\lambda) - \lambda(\bar{S} - r)}{1-\lambda}$$

$$\boxed{P \cdot F_K = r - \frac{\lambda}{1-\lambda}(\bar{S} - r)}$$

+ve

+ve

λ is possitive and $\bar{S} > r$

So from these two

$$\boxed{\frac{\lambda(\bar{S} - r)}{(1-\lambda)} > 0}$$

So finally we conclude that

$\boxed{P \cdot f_K < r}$ under monopoly

$\bar{S} > r$