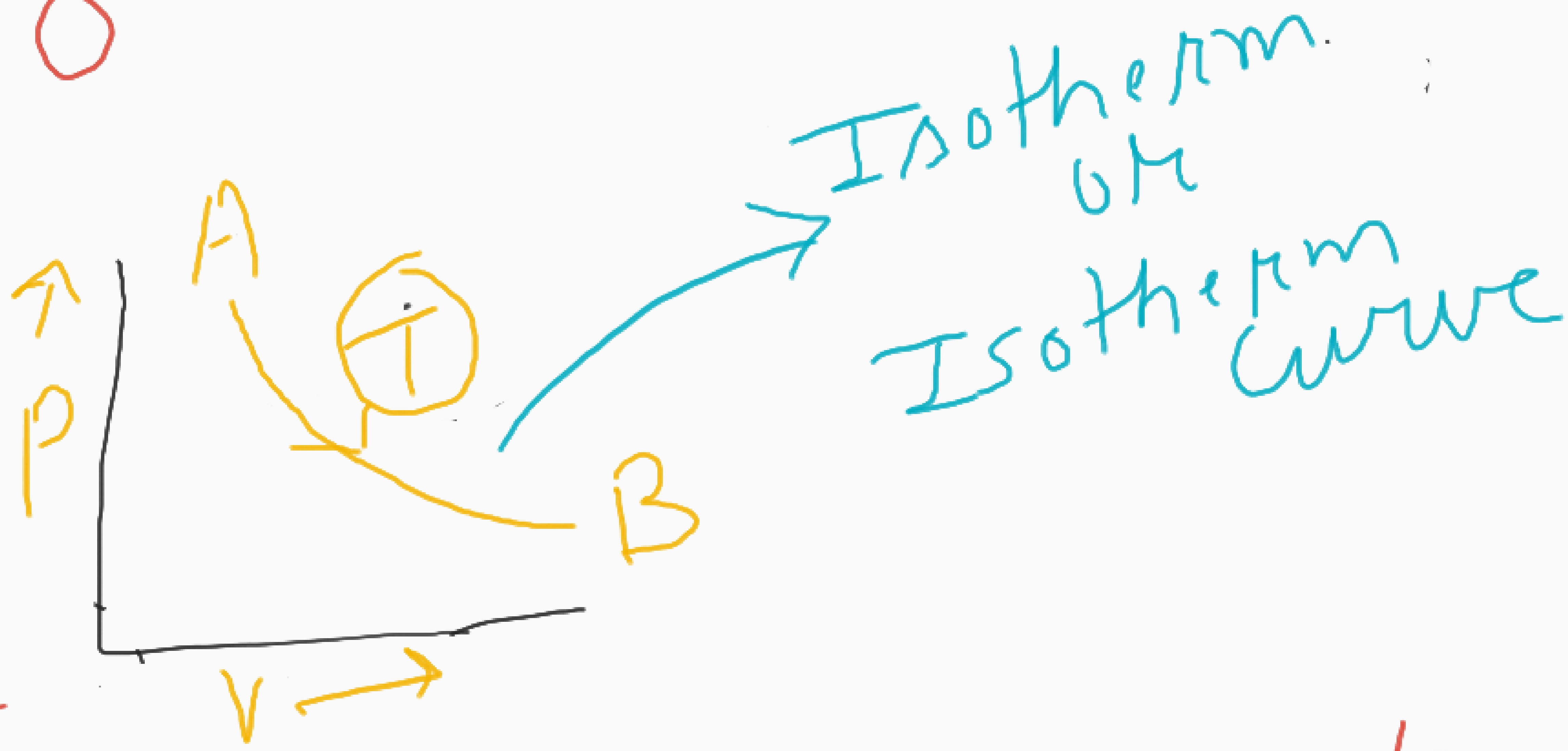
Various Processes

1. Adiabatic Process

$$\delta Q = 0$$

2. Isothermal process



$$PV = RT$$

$$PV = \text{const}$$

$$PV = RT = \text{const}$$

2. Isochoric Process

If the working substance is taken in a non expanding chamber, the heat supplied will increase the pressure and temerature. The volume of the substance will remain constant.

3. Isobaric Process

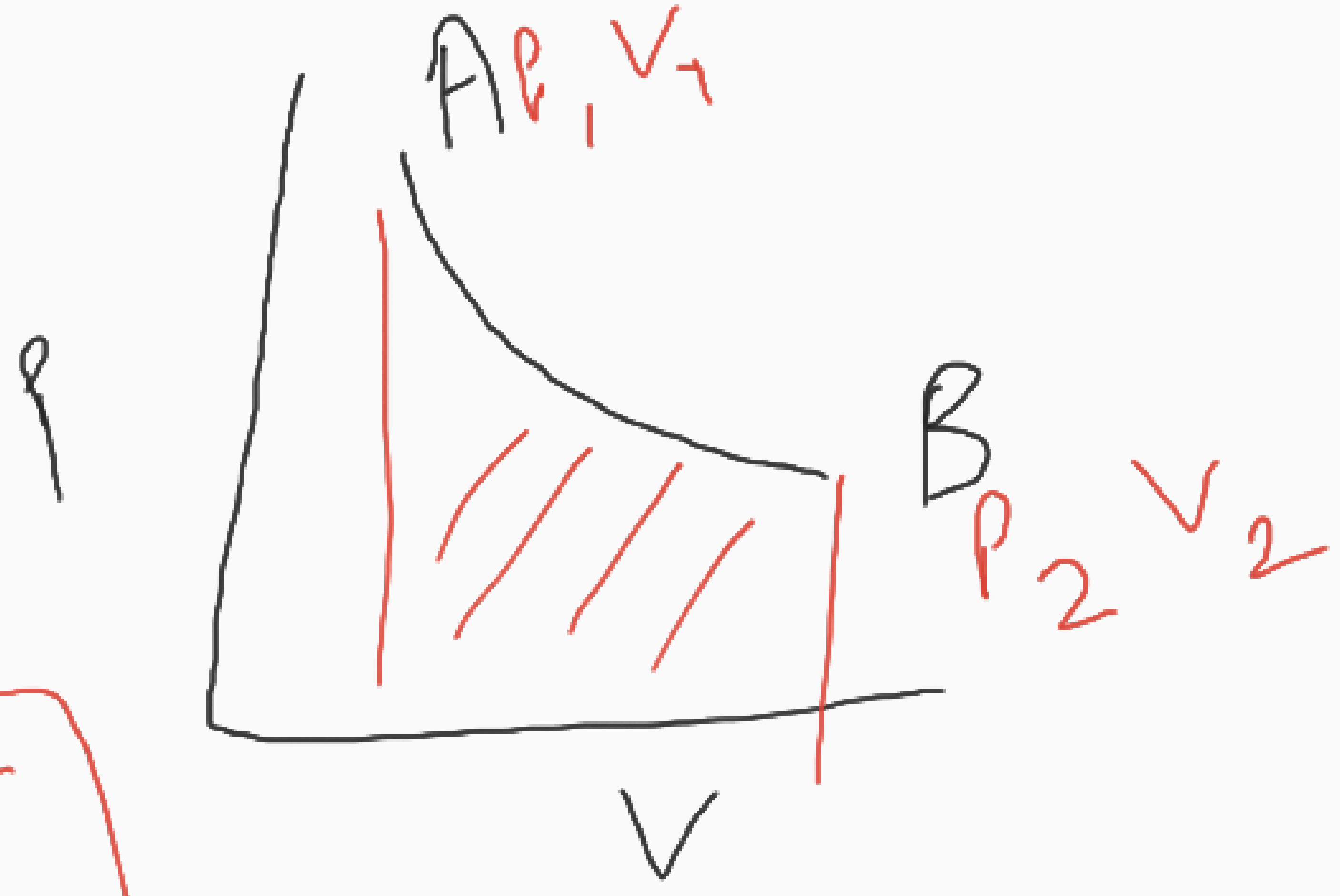
I f the working substance is taken in an expanding chamber kept at a constant pressure, the process is called an isobaric process.

Work done during Isothermal process

Total work done

$$W = \int_{V_1}^{V_2} P\,dV \quad \text{--- (1)}$$

$$PV = RT \Rightarrow \boxed{P = \frac{RT}{V}}$$



$$W = \int_{V_1}^{V_2} \frac{RT}{V}\,dV = RT \int_{V_1}^{V_2} \frac{dV}{V} = RT \left| \log_e V \right|_{V_1}^{V_2}$$

$$= RT \left[\log_e V_2 - \log_e V_1 \right] = RT \log_e \frac{V_2}{V_1}$$

$$W = nRT \log_e \frac{V_2}{V_1} = nRT \log_e \frac{P_1}{P_2}$$

Equations of state for adiabatic process

$$dQ = dU + dW$$

$$\Rightarrow dQ = C_v\, dT + P\,dV$$

$$0 = C_v\, dT + P\,dV \quad -(1)$$

$$PV = RT$$

$$\Rightarrow P\,dV + V\,dP = R\,dT$$

$$\Rightarrow dT = \frac{P\,dV + V\,dP}{R} \quad -(2)$$

$$0 = C_v \left[\frac{P\,dV + V\,dP}{R}\right] + P\,dV$$

$$PV^{\gamma} = \text{const}$$

$$TV^{\gamma-1} = \text{const}$$

$$\frac{P^{\gamma-1}}{T^{\gamma}} = \text{const}$$

$$\Rightarrow C_v P\,dV + C_v V\,dP + RP\,dV = 0$$

$$\Rightarrow (C_v + R) P\,dV + C_v V\,dP = 0$$

$C_p - C_v = R$

Divide $C_v PV$

$$\Rightarrow C_p P\,dV + C_v V\,dP = 0$$

$$\frac{C_p}{C_v}\frac{dV}{V} + \frac{dP}{P} = 0$$

$\frac{C_p}{C_v} = \gamma$

$$\Rightarrow \gamma \frac{dV}{V} + \frac{dP}{P} = 0$$

$$\gamma \int \frac{dV}{V} + \int \frac{dP}{P} = C$$

$$\gamma \log_e V + \log_e P = C$$

$$\log_e P V^{\gamma} = C$$

$$P V^{\gamma} = e^{C} = K$$

$$\boxed{P V^{\gamma} = K} \quad -(3)$$

$$K = \text{Const}$$

$$PV = RT \Rightarrow P = \frac{RT}{V}$$

$$\frac{RT}{V} V^{\gamma} = K$$

$$\Rightarrow T V^{\gamma - 1} = \frac{K}{R} = K' = \text{Const}$$

$$\Rightarrow \boxed{T V^{\gamma - 1} = \text{Const}} \quad \text{— (4)}$$

$$PV = RT \Rightarrow V = \frac{RT}{P}$$

$$(3) \Rightarrow P\left(\frac{RT}{P}\right)^{\gamma} = K$$

$$\Rightarrow \frac{T^{\gamma}}{P^{\gamma-1}} = \frac{K}{R^{\gamma}} = K_1$$

$$\Rightarrow \frac{P^{\gamma-1}}{T^{\gamma}} = \frac{1}{K_1} = K'$$

$$\Rightarrow \boxed{\frac{P^{\gamma-1}}{T^{\gamma}} = \text{Const}} \quad (5)$$

Work done in an adiabatic expansion

$$W = \int_{V_1}^{V_2} P\,dV$$

$$PV^{\gamma} = K$$

$$P = KV^{-\gamma}$$

$$W = \int_{V_1}^{V_2} KV^{-\gamma}\,dV = K\left[\frac{V^{-\gamma+1}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma}\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$$

$$W = \frac{1}{1-\gamma}\left[KV_2^{1-\gamma} - KV_1^{1-\gamma}\right] \quad \text{--- (1)}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma = K$$

$$W = \frac{1}{1-\gamma}\left[P_2 V_2^\gamma V_2^{1-\gamma} - P_1 V_1^\gamma V_1^{1-\gamma}\right]$$

$$\boxed{W = \frac{1}{1-\gamma}\left[P_2 V_2 - P_1 V_1\right]} \quad -(2)$$

$$P_1 V_1 = R T_1 \qquad P_2 V_2 = R T_2$$

$$W = \frac{1}{1-\gamma}\left[R T_2 - R T_1\right]$$

$$\boxed{W = \frac{R}{1-\gamma}\left[T_2 - T_1\right]} \quad -(3)$$

Slopes of adiabatic and isothermal

$$PV = const$$

$$\Rightarrow P\,dV + V\,dP = 0$$

$$\Rightarrow \boxed{\frac{dP}{dV} = -\frac{P}{V}}$$

$$PV^{\gamma} = const$$

$$\Rightarrow \gamma P V^{\gamma-1}\,dV + V^{\gamma}\,dP = 0$$

$$\Rightarrow \boxed{\frac{dP}{dV} = -\gamma\frac{P}{V}}$$