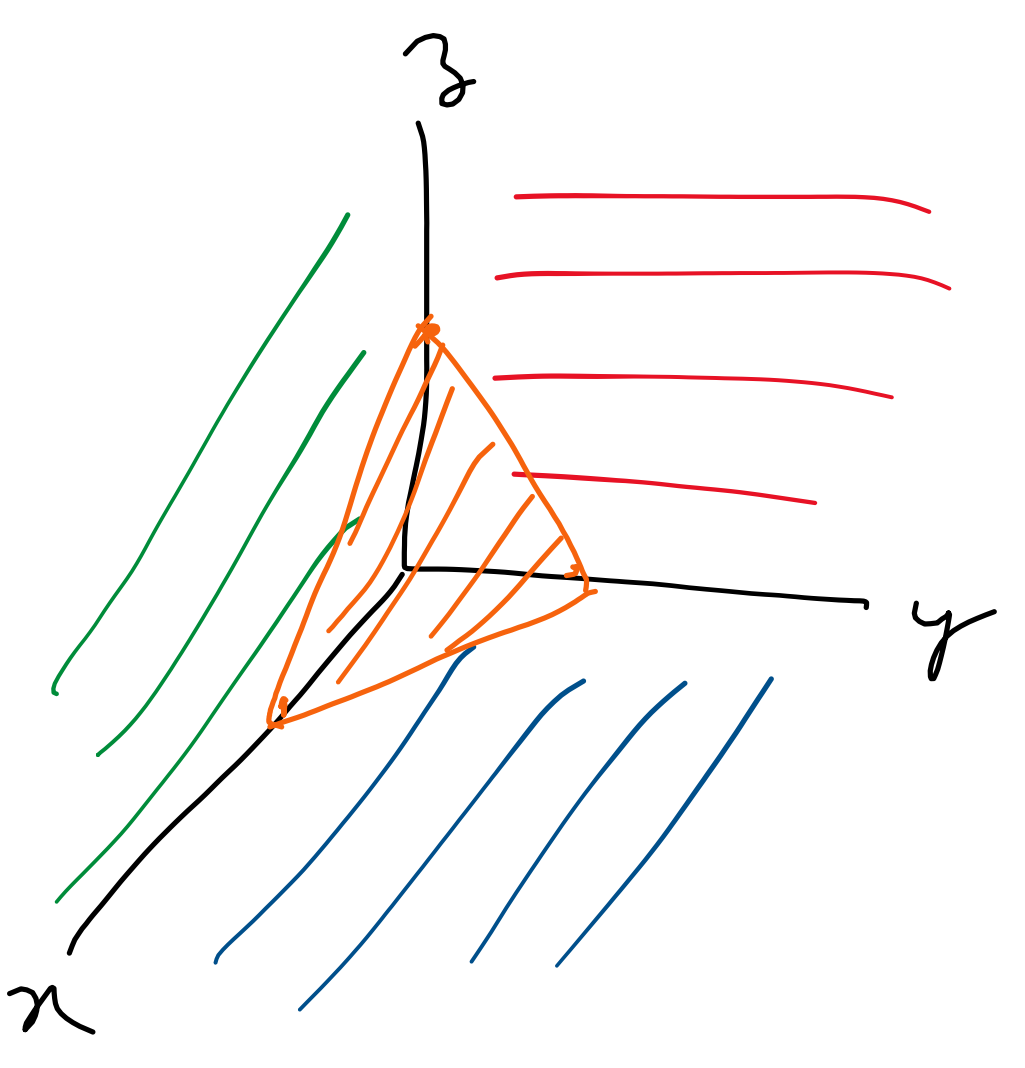
12.4 exercise

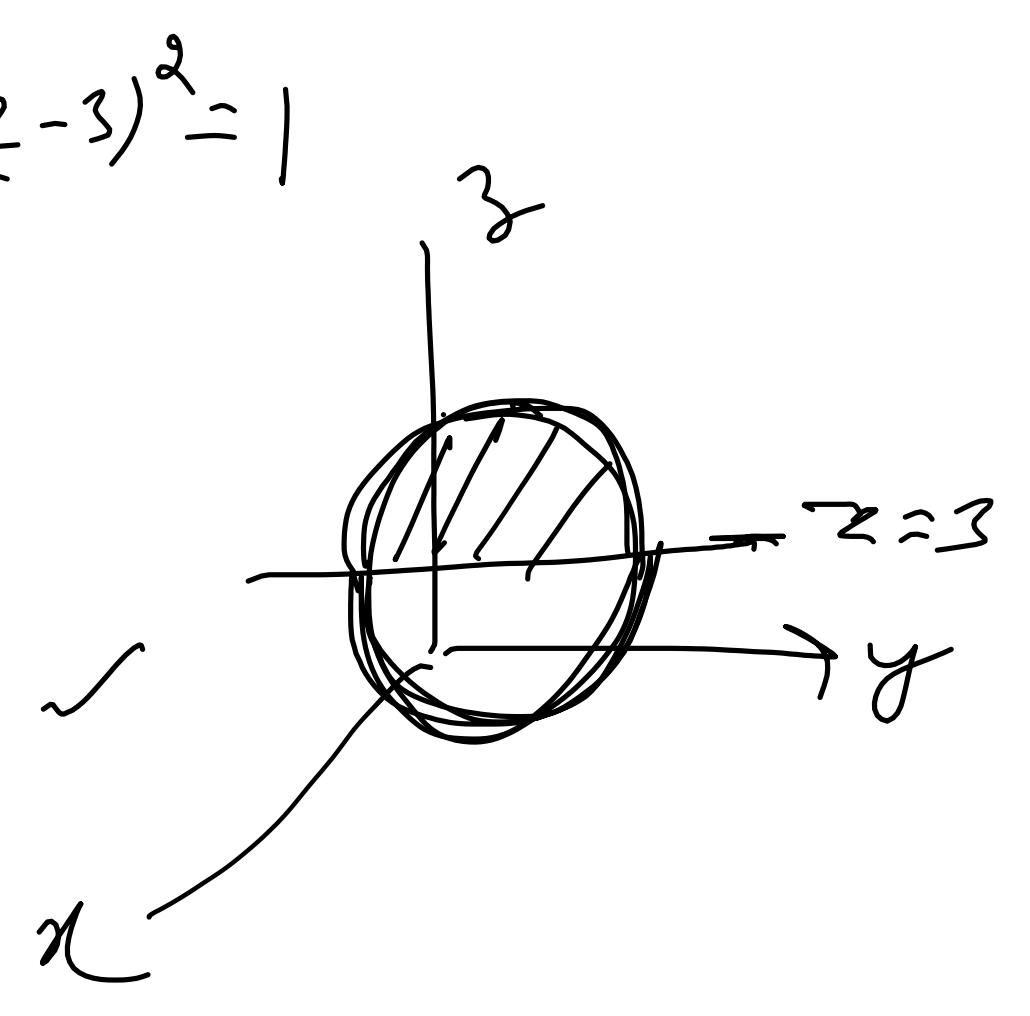
Thursday, 22 October 2020 11:09 AM



(17) $(x-1)^2+(y-2)^2+(z-3)^2=1$

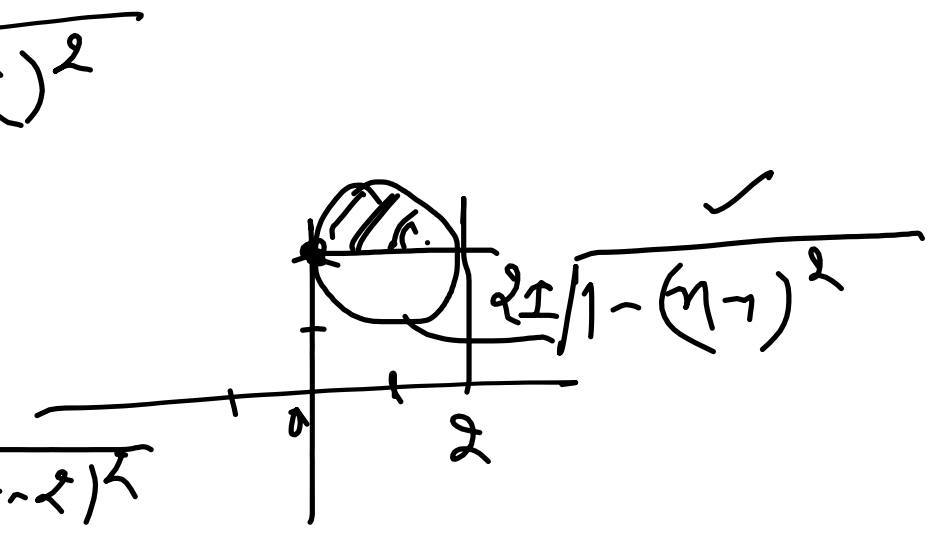
(1,2,3) center
radius (1)

$z=3\pm\sqrt{1-(x-1)^2-(y-2)^2}$ ✓



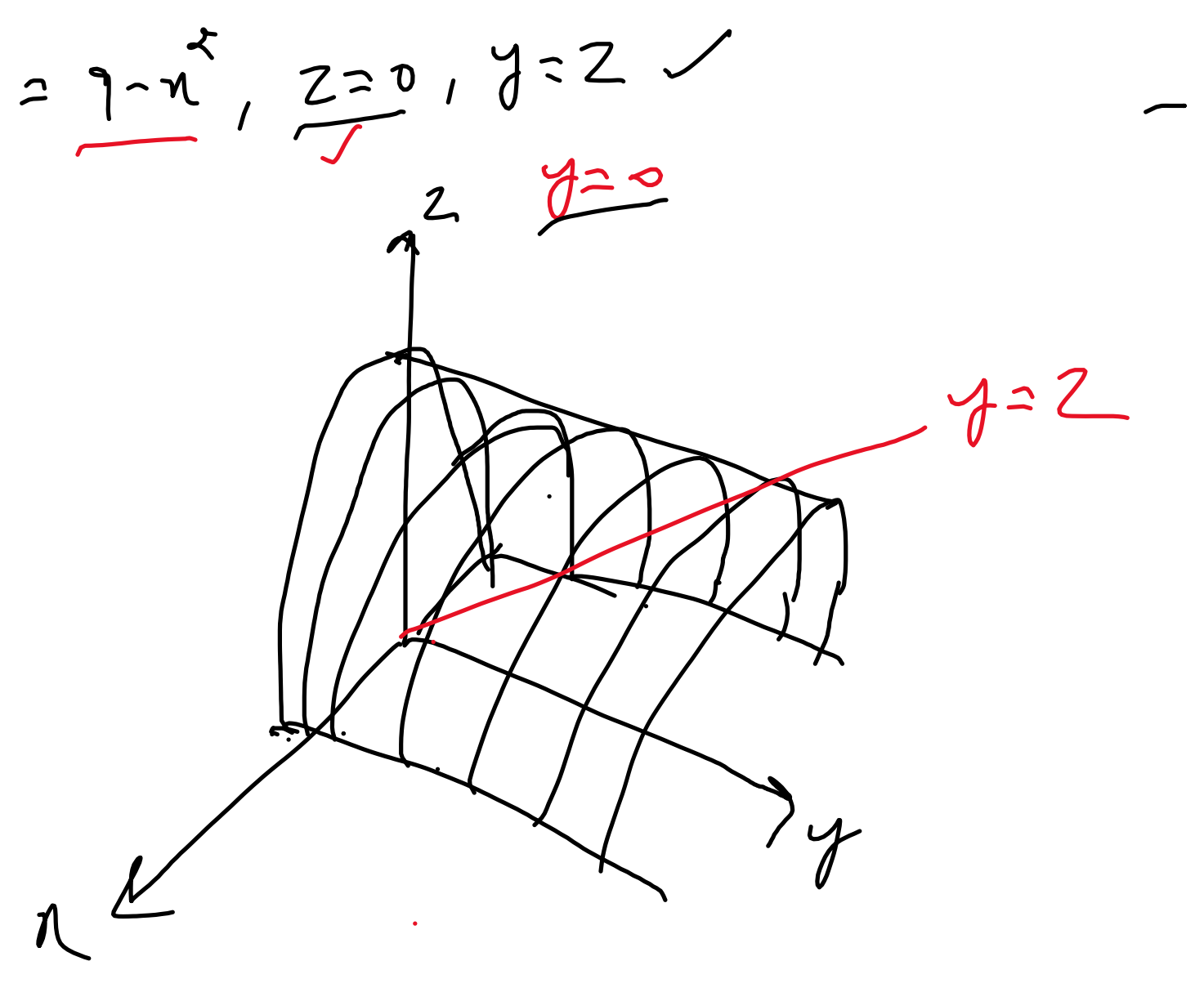
lower bding surface $z=3$
upper bding surface $z= 3+\sqrt{1-(x-1)^2-(y-2)^2}$

projection
$3=3+\sqrt{1-(x-1)^2-(y-2)^2}$
$\Rightarrow (x-1)^2+(y-2)^2=1$



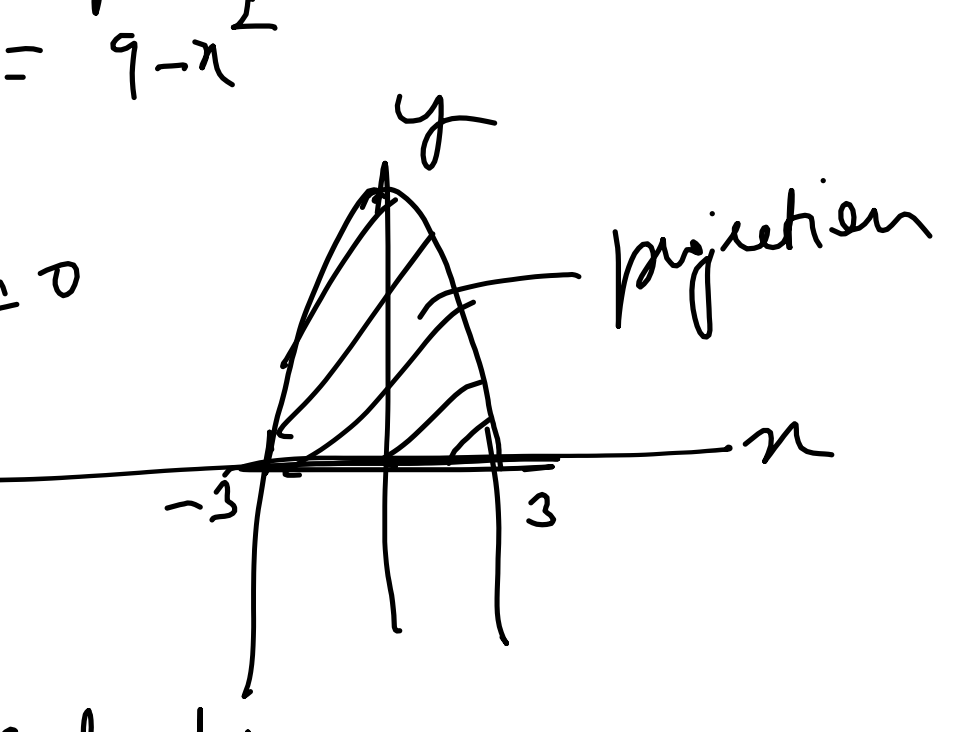
$$V=2\times2\int_0^2\int_2^{2+\sqrt{1-(x-1)^2}}\int_3^{3+\sqrt{1-(x-1)^2-(y-2)^2}} dz\,dy\,dx$$

(16) $z=9-x^2,\ z=0,\ y=2$ ✓ $y=0$



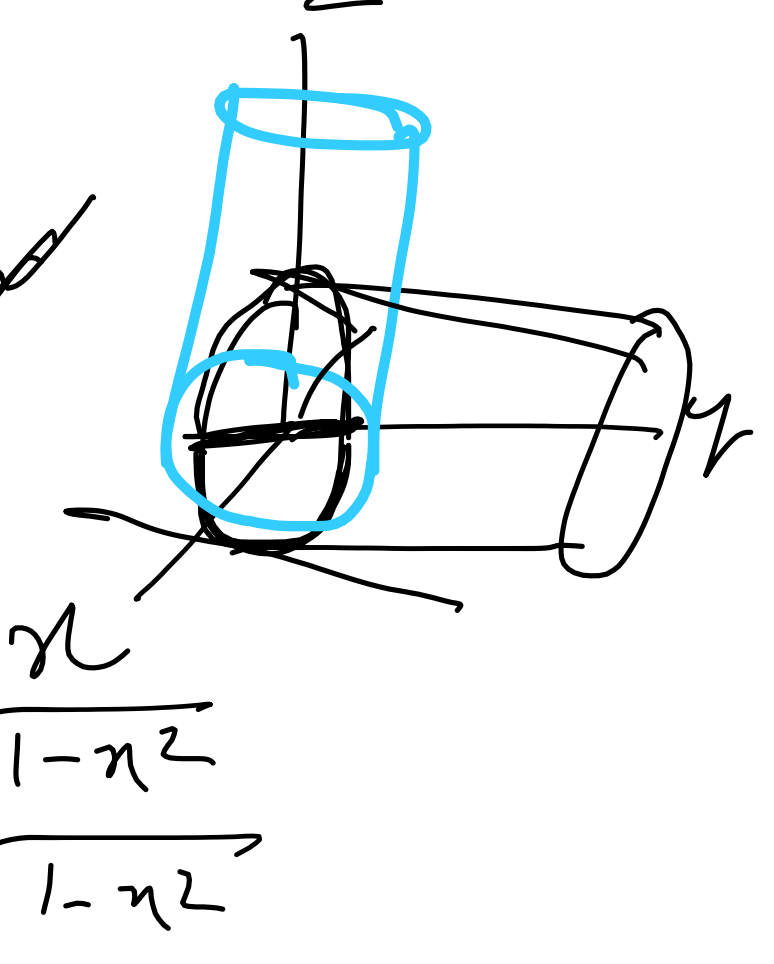
we have
lower bding surface $z=y$
upper bding surface $z=9-x^2$

projection $y=9-x^2,\ y=0$



$$V=\int_{-3}^{3}\int_0^{9-x^2}\int_y^{9-x^2} dz\,dy\,dx$$

(22) volume common to cylin $x^2+z^2=1$, $x^2+y^2=1$
$z=\pm\sqrt{1-x^2}$, $z=0$, $x=\pm1$



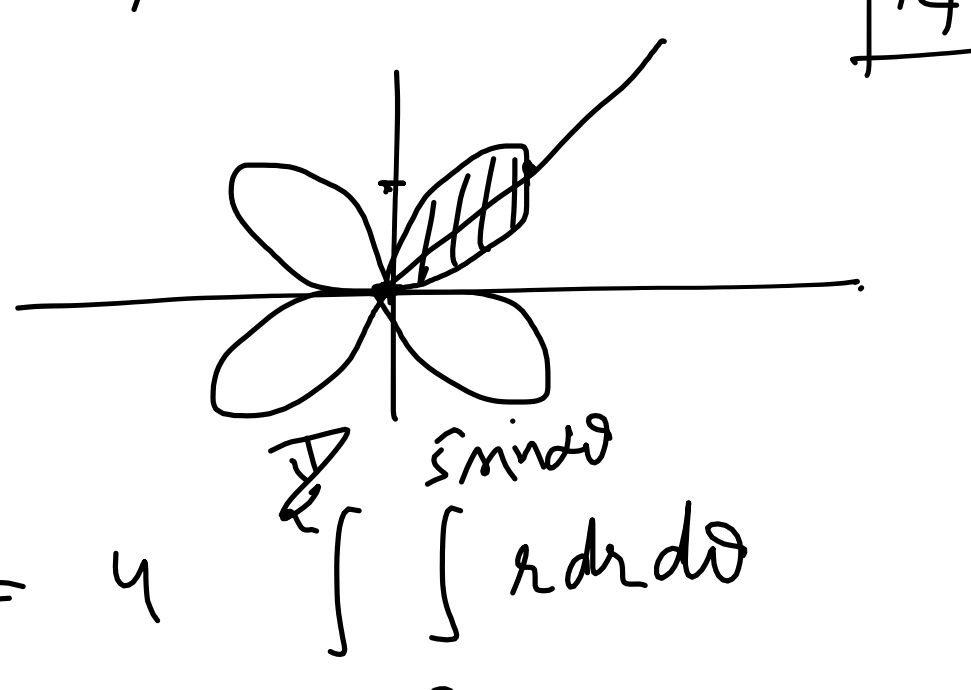
lower bding surface $z=-\sqrt{1-x^2}$
upper bding surface $z=+\sqrt{1-x^2}$
projection $x^2+y^2=1$

$$V=2\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2}} dz\,dy\,dx$$

$$=2\times2\times2\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2}} dz\,dy\,dx$$

12.3 (8) $r=5\sin 2\theta$

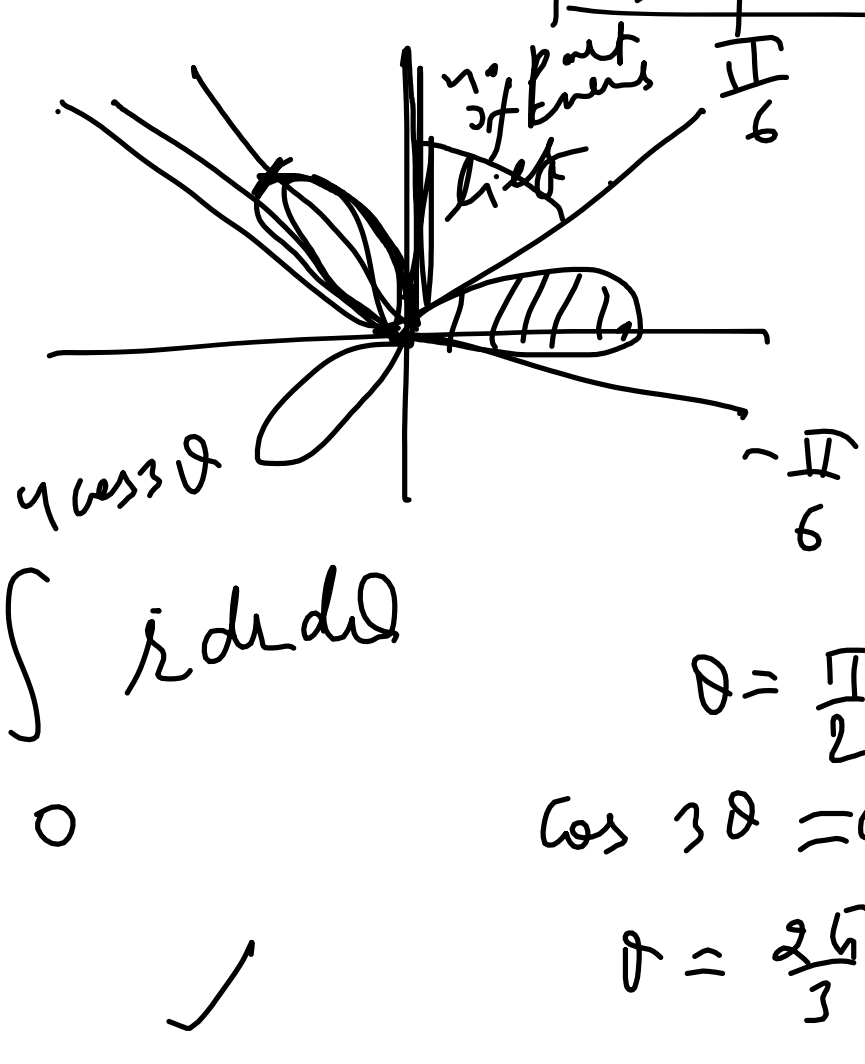
θ	0	$\frac{\pi}{4}$
r	0	5

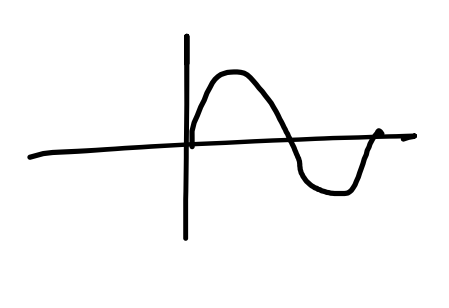


$$A=4\int_0^{\frac{\pi}{2}}\int_0^{5\sin2\theta} r\,dr\,d\theta$$

(7) $r=4\cos3\theta$

θ	0	$\frac{\pi}{6}$
r	4	





$$A=3\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_0^{4\cos3\theta} r\,dr\,d\theta$$

$\theta=\frac{\pi}{2}$
$\cos3\theta=\cos3\frac{\pi}{2}=0$
$\theta=\frac{2\pi}{3}$

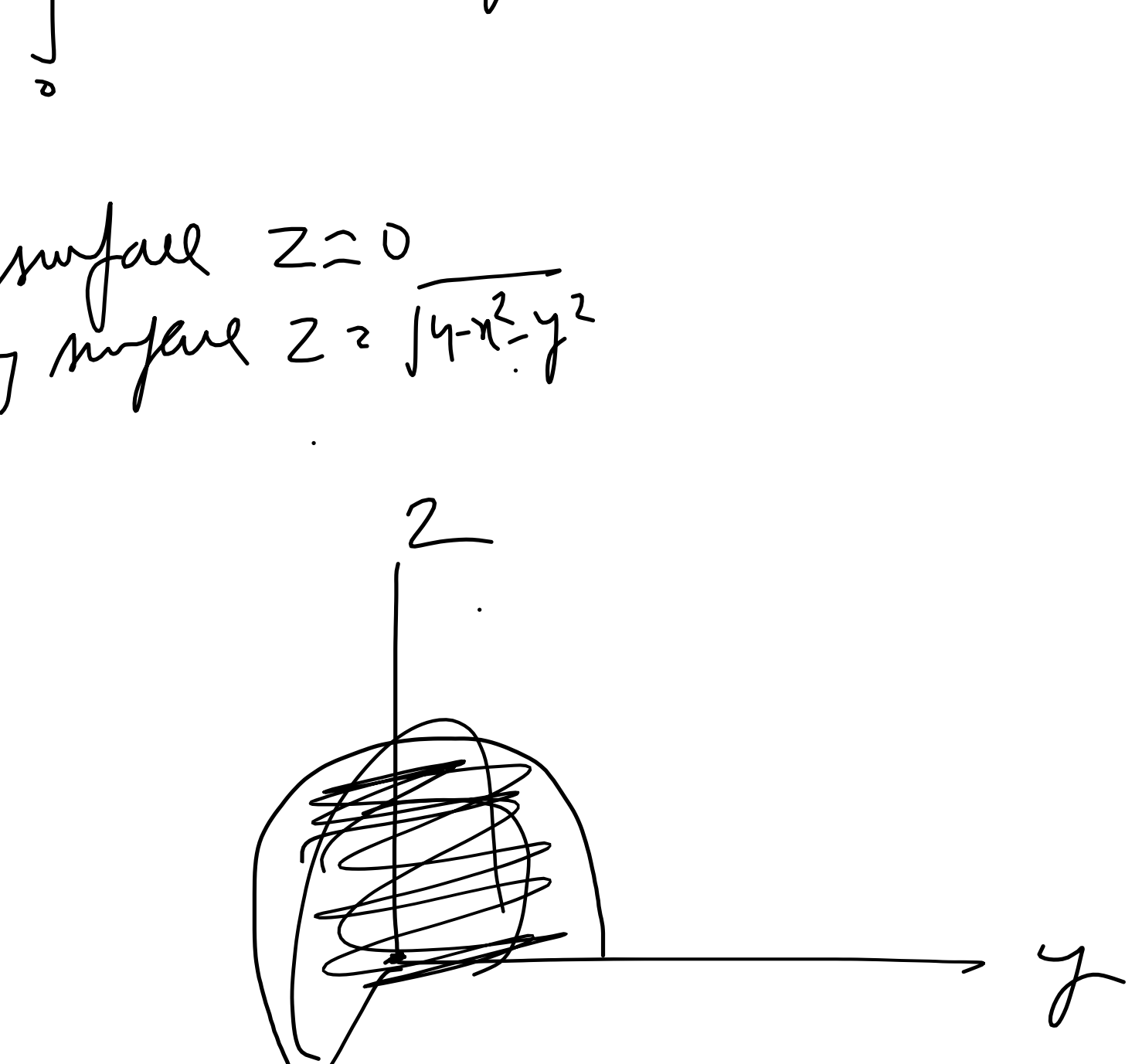
3θ	$3\frac{\pi}{2}$	2π	$5\frac{\pi}{2}$
r	0	1	0
θ	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$

12.4 (24)
$$\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} f(x,y,z)\,dz\,dy\,dx$$
change to $dx\,dy\,dz$

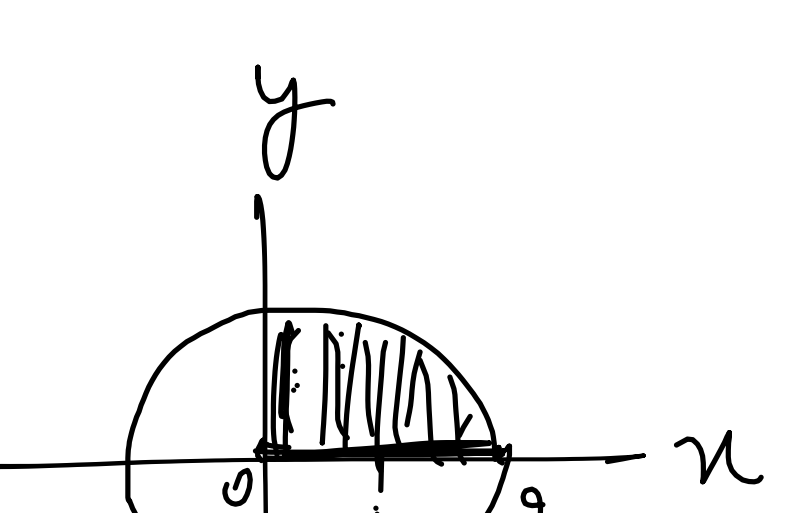
$x^2+y^2+z^2=4$

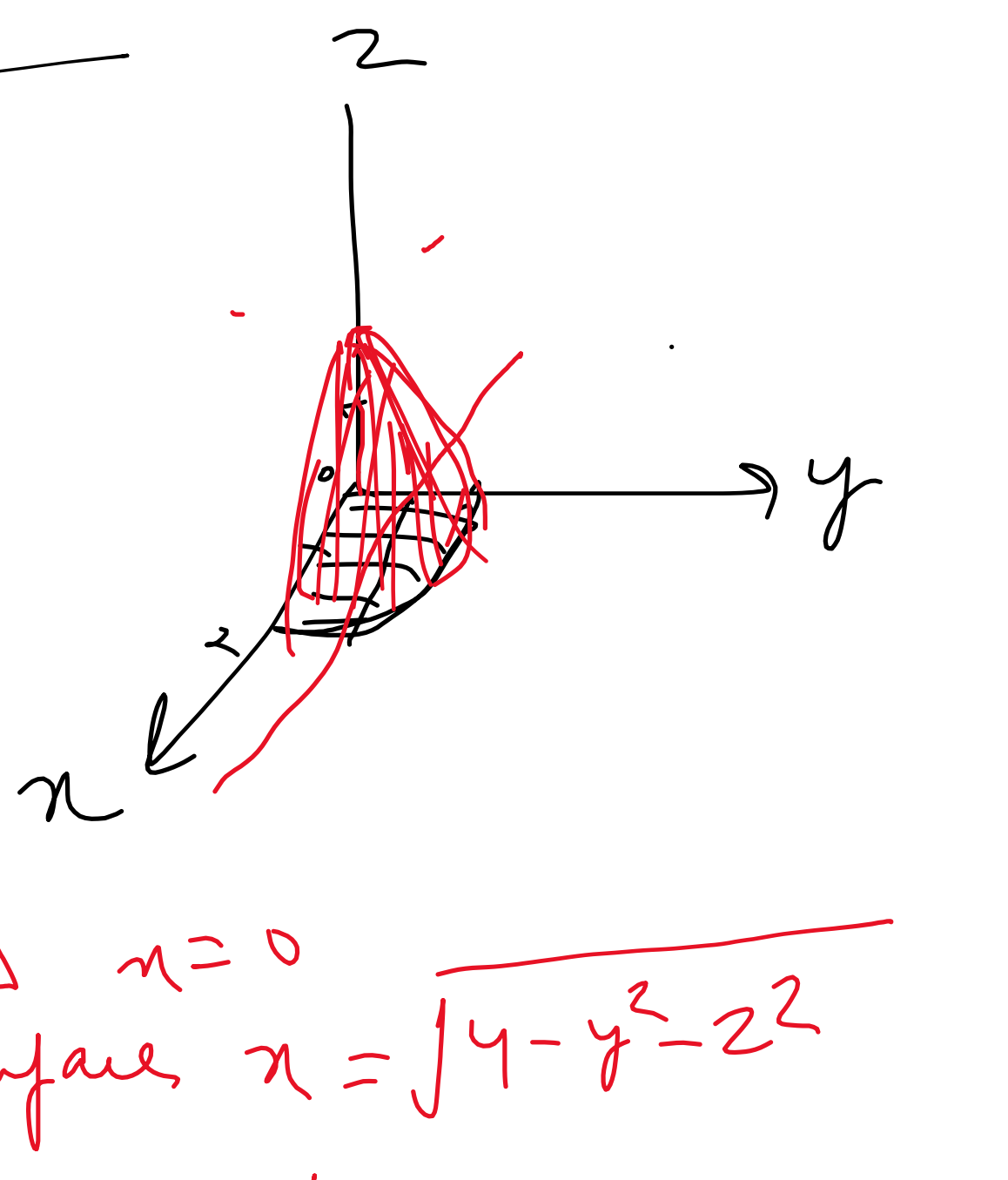
$$\int_0^2\int_0^{\sqrt{4-z^2}}\int_0^{\sqrt{4-y^2-z^2}} dx\,dy\,dz$$

given
lower bding surface $z=0$
upper bding surface $z=\sqrt{4-x^2-y^2}$



projection $x=0,2$
$y=0\to\sqrt{4-x^2}$
$x^2+y^2=4$
Integration is over $\frac{1}{4}$th of hemisphere





x lower bd $x=0$
x upper surface $x=\sqrt{4-y^2-z^2}$

projection on yz plane
$y=0,\ z=0$
$y^2+z^2=4$

$$\int_0^2\int_0^{\sqrt{4-z^2}}\int_0^{\sqrt{4-y^2-z^2}} f(x,y,z)\,dx\,dy\,dz$$