Kinetic theory of gases

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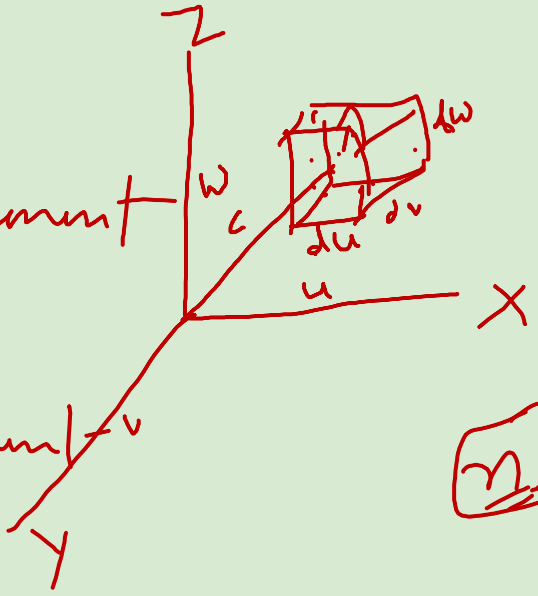
Maxwell’s law of distribution of velocities

$\underline{n}$ mol. per unit vol

$n_u du$

The prob of a mol having x-vel component lying betn u & $u+du$ is $f(u)\,du$



The prob of a mol. having y-vel component lying betn v & $v+dv$ is $f(v)\,dv$

$\boxed{n}$

Illy $\longrightarrow f(w)\,dw$

n_u

u & $u+du$, v & $v+dv$ and w & $w+dw$

$\boxed{u \;\&\; u+du}$

$$\boxed{f(u)\,f(v)\,f(w)\,du\,dv\,dw}$$

$$dn = n\, f(u) f(v) f(w)\, du\, dv\, dw \quad \text{— } 1$$

$du\, dv\, dw$

$$c^2 = u^2 + v^2 + w^2$$

$$n\, f(u) f(v) f(w)\, du\, dv\, dw$$

$$n\, F(c)\, du\, dv\, dw$$

$$\therefore n\, f(u) f(v) f(w) = n\, F(c) = n\, \phi(u^2+v^2+w^2)$$

$$\therefore \phi(u^2+v^2+w^2) = f(u) f(v) f(w) \quad \text{— } 2$$

$c \rightarrow$ fixed $\qquad d\left[\phi(c^2)\right] = 0$

$$\Rightarrow d\left[f(u) f(v) f(w)\right] = 0$$

$$\therefore f'(u) f(v) f(w)\, du + f'(v)\, dv\, f(u) f(w) + f'(w)\, dw\, f(u) f(v) = 0$$

$f(u)\,f(v)\,f(w)$

$$\frac{f'(u)}{f(u)}\,du + \frac{f'(v)}{f(v)}\,dv + \frac{f'(w)}{f(w)}\,dw = 0 \quad - (3)$$

$c \to$ fixed

$$\to u^2 + v^2 + w^2 = c^2$$

$$2u\,du + 2v\,dv + 2w\,dw = 0$$

$$\Rightarrow u\,du + v\,dv + w\,dw = 0 \quad - (4)$$

$$\left[\frac{f'(u)}{f(u)} + \lambda u\right] du + \left[\frac{f'(v)}{f(v)} + \lambda v\right] dv + \left[\frac{f'(w)}{f(w)} + \lambda w\right] dw = 0 \quad - (5)$$

$$\frac{f'(u)}{f(u)} + \lambda u = 0 \quad - (6)$$

$$\frac{f'(v)}{f(v)} + \lambda v = 0 \quad - (7)$$

$$\frac{f'(w)}{f(w)} + \lambda w = 0 \quad - (8)$$

$$\log f(u) = -\frac{\lambda u^2}{2} + \log a$$

$$\Rightarrow f(u) = a e^{-\lambda u^2/2}$$

or $f(u) = a e^{-bu^2}$ where $b = \frac{\lambda}{2}$

||ly $f(v) = a e^{-bv^2}$

$f(w) = a e^{-bw^2}$

$$\boxed{dn = n a^3 e^{-b(u^2+v^2+w^2)} du\, dv\, dw} \quad -(9)$$

$$n \iiint_{-\infty}^{\infty} \underline{f(u) f(v) f(w) du\, dv\, dw} = n$$

$$\iiint_{-\infty}^{\infty} a^3 e^{-b(u^2+v^2+w^2)} \, du\, dv\, dw = 1 \quad - (10)$$

$$\int_{-\infty}^{\infty} e^{-bu^2} du = \sqrt{\frac{\pi}{b}}$$

$$\Rightarrow a^3 \left(\frac{\pi}{b}\right)^{3/2} = 1$$

$$\Rightarrow a^3 = \left(\frac{b}{\pi}\right)^{3/2}$$

$$\Rightarrow a = \sqrt{\frac{b}{\pi}} \quad - (11)$$

$$P = 2m \sum_0^{\infty} n_u u^2 \quad - (12)$$

$$n_u = n f(u) = n a e^{-bu^2}$$

$$= n \sqrt{\frac{b}{\pi}} e^{-bu^2} \quad - (13)$$

$$P = 2mn \sqrt{\frac{b}{\pi}} \underbrace{\int_0^{\infty} e^{-bu^2} u^2 du}_{\Downarrow}$$

$$\frac{1}{4}\sqrt{\frac{\pi}{b^3}} \quad ✓$$

$$P = nKT$$

$$nKT = \frac{mn}{2b} \Rightarrow b = \frac{m}{2KT} \quad \text{— (14)}$$

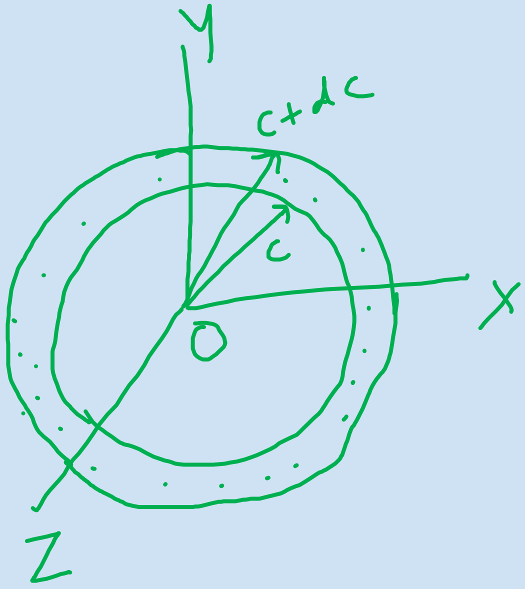
$$a = \sqrt{\frac{b}{\pi}} = \sqrt{\frac{m}{2\pi KT}} \quad \text{— (15)}$$

$$\boxed{dn = n\left(\frac{m}{2\pi KT}\right)^{3/2} e^{-\frac{m}{2}\left(\frac{u^2+v^2+w^2}{KT}\right)} du\,dv\,dw} \quad \text{— (16)}$$

$$P = \frac{dn}{n} = \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-\frac{m}{2}\left(\frac{u^2+v^2+w^2}{KT}\right)} du\,dv\,dw$$

— (17)

$$c^2 = u^2 + v^2 + \omega^2$$



$$\frac{4}{3}\pi\left[(c+dc)^3 - c^3\right] \quad - (18)$$

$$= 4\pi c^2 dc$$

$$dn = 4\pi n a^3 e^{-bc^2} c^2 dc$$

$$\boxed{dn_c = 4\pi n \left(\frac{m}{2\pi KT}\right)^{3/2} c^2 e^{-mc^2/2kT} dc} \quad - (19)$$

$$P(c) = \frac{dn_c}{n} = 4\pi \left(\frac{m}{2\pi KT}\right)^{3/2} c^2 e^{-mc^2/2kT} dc$$

$$- (20)$$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Thank you