DSC: CLASSICAL DYNAMICS
OBE

Problem 1. A bead of mass m slides without friction on a frictionless wire in the shape of a cycloid (Fig. 1) with equations

$$x = a(\theta - sin\theta), \quad y = a(1 + cos\theta)$$

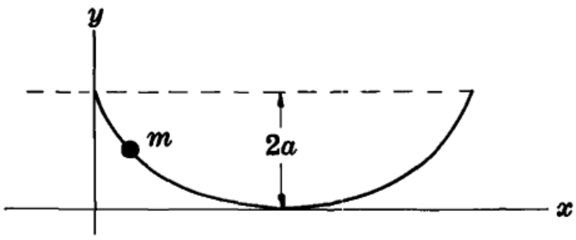

Figure 1

where $0 \leq \theta \leq 2\pi$. Find (a) the Lagrangian function, (b) the equation of motion.

Problem 2. A vertical spring (Fig. 2) has constant k and mass M. If a mass m is placed on the spring and set into motion, use Lagrange's equations to prove that the system will move with harmonic motion of period $2\pi\sqrt{\frac{M+3m}{3k}}$.

Figure 2

Problem 3. A pendulum is formed by suspending a mass m from the ceiling, using a spring of unstretched length l_0 & spring constant k.

(a) Choose, & show on a diagram, appropriate generalized coordinates, assuming that the pendulum moves in a fixed vertical plane.

(b) Set up the Lagrangian using your generalized coordinates.

(c) Write down the explicit Lagrange's equations of motion for your generalized coordinates.

Problem 4. The equations of motion for a particle of mass m and charge e moving in a uniform magnetic field B which points in the z-direction can be obtained from a Lagrangian

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \left(\frac{eB}{2c}\right)(x\dot{y} - y\dot{x})$$

(a) Find the momenta (p_x, p_y, p_z) conjugate to (x, y, z).

(b) Find the Hamiltonian, expressing your answer first in terms of $(x,\ y,\ z,\ \dot{x},\ \dot{y},\ \dot{z})$ and then in terms of (x, y, z, p_x, p_y, p_z).

(c) Evaluate the Poisson brackets

  i) $[m\dot{x},\ m\dot{y}]$

  ii) $[m\dot{x},\ H]$

Problem 5. A particle of mass m moves along x-axis under the influence of potential energy

$V(x) = -(Kx)\,e^{-(\beta x)}$, where k & β are constants. Find the equilibrium position & frequency of oscillation.

Problem 6. Two blocks and three springs are shown in Fig. 3. These blocks can execute longitudinal simple harmonic oscillation only. When the blocks are at rest, all springs are unstretched.

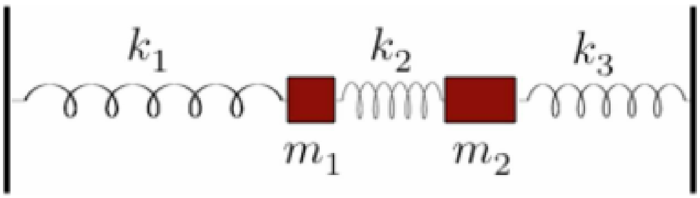

Figure 3. A system of masses and springs.

a) Choosing the displacement of each block from its equilibrium position as generalized coordinates, write the Lagrangian of this system.

b) Find the T (kinetic energy) & V (Potential energy) matrices.

c) Suppose $m_1 = 2m$, $m_2 = m$, $k_1 = 4k$, $k_2 = k$, $k_3 = 2k$, find the frequencies of small oscillation.

d) Find the normal modes of the oscillation.

Problem 7. A simple pendulum of length *l* and mass m is suspended from a pivot of mass *M* (Fig. 4) that is free to slide on a frictionless wire frame in the shape of a parabola $y = ax^2$. The pendulum moves in the plane of the frame.

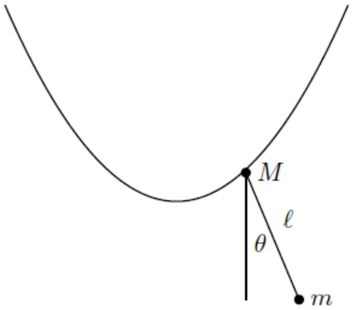

Figure 4

(a) Write down the cartesian coordinates of both masses in terms of x and θ.

(b) Calculate the time derivatives of the cartesian coordinates.

(c) Write down the kinetic and potential energies.

(d) Write down the Lagrangian using the approximation that x, θ and their derivatives are small, and solve the corresponding linear Lagrange equations.

Problem 8. Solve the Lagrangian equation for coupled pendulum (two pendulums coupled with a spring Fig. 5).

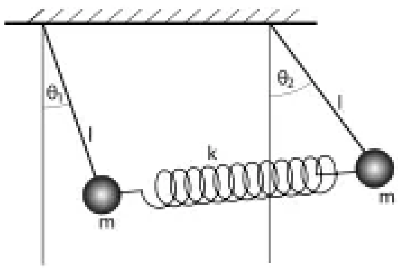

Figure 5