Statements of theorem (chapter - Permutation)

Theorem ① Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles

② If the pair of cycles $\alpha = (a_1, a_2, ---, a_m)$ and $\beta = (b_1, b_2, --- b_n)$ have no entries in common then $\alpha\beta = \beta\alpha$

③ The order of permutation of a finite set written in disjoint cycle form is the least common multiple of lengths of the cycles.

④ Every permutation in S_n, $n > 1$, is a product of 2 cycles.

⑤ If $\epsilon = \beta_1 \beta_2 --- \beta_r$, where β's are 2 cycles then r is even

⑥ If a permutation α can be expressed as a product of an even number of 2 cycles then every decomposition of α into a product of 2 cycles must have an even number of 2 cycles.

i.e If $\alpha = \beta_1 \beta_2 --- \beta_r$ & $\alpha = \gamma_1, \gamma_2, ..., \gamma_s$

where β's & γ's are 2 cycles

then r and s both are even or both odd.

(7) The set of even permutation in S_n form a subgrp of S_n.

(8) For $n>1$, A_n has order $\frac{n!}{2}$

(example) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 7 & 4 & 1 & 9 & 8 & 5 & 6 & 2 \end{pmatrix} \in S_9$

express f as product of disjoint cycles

$f = (1\,3\,4)(2\,7\,5\,9)(6\,8)$

Remark: The decomposition into transposition is Not Unique

$(1\,2\,3\,4) = (1\,4)(1\,3)(1\,2)$

$(1\,2\,3\,4) = (1\,4)(2\,3)(1\,3)$