

Practical 4(a) – Bisection Method

In[7]:=

Out[7]= -Bisection Method + 4 a Practical

In[8]:=

```
BM[a0_, b0_, n_, f_] :=
Module[{a = N[a0], b = N[b0]}, c =  $\frac{a + b}{2}$ ;
i = 0;
If[f[a] * f[b] > 0,
Print["we cannot find the roots in the given interval "];
Return []];
OutputDetails = {{i, a, b, c, f[c]}};
While[i < n,
If[f[a] * f[c] < 0, b = c, a = c];
c =  $\frac{(a + b)}{2}$ ;
i = i + 1;
OutputDetails = Append[OutputDetails, {i, a, b, c, f[c]}];
Print[NumberForm[TableForm[OutputDetails,
TableHeadings -> {None, {"i", "ai", "bi", "ci", "f[ci]"}}, 7]];
Print["root after ", n, " iteration is ", NumberForm[c, 6]];];
```

In[9]:=

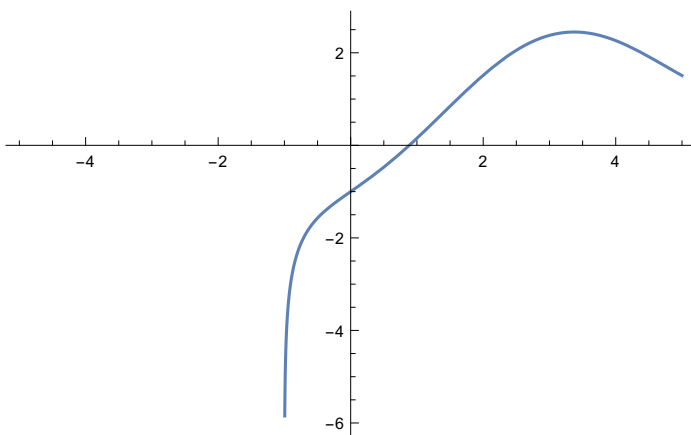
In[10]:= Q. 1 Use the Bisection method upto 15 iterations to obtain the root of the equation $\log(1+x) - \cos(x) = 0$ in the interval $(0, 1)$

••• Syntax : "(" cannot be followed by "0, 1)".

In[10]:=

```
f[x_] := Log[1 + x] - Cos[x]
Plot[f[x], {x, -5, 5}]
BM[0, 1, 15, f]
```

Out[11]=



i	ai	bi	ci	f[ci]
0	0.	1.	0.5	-0.4721175
1	0.5	1.	0.75	-0.1720731
2	0.75	1.	0.875	-0.0123882
3	0.875	1.	0.9375	0.06959341
4	0.875	0.9375	0.90625	0.0284359
5	0.875	0.90625	0.890625	0.007981228
6	0.875	0.890625	0.8828125	-0.002214254
7	0.8828125	0.890625	0.8867188	0.002880809
8	0.8828125	0.8867188	0.8847656	0.0003326059
9	0.8828125	0.8847656	0.8837891	-0.0009409923
10	0.8837891	0.8847656	0.8842773	-0.0003042352
11	0.8842773	0.8847656	0.8845215	0.00001417485
12	0.8842773	0.8845215	0.8843994	-0.0001450328
13	0.8843994	0.8845215	0.8844604	-0.00006542963
14	0.8844604	0.8845215	0.884491	-0.00002562756
15	0.884491	0.8845215	0.8845062	-5.726396 × 10 ⁻⁶

root after 15 iterations is 0.884506

In[13]:=

In[14]:= **Q.2 Use the Bisection method upto 20 iterations to obtain the root of the equation $f(x) = x^3 - 17 = 0$ in the interval (2, 3)**

Syntax : "(" cannot be followed by "2, 3)".

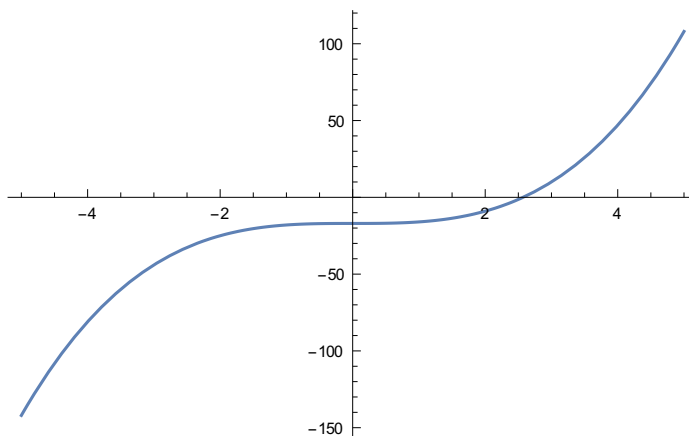
In[14]:=

f[x_] := x³ - 17

Plot[f[x], {x, -5, 5}]

BM[2, 3, 20, f]

Out[15]=



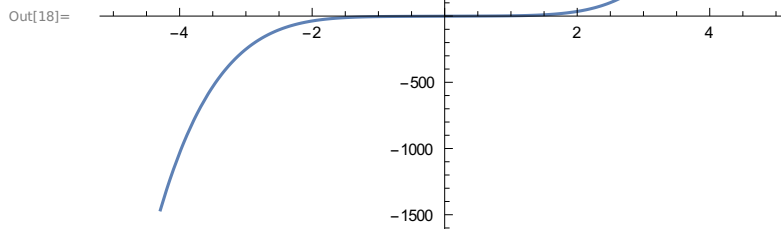
i	ai	bi	ci	f[ci]
0	2.	3.	2.5	-1.375
1	2.5	3.	2.75	3.796875
2	2.5	2.75	2.625	1.087891
3	2.5	2.625	2.5625	-0.173584
4	2.5625	2.625	2.59375	0.4495544
5	2.5625	2.59375	2.578125	0.136097
6	2.5625	2.578125	2.570313	-0.01921415
7	2.570313	2.578125	2.574219	0.05832356
8	2.570313	2.574219	2.572266	0.01952527
9	2.570313	2.572266	2.571289	0.0001482004
10	2.570313	2.571289	2.570801	-0.009534815
11	2.570801	2.571289	2.571045	-0.004693767
12	2.571045	2.571289	2.571167	-0.002272898
13	2.571167	2.571289	2.571228	-0.001062378
14	2.571228	2.571289	2.571259	-0.0004570958
15	2.571259	2.571289	2.571274	-0.0001544495
16	2.571274	2.571289	2.571281	-3.124975×10^{-6}
17	2.571281	2.571289	2.571285	0.00007253762
18	2.571281	2.571285	2.571283	0.00003470629
19	2.571281	2.571283	2.571282	0.00001579065
20	2.571281	2.571282	2.571282	6.332837×10^{-6}

root after 20 iterations is 2.57128

In[17]:= **Q. 3 Use the Bisection method upto 20 iterations to obtain the root of the equation $f(x) = x^5 + 2x - 1 = 0$ in the interval $(0, 1)$**

Syntax : "(" cannot be followed by "0, 1)".

In[17]:= **f[x_] := x⁵ + 2 x - 1**
Plot[f[x], {x, -5, 5}]
BM[0, 1, 20, f]



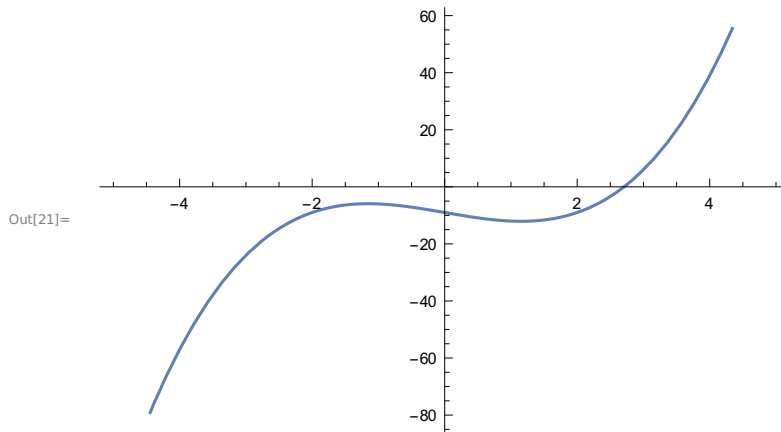
i	ai	bi	ci	f[ci]
0	0.	1.	0.5	0.03125
1	0.	0.5	0.25	-0.4990234
2	0.25	0.5	0.375	-0.2425842
3	0.375	0.5	0.4375	-0.1089716
4	0.4375	0.5	0.46875	-0.03986886
5	0.46875	0.5	0.484375	-0.004587025
6	0.484375	0.5	0.4921875	0.0132587
7	0.484375	0.4921875	0.4882813	0.004318076
8	0.484375	0.4882813	0.4863281	-0.0001388627
9	0.4863281	0.4882813	0.4873047	0.002088503
10	0.4863281	0.4873047	0.4868164	0.000974545
11	0.4863281	0.4868164	0.4865723	0.0004177725
12	0.4863281	0.4865723	0.4864502	0.0001394377
13	0.4863281	0.4864502	0.4863892	2.832053×10^{-7}
14	0.4863281	0.4863892	0.4863586	-0.00006929084
15	0.4863586	0.4863892	0.4863739	-0.00003450408
16	0.4863739	0.4863892	0.4863815	-0.00001711051
17	0.4863815	0.4863892	0.4863853	-8.413667×10^{-6}
18	0.4863853	0.4863892	0.4863873	-4.065235×10^{-6}
19	0.4863873	0.4863892	0.4863882	-1.891016×10^{-6}
20	0.4863882	0.4863892	0.4863887	-8.039056×10^{-7}

root after 20 iterations is 0.486389

In[20]:= **Q. 4 Use the Bisection method upto 20 iterations to obtain the root of the equation $x^3 - 4x - 9 = 0$ in the interval (0, 4)**

Syntax : "(" cannot be followed by "0, 4)".

In[20]:= **f[x_] := x^3 - 4x - 9**
Plot[f[x], {x, -5, 5}]
BM[0, 4, 20, f]



i	ai	bi	ci	f[ci]
0	0.	4.	2.	-9.
1	2.	4.	3.	6.
2	2.	3.	2.5	-3.375
3	2.5	3.	2.75	0.796875
4	2.5	2.75	2.625	-1.412109
5	2.625	2.75	2.6875	-0.3391113
6	2.6875	2.75	2.71875	0.2209167
7	2.6875	2.71875	2.703125	-0.06107712
8	2.703125	2.71875	2.710938	0.07942343
9	2.703125	2.710938	2.707031	0.009049237
10	2.703125	2.707031	2.705078	-0.0260449
11	2.705078	2.707031	2.706055	-0.008505573
12	2.706055	2.707031	2.706543	0.0002698962
13	2.706055	2.706543	2.706299	-0.004118322
14	2.706299	2.706543	2.706421	-0.001924334
15	2.706421	2.706543	2.706482	-0.0008272491
16	2.706482	2.706543	2.706512	-0.000278684
17	2.706512	2.706543	2.706528	-4.395767×10^{-6}
18	2.706528	2.706543	2.706535	0.0001327498
19	2.706528	2.706535	2.706532	0.00006417688
20	2.706528	2.706532	2.70653	0.00002989053

root after 20 iterations is 2.70653

In[29]:= **Q. 5 Use the Bisection method upto 15 iterations to obtain the root of the equation $x \log(10, x) - 1.2 = 0$ in the interval $(0, 4)$**

Clear [

f,

c]

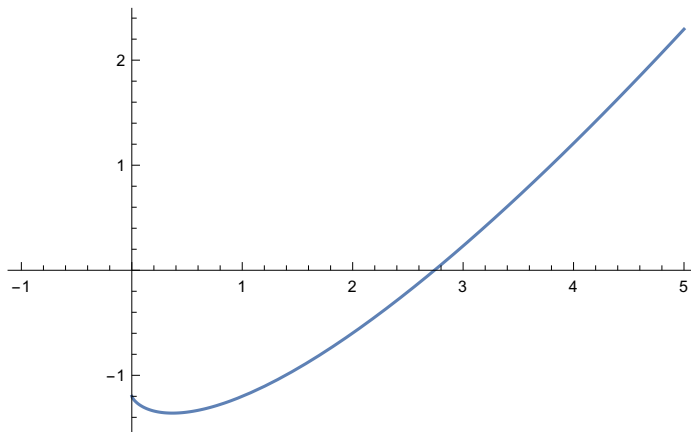
Syntax : "(" cannot be followed by "10, x)".

In[29]:= **f[x_] := x Log[10, x] - 1.2**

Plot[f[x], {x, -1, 5}]

BM[0, 4, 15, f]

Out[30]=



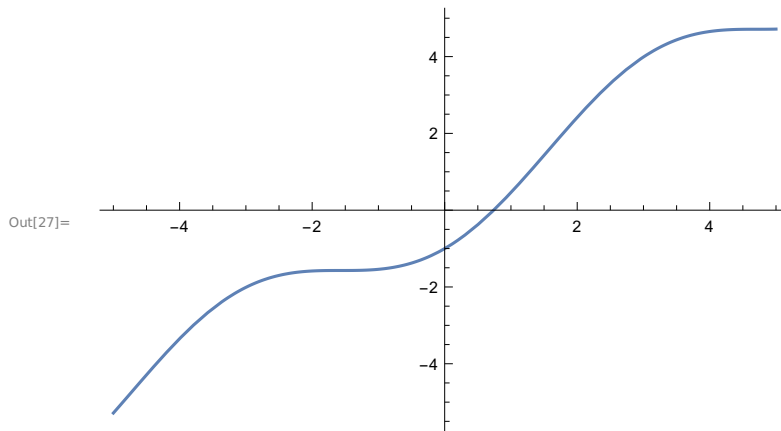
i	ai	bi	ci	f[ci]
0	0.	4.	2.	-0.59794
1	0.	4.	2.	-0.59794
2	0.	4.	2.	-0.59794
3	0.	4.	2.	-0.59794
4	0.	4.	2.	-0.59794
5	0.	4.	2.	-0.59794
6	0.	4.	2.	-0.59794
7	0.	4.	2.	-0.59794
8	0.	4.	2.	-0.59794
9	0.	4.	2.	-0.59794
10	0.	4.	2.	-0.59794
11	0.	4.	2.	-0.59794
12	0.	4.	2.	-0.59794
13	0.	4.	2.	-0.59794
14	0.	4.	2.	-0.59794
15	0.	4.	2.	-0.59794

root after 15 iterations is 2.

In[26]:= **Q .6 Use the Bisection method upto 15 iterations to obtain the root of the equation $x - \cos(x) = 0$ in the interval $(0, 2)$**

Syntax : "(" cannot be followed by "0, 2)".

In[26]:= **f[x_] := x - Cos[x]**
Plot[f[x], {x, -5, 5}]
BM[0, 2, 15, f]



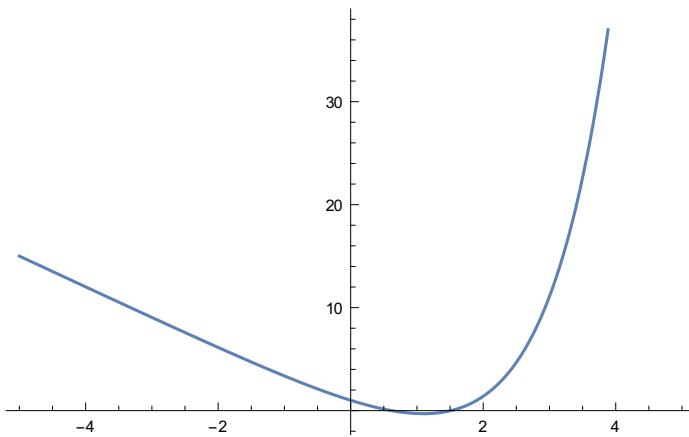
i	ai	bi	ci	f[ci]
0	0.	2.	1.	0.4596977
1	0.	1.	0.5	-0.3775826
2	0.5	1.	0.75	0.01831113
3	0.5	0.75	0.625	-0.1859631
4	0.625	0.75	0.6875	-0.08533495
5	0.6875	0.75	0.71875	-0.03387937
6	0.71875	0.75	0.734375	-0.007874725
7	0.734375	0.75	0.7421875	0.005195712
8	0.734375	0.7421875	0.7382813	-0.00134515
9	0.7382813	0.7421875	0.7402344	0.001923873
10	0.7382813	0.7402344	0.7392578	0.0002890091
11	0.7382813	0.7392578	0.7387695	-0.0005281584
12	0.7387695	0.7392578	0.7390137	-0.0001195967
13	0.7390137	0.7392578	0.7391357	0.00008470073
14	0.7390137	0.7391357	0.7390747	-0.00001744935
15	0.7390747	0.7391357	0.7391052	0.00003362535

root after 15 iterations is 0.739105

Q. 7 Use the Bisection method upto 15 iterations to obtain the root of the equation $e^x - 3x = 0$ in the interval $(0, 1)$

```
In[32]:= f[x_] := E^x - 3 x
Plot[f[x], {x, -5, 5}]
BM[0, 1, 15, f]
```

Out[33]=

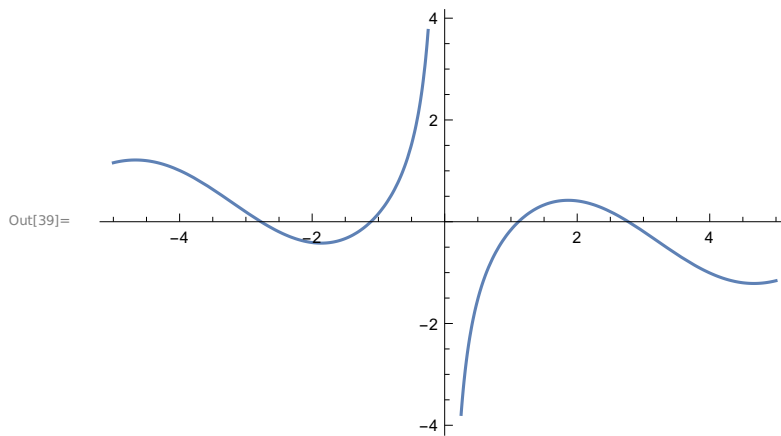


i	ai	bi	ci	f[ci]
0	0.	1.	0.5	0.1487213
1	0.5	1.	0.75	-0.133
2	0.5	0.75	0.625	-0.006754043
3	0.5	0.625	0.5625	0.06755466
4	0.5625	0.625	0.59375	0.02951607
5	0.59375	0.625	0.609375	0.01115649
6	0.609375	0.625	0.6171875	0.002144652
7	0.6171875	0.625	0.6210938	-0.002318893
8	0.6171875	0.6210938	0.6191406	-0.0000906632
9	0.6171875	0.6191406	0.6181641	0.00102611
10	0.6181641	0.6191406	0.6186523	0.0004675019
11	0.6186523	0.6191406	0.6188965	0.000188364
12	0.6188965	0.6191406	0.6190186	0.00004883657
13	0.6190186	0.6191406	0.6190796	-0.00002091678
14	0.6190186	0.6190796	0.6190491	0.00001395903
15	0.6190491	0.6190796	0.6190643	-3.479088×10^{-6}

root after 15 iteration is 0.619064

Q. 8 Use the Bisection method upto 15 iterations to obtain the root of the equation $\sin(x) - 1/x = 0$ in the interval $(-2, -0.5)$

```
In[38]:= f[x_] := Sin[x] - 1/x
Plot[f[x], {x, -5, 5}]
BM[-2, -0.5, 15, f]
```



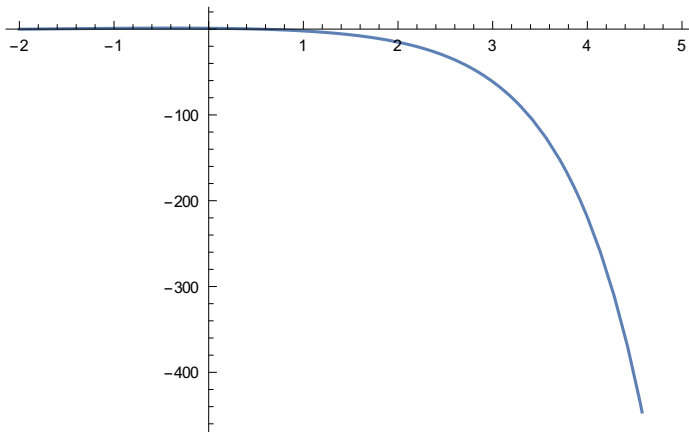
i	ai	bi	ci	f[ci]
0	-2.	-0.5	-1.25	-0.1489846
1	-1.25	-0.5	-0.875	0.3753136
2	-1.25	-0.875	-1.0625	0.06760154
3	-1.25	-1.0625	-1.15625	-0.05043428
4	-1.15625	-1.0625	-1.109375	0.005987853
5	-1.15625	-1.109375	-1.132813	-0.02284997
6	-1.132813	-1.109375	-1.121094	-0.008590369
7	-1.121094	-1.109375	-1.115234	-0.001341425
8	-1.115234	-1.109375	-1.112305	0.002313129
9	-1.115234	-1.112305	-1.11377	0.0004833358
10	-1.115234	-1.11377	-1.114502	-0.000429673
11	-1.114502	-1.11377	-1.114136	0.00002667424
12	-1.114502	-1.114136	-1.114319	-0.0002015387
13	-1.114319	-1.114136	-1.114227	-0.00008744204
14	-1.114227	-1.114136	-1.114182	-0.00003038636
15	-1.114182	-1.114136	-1.114159	-1.856675 × 10 ⁻⁶

root after 15 iterations is -1.11416

Q. 8 Use the Bisection method upto 15 iterations to obtain the root of the equation $\cos(x) - xe^x = 0$ in the interval $(-3, -1)$

```
In[44]:= f[x_] := Cos[x] - x E^x
Plot[f[x], {x, -2, 5}]
BM[-3, -1, 15, f]
```

Out[45]=



i	a _i	b _i	c _i	f[c _i]
0	-3.	-1.	-2.	-0.1454763
1	-2.	-1.	-1.5	0.4054324
2	-2.	-1.5	-1.75	0.1258583
3	-2.	-1.75	-1.875	-0.01199294
4	-1.875	-1.75	-1.8125	0.05652537
5	-1.875	-1.8125	-1.84375	0.02214668
6	-1.875	-1.84375	-1.859375	0.005044803
7	-1.875	-1.859375	-1.867188	-0.003482357
8	-1.867188	-1.859375	-1.863281	0.0007791853
9	-1.867188	-1.863281	-1.865234	-0.001352099
10	-1.865234	-1.863281	-1.864258	-0.000286585
11	-1.864258	-1.863281	-1.86377	0.0002462682
12	-1.864258	-1.86377	-1.864014	-0.00002016639
13	-1.864014	-1.86377	-1.863892	0.0001130489
14	-1.864014	-1.863892	-1.863953	0.00004644077
15	-1.864014	-1.863953	-1.863983	0.00001313706

root after 15 iterations is -1.86398