Chapter 7

Coset of a Subgroup:

Let G be a group and H be a subgroup of G. Then for any $a \in G$.

(i) The set $Ha = \{ha \mid h \in H\}$ is called the right coset of H in G containing a.

(ii) The set $aH = \{ah \mid h \in H\}$ is called the left coset of H in G containing a.

Example (1): Let $G = S_3$

$$H = \{(1), (1\,2)\}$$

$$(1) = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}, \quad (1\,2) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}$$

$H = \{(1), (12)\}$

the left cosets of H in G are:

$G = S_3$

$G = \{(1), (12), (13), (23), (123), (132)\}$

$(1)H = H$

$$G = S_3$$
$$G = \{(1), (12), (13), (23), (123), (132)\}$$

$$(1)H = H$$

$$(12)H = \{(12)h \mid h \in H\}$$
$$= \{(12)(1), (12)(12)\}$$
$$= \{(12), (1)\} = \{(1), (12)\} = H.$$

$$(13)H = \{(13)h \mid h \in H\}$$
$$= \{(13)(1), (13)(12)\}$$
$$= \{(13), (123)\}$$

$$(123) = (13)(12)$$

$$(23)H = \{(23)(1), (23)(12)\}$$
$$= \{(23), (23)(21)\}$$
$$= \{(23), (213)\} = \{(23), (132)\}$$

$$(123)H = \{(123)(1), (123)(12)\}$$
$$= \{(123), (13)\}.$$

$$(132)H = \{(132)(1), (132)(12)\}$$
$$= \{(132), (23)\}.$$

Example ②:- Let $G = \mathbb{Z}_6$

$H = \{0, 2, 4\}$.

aH.
$a * H$.

left cosets of H in G are.

$0 +_6 H = 0 + H = \{0+0, 0+2, 0+4\}$
$= \{0, 2, 4\} = H$

$1 + H = \{1+0, 1+2, 1+4\} = \{1, 3, 5\}$
$2 + H = \{2+0, 2+2, 2+4\} = \{2, 4, 0\}$
$= \{0, 2, 4\} = H$
$3 + H = \{3+0, 3+2, 3+4\} = \{3, 5, 1\} = \{1, 3, 5\}$
$= 1 + H$
$4 + H = \{4, 6, 8\} = \{4, 0, 2\} = \{0, 2, 4\} = H$.
$5 + H = \{5+0, 5+2, 5+4\} = \{5, 1, 3\}$
$= \{1, 3, 5\} = 1 + H$

$\therefore$ the distinct left cosets of H in G are H and $1 + H$.

Example 3:

Let $G = D_4 = \{R_0, R_{90}, R_{180}, R_{270}, H, V, D, D'\}$

and $K = \{R_0, R_{180}\}$

then, right cosets of H in G are:

$$KR_0 = \{kR_0 \mid k \in K\} = \{R_0R_0,\ R_{180}R_0\}$$
$$= \{R_0, R_{180}\} = K$$

$$KR_{90} = \{kR_{90} \mid k \in K\} = \{R_0R_{90},\ R_{180}R_{90}\}$$
$$= \{R_{90}, R_{270}\}.$$

$$KR_{180} = \{kR_{180} \mid k \in K\} = \{R_0R_{180},\ R_{180}R_{180}\}$$
$$= \{R_{180}, R_0\} = \{R_0, R_{180}\} = K$$

$$KR_{270} = \{R_0R_{270},\ R_{180}R_{270}\} = \{R_{270}, R_{90}\}$$
$$= \{R_{90}, R_{270}\} = KR_{90}.$$

$$KH = \{R_0H,\ R_{180}H\} = \{H, V\}$$

$$KV = \{R_0V,\ R_{180}V\} = \{V, H\} = \{H, V\} = KH$$

$$KD = \{R_0D,\ R_{180}D\} = \{D, D'\}$$