increases. This can be explained by the relativistic law of variation of mass with velocity $m = m_0/\sqrt{1-v^2/c^2}$. With the experimental value of e/m for the different values of v, Kaufmann calculated the value of e/m_0. The value of e/m_0 was found to be a constant for all values of v.

20.10 BETA RAY SPECTRUM

Theory. The energies of β-particles from radio-active elements are determined by measuring the radii of curvature of their paths in a magnetic field of known flux density B. The circular path traversed by the β-particles of velocity v is governed by the relation

$$Bev = \frac{mv^2}{r}.$$

$$\therefore \quad v = Br\,(e/m).$$

From the geometry of the arrangement, the radius of the circular path r can be found. The value of e/m can be assumed. Hence, the velocity v can be calculated.

For particles moving with very high velocities, the kinetic energy of the particle,

$$E_k = mc^2 - m_0c^2 = m_0c^2\left[\frac{1}{(1-v^2/c^2)^{1/2}} - 1\right]$$

Magnetic spectrograph

Experimental study. The energy spectrum of β-particles is studied with the help of a magnetic spectrograph (Fig. 20.17).

The source is the radioactive substance S under study, coated on a fine wire. A and B are diaphragms used for limiting the beam of β-particles. G is a Geiger counter.

β-rays from the source at S are bent around by the magnetic field and focused on the aperture O.

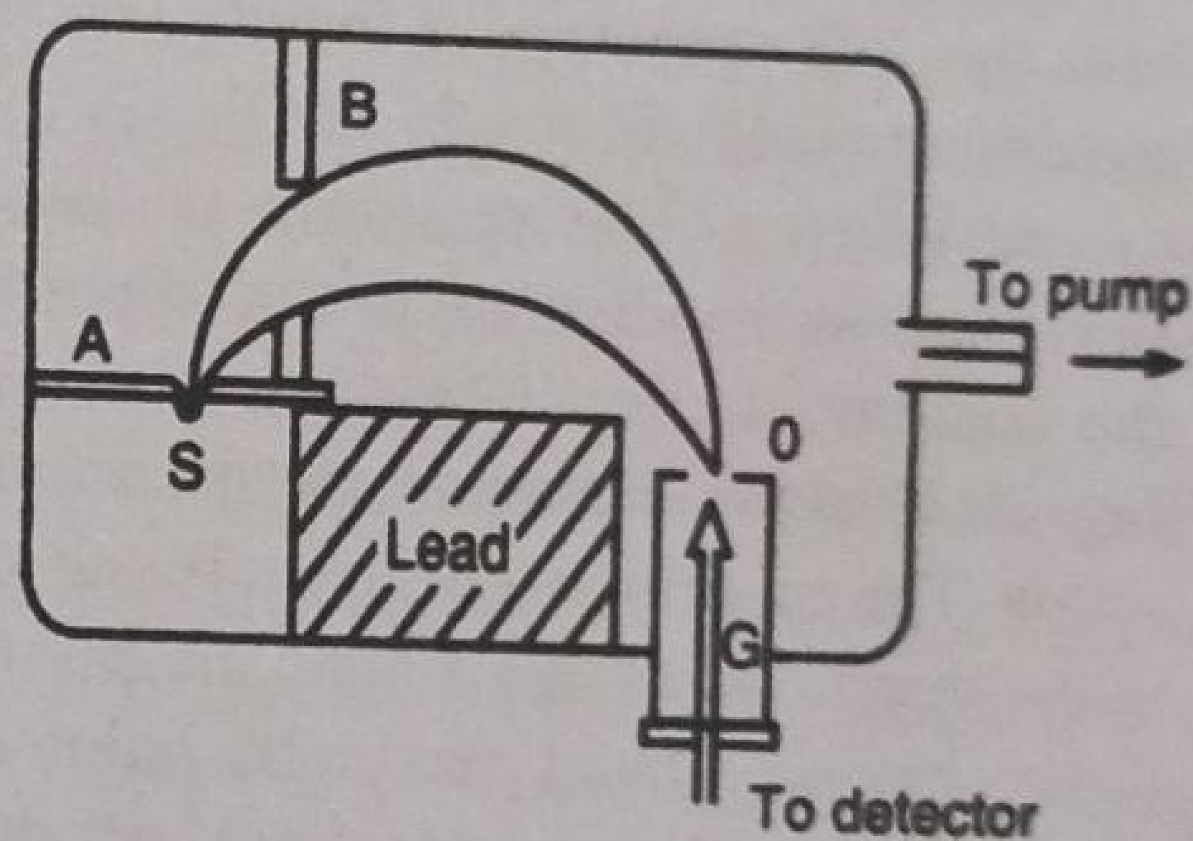

Fig. 20.17

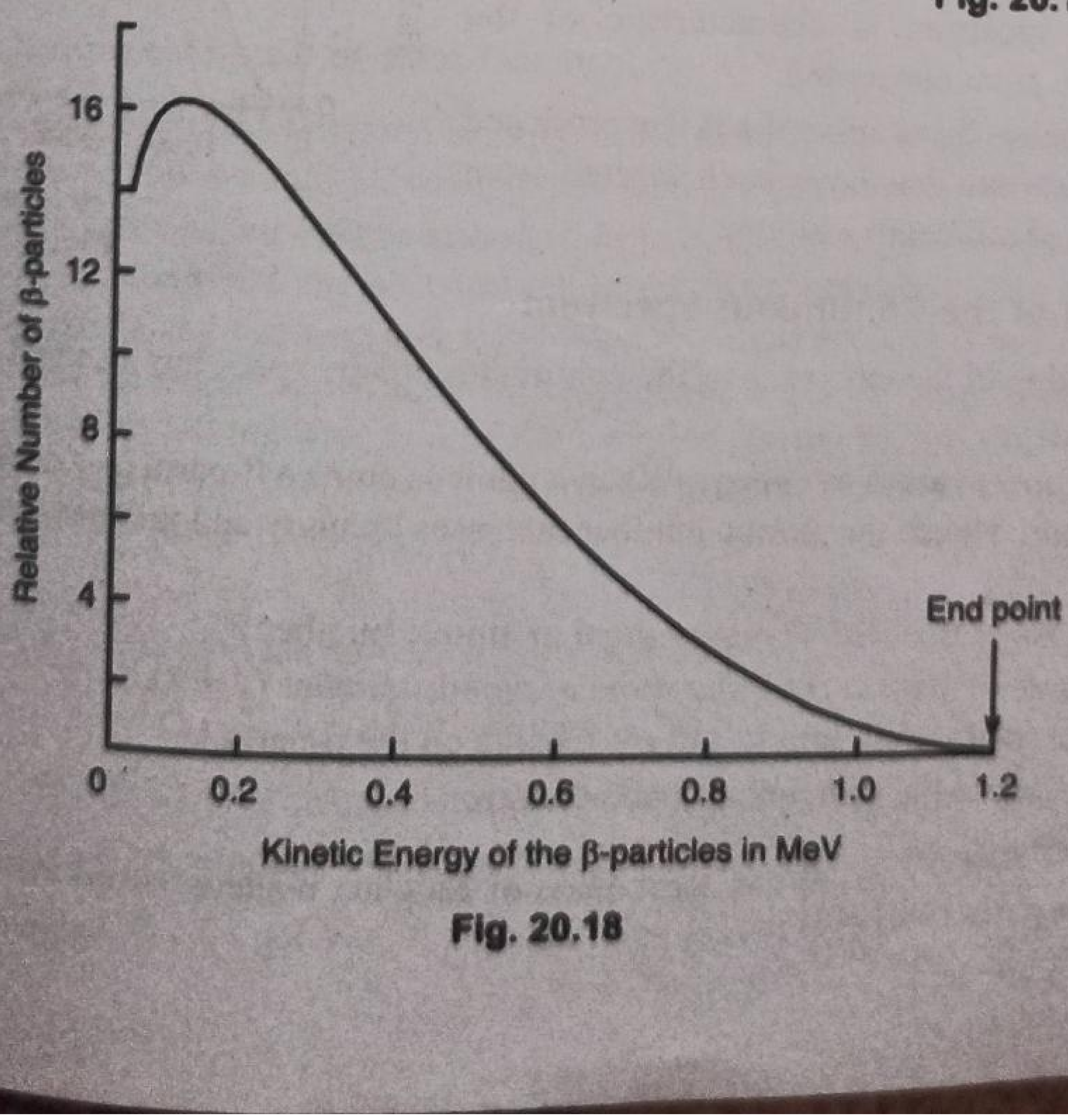

Fig. 20.18

Working. The number of β-particles entering the aperture O is counted at a given value of the magnetic field of intensity B. The intensity of the magnetic field is then changed to a new value and the number of β-particles entering the aperture O is again counted. In this manner, the velocity distribution of the β-particles is determined.

Fig. 20.18 shows the continuous β-ray spectrum of radium E.

- The curve first rises to a maximum and then decreases to zero at a well defined velocity which corresponds to the maximum velocity of the β-particles.
- Below the maximum energy (E_{max}), the β-ray particles give a continuous spectrum.

End point energy. The β-spectrum is continuous, having energies ranging from zero to a certain well defined limit called *End point energy*, which is characteristic of the β-emitter. The energy distribution has a well defined maximum. The height and position of the maximum also depends upon the nature of the radioactive substance emitting the β-particles.

Thus end point energy is the maximum energy with which a β-particle is emitted from a radioactive nuclide.

Results. The results of these measurements show that there are apparently two distinct types of β-ray spectrum, one a *sharp line spectrum* and the other a *continuous spectrum.*

- The lines correspond to β-particles emitted with discrete energies.

The sharp line spectra are due to electrons that have been ejected from the K, L, M and N shells of the atom by the process of *internal conversion.*

The continous spectrum of β-particle energies found for RaE is shown in Fig. 20.18. In this case, no line spectrum is found. The upper limit or maximum energy is at 1.17 MeV.

- When a line spectrum is also present, the lines appear as distinct peaks superimposed upon the continuous distribution curve, as in Fig. 20.19.

Fig. 20.19 shows Beta-particle spectrum of Au^{198}.

- The continuous spectrum is due to a continuous spread of energy among the emitted β-particles having a fixed maximum value of the energy. The upper energy limit of the continuous β-particle spectrum is characteristic of the radioactive atom concerned.
- The continuous β-ray spectrum is that produced by the electrons that have been ejected from the nuclei of radioactive atoms.

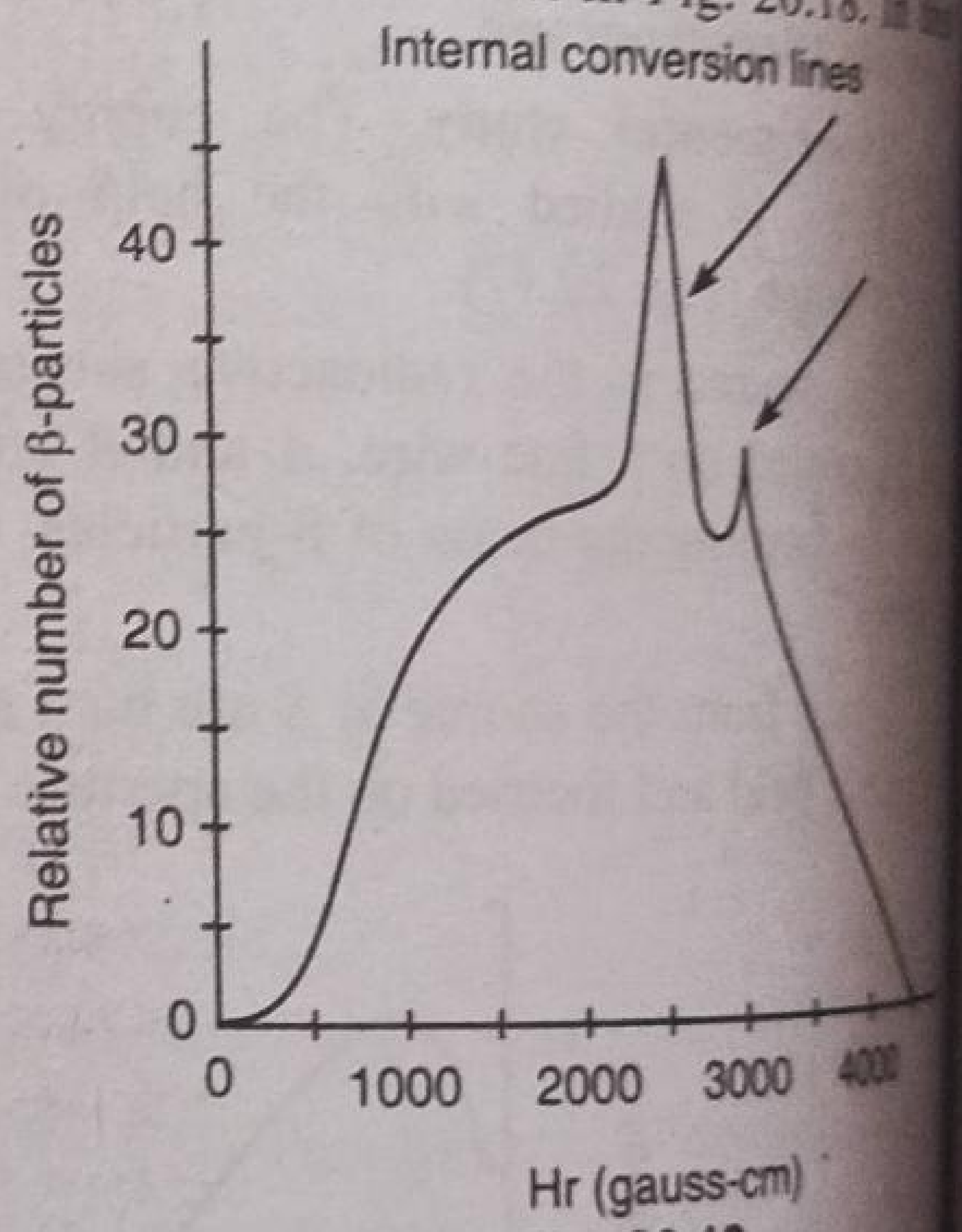

Fig. 20.19

20.10.1. Origin of the Continuous Spectrum

The explanation of the existence of the continuous energy spectrum for the β-particles presents some difficulties.

(1) Law of conservation of energy. When a nucleus emits a β-particle, a neutron in the nucleus changes to a proton. Hence the atomic number increases by unity and the mass number remains the same.

Let M_1 = mass of the *neutral parent atom* of atomic number Z,

M_2 = mass of *neutral daughter atom of atomic number* $(Z+1)$,

m = mass of the β-particle and e = Charge on the β-particle.

Then, according to the principle of mass energy,

$$\left.\begin{matrix}\text{Rest mass of}\\ \text{the parent nucleus}\end{matrix}\right\} = \text{Rest mass of daughter nucleus} + \text{Rest mass of electron} + \text{Energy of the electron}$$

$$(M_1 - Zm)\,c^2 = [M_2 - (Z+1)\,m]\,c^2 + mc^2 + Q$$

or

$\therefore$ Energy of the electron $= Q = (M_1 - M_2)\,c^2$.

Hence all the β-particles from a given radioactive substance must be emitted with the same K.E. But actual measurements show that only a few β-particles are emitted with this maximum value of energy. The majority of β-particles are emitted with smaller energies. What happens to the remaining energy?

(2) **Law of conservation of angular momentum.** Another difficulty comes in the conservation of angular momentum. Every nucleus has an angular momentum (nuclear spin) which is an odd multiple of $\frac{1}{2}\hbar$ for nuclei of odd mass number and an even multiple of $\frac{1}{2}\hbar$ for nuclei of even mass number. The electron has an angular momentum $\frac{1}{2}\hbar$. In β-decay, mass number remains unchanged. How is it possible for a nucleus of even mass number and therefore an integral spin to give rise to a daughter nucleus of the same mass number and also an integral spin and yet emit an electron of spin $\frac{1}{2}\hbar$? The same is the difficulty for a nucleus of odd mass number.

(3) There is also an apparent failure to conserve linear momentum in β-decay.

Pauli's Neutrino Hypothesis

In 1930, Pauli proposed that if an uncharged particle of zero mass and spin $\frac{1}{2}$ is emitted in β-decay together with the electron, the energy, angular momentum and linear momentum discrepancies discussed above would be removed. The particle was named *neutrino*. It was supposed that neutrino carries off an energy equal to the difference between Q and the actual electron K.E. Subsequently it was found that there are two kinds of neutrino involved in β-decay, the neutrino itself (symbol ν) and the anti-neutrino (symbol $\bar{\nu}$). The reason neutrinos were not experimentally detected until recently is that their interaction with matter is extremely feeble. Lacking charge and mass, and not electromagnetic in nature, the neutrino can pass unimpeded through vast amounts of matter. A neutrino would have to pass through over 100 light-years of solid iron on the average before interacting.

20.10.2. The Neutrino Theory of Beta Decay

In 1934, Fermi developed a theory to explain the continuous β-ray spectrum. This theory is called neutrino theory of β-decay. According to this theory, a β-particle and a neutrino are created in the nucleus and both are emitted simultaneously. The *total energy* of these two particles is a constant which is equal to the end-point energy observed in the β-ray spectrum. This maximum energy is shared by the β-particle, the neutrino and also by the recoiling nucleus. The electron will carry the maximum energy when the energy of the neutrino is zero. In all other cases, electron will carry an energy less than the maximum. The sum of the energies carried by the electron and the neutrino will always be the same. This energy may be shared by the two particles in any proportion. Hence it explains the continuous β-ray spectrum.

When the nucleon shifts from the neutron quantum state to the proton quantum state, electron and antineutrino are emitted. This process is represented by

$$n \rightarrow p + e^- + \bar{\nu}.$$

In ordinary beta decay, it is an antineutrino that is emitted.

Positron emission corresponds to the conversion of a nuclear proton into a neutron, a positron, and a neutrino.

$p = n + e^+ + \nu$

$$p \rightarrow n + e^{+} + \nu.$$

Positron emission leads to a daughter nucleus of lower atomic number Z while leaving the mass number A unchanged. Thus negative and positive beta decays may be represented as

$$Z^{X^A} \rightarrow Z+1^{X^A} + e^{-} + \overline{\nu}$$

$$Z^{X^A} \rightarrow Z-1^{X^A} + e^{+} + \nu$$

The electron, neutrino and product nucleus share among them the energy, angular momentum and linear momentum available from the nuclear transitions. Thus the neutrino theory of β-decay successfully explains the continuous energy spectrum of β-rays.

20.10.3. Detection of Neutrino

Reines and Cowan made use of the abundant supply of antineutrinos emitted from a nuclear reactor. Energetic antineutrinos can be detected by the reaction

$$\overline{\nu} + p \rightarrow n + \beta^{+}$$

The experimental arrangement is shown in Fig. 20.20.

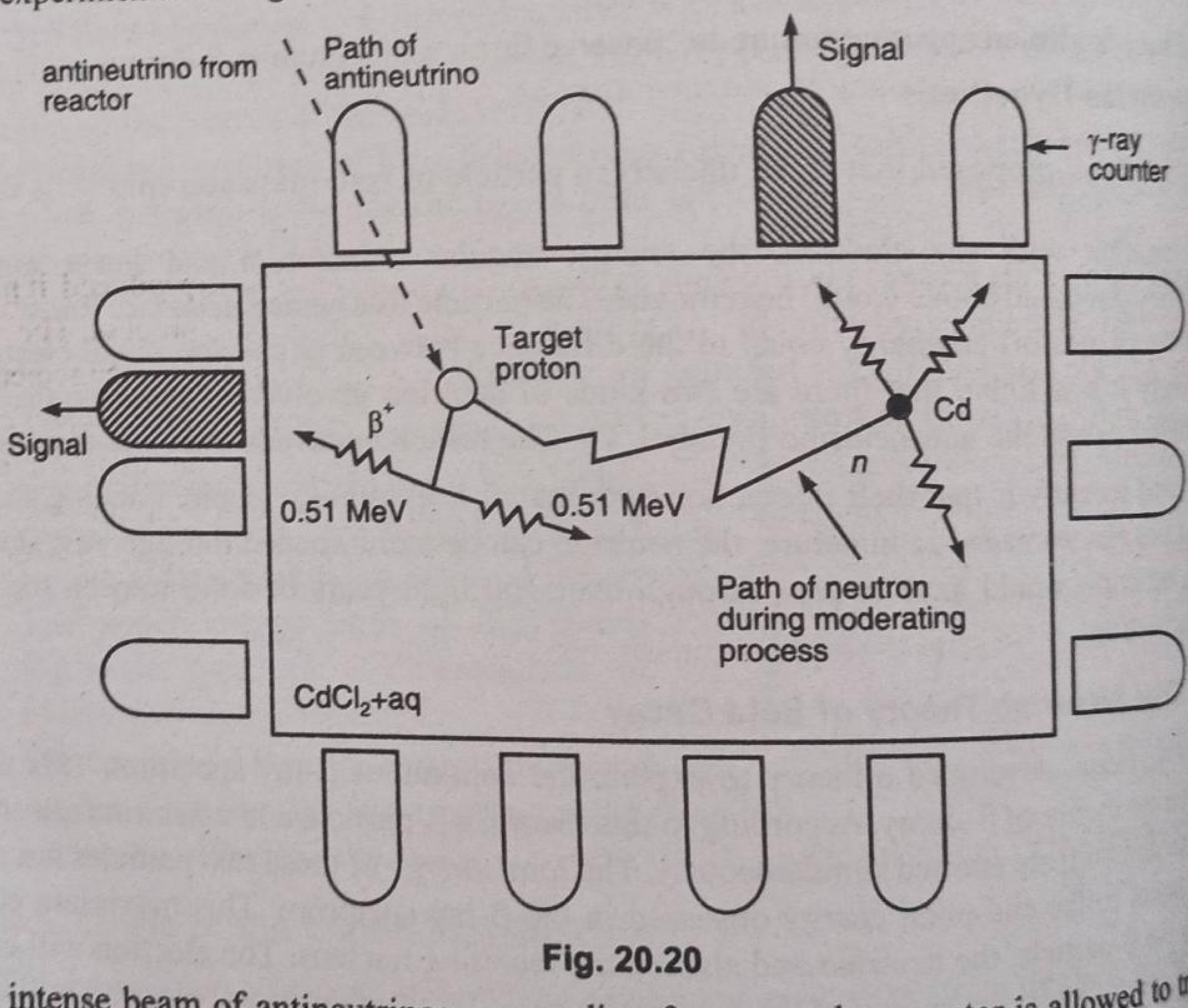

Fig. 20.20

An intense beam of antineutrinos, proceeding from a nuclear reactor is allowed to traverse a large plastic tank filled with an aqueous solution of cadmium chloride, $CdCl_2$. The tank is surrounded by many photomultiplier tubes. Suppose an antineutrino is absorbed by a proton and this system decays by β^{+} emission to become a neutron. Thus

$${}_1H^1 + \overline{\nu} \rightarrow {}_0n^1 + {}_1e^0.$$

The positron combines with an electron to create two γ photons, which are detected by the photomultiplier tubes. The neutron, moderated, *i.e.*, slowed down by collisions with protons, is finally captured by a cadmium nucleus when three γ photons are emitted. Thus,

$${}_{48}Cd^{113} + {}_0n^1 \rightarrow {}_{48}Cd^{114} + 3\gamma.$$