Numerical Problem's on Surface Tension

1. A capillary tube of internal diameter 0.21 mm is dipped into a liquid whose density is 0.79 g cm^{-3}. The liquid rises in this capillary to a height of 6.30 cm. Calculate the surface tension of the liquid. (g = 980 cm sec^{-2}).

Solution: $\gamma = \frac{hrdg}{2}$ ——— (1)

Given that $h = 6.30\ cm$ (height)

$g = 980\ cm s^{-2}$ (acceleration due to gravity)

$d = 0.79\ g cm^{-3}$

diameter = 0.21 mm

$r = \frac{diameter}{2} = \frac{0.21}{2} \times \frac{1}{10} = 0.0105\ cm$

Put all those values in equ. (1)

$$\gamma = \frac{6.30\ cm \times 0.0105\ cm \times 0.79\ g cm^{-3} \times 980\ cm s^{-2}}{2}$$

$\gamma = 25.6\ g cm s^{-2}/cm$

$\boxed{\gamma = 25.6\ dynes/cm}$

2. How high will sap rise in a plant if the capillaries are 0.01 mm diameter, the density of the fluid is 1.3 g cm^{-3} and its surface tension 0.065 Nm^{-1}. (g = 981 cm s^{-2})

Solution: $\gamma = \frac{hrdg}{2}$

$h = \frac{2\gamma}{rdg}$ ——— (1)

Given that

$\gamma = 0.065\ Nm^{-1}$

$g = 981\ cm/s^2 = 9.81\ m/s^2$

$d = 1.3\ g/cm^3 = 1.3 \times 10^3\ kg/m^3$

$r = \frac{d}{2} = \frac{0.01\ mm}{2} = \frac{10^{-5}}{2}\ m$

$$h = \frac{2 \times 2 \times 0.065}{10^{-5} \times 1.3 \times 10^3 \times 9.81}$$

$\boxed{h = 2.04\ m}$

3. In the determination of surface tension of a liquid by the drop-number method, it gives 55 drops while water gave 25 drops for the same volume. The densities of the liquid and water are 0.996 and 0.800 g/cm^3 respectively. Find the surface tension of the liquid if that of water is 72.0 dynes/cm.

Solution:

$$\frac{\gamma_1}{\gamma_2} = \frac{n_2 d_1}{n_1 d_2}$$

γ_1 = surface tension of liquid

γ_2 = surface tension of water = 72.0 dynes/cm

n_1 = number of drops of liquid = 55

n_2 = " " " " water = 25

$d_1 = 0.996 \, g/cm^3$

$d_2 = 0.800 \, g/cm^3$

$$\gamma_1 = \frac{\gamma_2 \times n_2 d_1}{n_1 d_2} = \frac{72 \times 25 \times 0.996}{0.800 \times 55}$$

$$\gamma_1 = 40.7 \text{ dynes/cm}$$

4. The surface tension of ethanol at 30 ºC is 2.189 X 10^{-2} N m^{-1} and its density = 0.780 g/cc. To what height will this liquid rise in a capillary tube of radius 0.002 cm? What pressure is needed to push the miniscus level back with the surrounding liquid?

Solution:

$$h = \frac{2\gamma}{rdg}$$

Given that $d = 0.780 \, g/cc$

$= 0.78 \times 10^3 \, kg/m^3$

$r = 0.002 \, cm = 2 \times 10^{-4} \, m$

$g = 9.8 \, m/s^2$

$\gamma = 2.189 \times 10^{-2} \, Nm^{-1}$

$$h = \frac{2 \times 2.189 \times 10^{-2}}{2.0 \times 10^{-4} \times 0.78 \times 10^3 \times 9.8} = 0.0286 \, m$$

$$h = 0.0286 \, m$$

We know that $p = hdg$

(pressure) $= 0.0286 \, m \times 0.78 \times 10^3 \times 9.8$

$$p = 218.6 \, N/m^2$$

5. The surface tension of water at 21ºC is 72.75 X 10^{-3} N m^{-1}. A 33.24% (vol./vol.) solution of ethanol has γ = 33.24 X 10^{-3} N m^{-1} at the same temperature. Given density (solution) = 0.9614 X 10^3 kg m^{-3} and density (water) = 0.9982 X 103 kg m^{-3}. How much less will the alcohol solution rise in the same capillary? Angle of contact, ϴ= 0º.

Solution: $\gamma = \frac{hrdg}{2}$

As the same capillary is used for both CH_3CH_2OH & H_2O r and g are the same in both

$$h = \frac{2\gamma}{rdg}$$

$\gamma_1 = 33.24 \times 10^{-3}\ N/m$

$\gamma_2 = 72.75 \times 10^{-3}\ N/m$

$d_1 = 0.9614 \times 10^{3}\ kg/m^3$

$d_2 = 0.9982 \times 10^{3}\ kg/m^3$

CH_3CH_2OH: $h_1 = \frac{2\gamma_1}{rd_1g}$

H_2O: $h_2 = \frac{2\gamma_2}{rd_2g}$

$$\frac{h_1}{h_2} = \frac{\gamma_1}{\gamma_2} \times \frac{d_2}{d_1}$$

$$\frac{h_1}{h_2} = \frac{33.24 \times 10^{-3}}{72.75 \times 10^{-3}} \times \frac{0.9982 \times 10^{3}}{0.9614 \times 10^{3}}$$

$$\frac{h_1}{h_2} = 0.474$$

or $\boxed{h_1 = 0.474\ h_2}$

Ethanol will rise only 47.4% of H_2O

6. In the determination of the surface tension of a liquid A by the drop number method, equal volumes of A and water gave 60 and 20 drops, respectively. Calculate the surface tension A if density of A and water are 0.896 and 0.964 g/cm^{-3} respectively. Given surface tension of water is 72.75×10^{-3} Nm^{-1}.

Solution: $\frac{\gamma_1}{\gamma_2} = \frac{n_2 d_1}{n_1 d_2}$

$\gamma_w = 72.75 \times 10^{-3}\ N/m$

$n_w = 20$

$n_A = 60$

$d_A = 0.896 \times 10^{3}\ kg/m^3$

$d_w = 0.964 \times 10^{3}\ kg/m^3$

$$\frac{\gamma_A}{\gamma_w} = \frac{n_w \times d_A}{n_A \times d_w}$$

$$\gamma_A = \frac{\gamma_w \times n_w \times d_A}{n_A \times d_w}$$

$$\gamma_A = \frac{72.75 \times 10^{-3} \times 20 \times 0.896 \times 10^{3}}{60 \times 0.964 \times 10^{3}}$$

$$\boxed{\gamma_A = 22.53 \times 10^{-3}\ N/m}$$