Dear Students

Sharing with you solutions to some typical questions from the chapter Application of Calculus.

Answer 19 : Ex. 7.2 JKT

$$x = \frac{200t - t^2}{20}$$

$$p = -0.1x + 70$$

$$px = -0.1x^2 + 70x$$

$$\frac{dR}{dx} = -0.2x + 70$$

$$\frac{dx}{dt} = \frac{1}{20}(200 - 2t)$$

$$\frac{dR}{dx} \cdot \frac{dx}{dt} = \left[-0.2\left(\frac{200t - t^2}{20}\right) + 70\right]\left[10 - \frac{t}{10}\right]$$

Put $t = 40$

$$MRP = \left[\left[\frac{-1}{100}(8000 - 1600)\right] + 70\right]\left[10 - \frac{40}{10}\right]$$

$$= [-64 + 70][10 - 4]$$

$$= (6)(6) = 36$$

Marginal Revenue Product is $\frac{dR}{dt}$

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(15) Answer Ex 7.2 JKT

$$p = 10e^{-x/400}$$

$$R = x\,10e^{-x/400}$$

$$MR = \frac{dR}{dx} = x \cdot \left(10e^{-x/400} \cdot -\frac{1}{400}\right) + 10e^{-x/400}$$

$$= 10e^{-x/400}\left(-\frac{x}{400} + 1\right) = 0$$

$$\therefore \quad x = 400$$

Ex 7.4 JKT Q.21 b

21(b) $x = 0.4K^{1.06}$

elasticity of x w.r.t. K = $\frac{\%\text{ change in } x}{\%\text{ change in } K}$,

$e_{xK} = \frac{dx}{dK} \cdot \frac{K}{x}$ {Diff. x w.r.t. K and substitute the value of x

we get

$\therefore e_{xK} = (\cancel{0.4}) K^{0.6} \cdot \frac{K \cdot (1.06)}{\cancel{0.4} K^{1.06}} = 1.06$

Thus, $\frac{3\%}{\%\text{ change in } K} = 1.06 \quad \therefore \%\text{ change in } K = \frac{3}{1.06} = 2.83\%$

Exercise 7.4

(16) Given cost in Rs./hour.
We need to know cost in Rs./Km.

$$\text{cost in Rs./Km} = \frac{\text{cost in Rs./hour}}{\text{Speed}}$$

$$\therefore \frac{Rs.}{hour \times \frac{Km.}{hour}} = \frac{Rs.}{Km.}$$

$c \propto s^2$ given.
$\Rightarrow c = k \cdot s^2$ (k is const.)
At $s = 15$, $c = 45$

$45 = k(15)^2 \quad \therefore k = \frac{45}{225} = \frac{1}{5}$.

$c = \frac{1}{5}s^2$ (cost in Rs./hour)

$C_1 = \frac{1}{5}s^2 \times \frac{1}{s}$ (running cost in Rs./km.)

$C_1 = \frac{1}{5}s$. (running cost in Rs./Km.)

$C_2 = \frac{500}{s}$ other costs in Rs./Km)

$\therefore$

$TC = \text{Total cost} = \frac{1}{5}s + \frac{500}{s}$

To find s, for TC to be minimum.

$$\frac{dTC}{ds} = \frac{1}{5} - \frac{500}{s^2} = 0$$

$$\Rightarrow s^2 = 2500 \Rightarrow s = 50 \text{ km/hour}$$

$$\frac{d^2TC}{ds^2} = \frac{1000}{s^3} > 0 \therefore TC \text{ is min at } s = 50 \text{ Km/hour}$$

$\therefore$ Min cost/Km. $= \frac{1}{5}(50) + \frac{500}{50} = 20$ Rs./Km.

For 100 km., cost $= 20 \times 100 =$ Rs. 2000.

Ex 7.4 , Q.2

$$x = 10000\, e^{-0.02p}$$

$$R = p \cdot x = 10000\, e^{-0.02p} \cdot p$$

$$\frac{dR}{dp} = p \cdot 10{,}000\, e^{-0.02p}(-0.02) + 10000\, e^{-0.02p} \cdot 1$$

$$\frac{dR}{dp} = p \cdot 10{,}000\, e^{-0.02p}(-0.02) + 10000\, e^{-0.02p} = 0 \quad \left[\text{for max}\right.$$

$$\Rightarrow -0.02p = -1$$

$$\therefore p = \frac{1}{0.02} = 50.$$

$\frac{d^2R}{dp^2}$ should be negative,

$$\frac{dR}{dp} = \left[p \cdot 10000(-0.02p)e^{-0.02p} + 10000\, e^{-0.02p}\right]$$

differentiate it w.r.t. to. p. & show it is -ve.
Apply product rule,

21(a) Ex 7.4 Q. 21(a)

$$R = p \cdot x$$

$$\frac{dR}{dx} = p \cdot \frac{dx}{dx} + x \frac{dp}{dx}$$

$$MR = AR + x \frac{dp}{dx} \times \frac{p}{p} = AR + \frac{AR}{-\eta}$$

$$\therefore MR = AR + \frac{AR}{-\eta} = AR\left[1 - \frac{1}{\eta}\right] \qquad \left[\because \eta = -\frac{p}{x}\frac{dx}{dp}\right.$$

$$\therefore MR - AR = \frac{AR}{-\eta}$$

$$\therefore \eta = \frac{AR}{AR - MR}$$

Note - you can be asked to verify for a given function & not the proo

Ex. 7.4 J.KT Q. 9

$p_x x + p_y y = TR.$

$p_x \cdot \frac{dx}{dx} + p_y \cdot \frac{dy}{dx} = \frac{dTR}{dx}$ [diff. both sides w.r.t. x]

$p_x + p_y \frac{dy}{dx} = 0$ For maxima

$\Rightarrow \frac{dy}{dx} = -\frac{P_x}{P_y} = f'(x)$

or $-f'(x) = \frac{P_x}{P_y}.$

Answer to Q.5

$$(15000-100x)(60+x)=R$$

$$9{,}00{,}000-6000x+15000x-100x^2=R$$

$$900000+9000x-100x^2=R$$

$$\frac{dR}{dx}=+9000-200x=0$$

$$\therefore \quad x=\frac{9000}{200}=45$$

$$\frac{d^2R}{dx^2}=-200<0 \Rightarrow$$

Rev. is max at $x=45$

$\therefore$ Optimum order size is

$60+45=105$ sets.