## Duration of Portfolio :-

Suppose there are m fixed-income securities with prices and durations of $P_i$ and $D_i$, respectively, $i = 1, 2, -, m$. all computed at a common yield. The portfolio consisting of the aggregate of these securities has price P and duration D, given by

$$P = P_1 + P_2 + \cdots + P_m$$

$$D = w_1 D_1 + w_2 D_2 + \cdots + w_m D_m$$

where $w_i = \dfrac{P_i}{P}$, $i = 1, 2, \ldots, m$.

**Immunization** :- The procedure of structuring of a bond portfolio to protect against interest rate risk is termed immunization.

Ex:- Suppose we have to pay some one \$108 next year, ~~then~~ and we ~~can~~ have two options,

One, to fix deposit some amount that will give 108 next year (1 year)

At 10% interest rate, we need to invest 98.18 today.

Second option, is to buy zero coupon bond of maturity 10 years with face value 300 and yield 11%.

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Then the price of the bond is 96.5, and if sell the bond next year at 12% yield, we get 108. But if the yield increases next year, the price of bond also decreases and we get 85.27 (for yield 15%.)

Now suppose we face a series of cash obligation and we wish to acquire a portfolio that ~~you~~ we will use to pay these obligations as they arise.
One way, is to buy zero coupon bonds (Not feasible)
And other is to buy bonds of present value same as present value of obligations (Risk involve)
Immunization solves this problem by matching durations as well as present values. If the duration of the portfolio matches that of obligation cash flow streams, then the cash value of portfolio and present value of the obligation stream will respond identically to a change in yield.

__Example!__- Suppose X corporation has an obligation to pay $ 1 million in 10 years. It wishes to invest money now that will be sufficient to meet this obligation.

## Bond choices

| | Rate | Maturity | Price | Yield |
|---|---|---|---|---|
| Bond 1 | 6% | 30 yr | 69.04 | 9.00% |
| Bond 2 | 11% | 10 yr | 113.01 | 9.00% |
| Bond 3 | 9% | 20 yr | 100.00 | 9.00% |

Now, $D_1 = 11.44$ $\quad D_2 = 6.54$ $\quad D_3 = 9.61$

If we have to choose only two bonds then we cannot choose $D_2$ & $D_3$ as then the duration of portfolio cannot become 10.

So, we choose $D_1$ and $D_2$

then Net present value of obligation is \$414,643

then Value of bond 1 + Value of bond 2 = 414,643.

$$V_1 + V_2 = 414,643 \quad \text{—(1)}$$

also, duration of portfolio = duration of obligation

$$\frac{D_1 V_1 + D_2 V_2}{PV} = 10$$

$$D_1 V_1 + D_2 V_2 = 10 \times PV \quad \text{—(2)}$$

solving (1) & (2) for $V_1$ & $V_2$, we get

$V_1 = \$292,788$ $\qquad V_2 = \$121,854$

So, we should buy 4241 shares of bond (1) and 1078 shares of bond (2).

## Convexity :-

Modified duration measures the relative slope of the price-yield curve at a given point and this leads to a straight-line approximation to the price-yield curve.

An even better approximation can be obtained by including a second-order term. This second order term is based on convexity, which is relative curvature at a given point on the price yield curve.

So Convexity is the value defined as

$$C = \frac{1}{P}\frac{d^2P}{d\lambda^2}$$

which can be expressed in terms of cash flows

$$C = \frac{1}{P}\sum_{k=1}^{n}\frac{d^2PV_k}{d\lambda^2}$$

Assuming m coupons per periods, we have

then $$\frac{dPV_k}{d\lambda} = -\frac{k/m}{1+(\lambda/m)}PV_k$$

$$\frac{d^2PV_k}{d\lambda^2} = -\frac{k}{m}\left(\frac{(1+\lambda/m)\frac{dPV_k}{d\lambda} - PV_k \times 1/m}{(1+\lambda/m)^2}\right)$$

$$\frac{d^2PV_k}{d\lambda} = \frac{-k}{m}\left(\frac{-k/m\, PV_k - \frac{PV_k}{m}}{(1+\lambda/m)^2}\right)$$

$$= \frac{-k}{m}\left[\frac{-\frac{PV_k}{m}(k+1)}{(1+\lambda/m)^2}\right]$$

$$= \frac{k(k+1)\; PV_k}{m^2(1+\lambda/m)^2}$$

$$\therefore \quad C = \frac{1}{P}\sum_{k=1}^{n}\frac{d^2PV_k}{d\lambda^2}$$

$$\boxed{C = \frac{1}{P(1+\lambda/m)^2}\sum_{k=1}^{n}\frac{k(k+1)}{m^2}\frac{c_k}{(1+\lambda/m)^k}}$$

Convexity has unit of time squared.

$$\Delta P \approx -D_m P\Delta\lambda + \frac{PC}{2}(\Delta\lambda)^2 .$$

<u>Q5</u> Value to be paid by the Corporation to bond holder after 5 years = face value + 5%

= 105.

To make this call advantageous, Price of bond should be greater than 105. Thus.

$$\frac{100}{(1+\lambda)^{15}} - \frac{10}{\lambda}\left\{1 - \frac{1}{(1+\lambda)^{15}}\right\} > 105$$

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this gives $y \neq 9.366$

at $\lambda = 9$ $\quad P = 108.$

$\lambda = 9.3$ $\quad P = 105.54.$

Q12 (a) Bond A = 885.83
Bond B = 771.677
Bond C = 657.516
Bond D = 869.5665

(b) $D_A = 2.718$ year $\quad D_C = 3$ yrs
$D_B = 2.838$ year $\quad D_D = 1$ yr.

(c) $D_M(A) = 2.3634$

$\Delta P \approx 2.3634$

$D_M(B) = 2.467$

$\Delta P \approx 2.467$

$D_M(C) = 2.608$

$D_M(D) = 0.869.$

(d) $V_A + V_B + V_C + V_D = 1512.28$

and $D_A V_A + D_B V_B + D_C V_C + D_D V_D = 1512.28 \times 2.$

(v) We chose D,

$V_C + V_D = 1512.28$

$3V_C + V_D = 2 \times 1512.28$

$\Rightarrow$ $V_C = 756.14$ share = 1

$V_D = 756.14$ share = 1