Forward Contracts :-

A derivative can be defined as a financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables. Often the variables underlying derivatives are the prices of traded assets. Ex:- A stock option, is a derivative whose value is dependent on the price of a stock.

The main types of derivative securities are forward contracts, future contracts, options, options on futures and swaps.

Forward Contract :-

A forward contract on a commodity is a contract to purchase or sell a specific amount of the commodity at a specific price and at a specific time in future. for example, A forward contract to purchase 1,000kg of sugar at Rs40/kg next year. One of the parties to a forward contract assumes a long position and agrees to buy the underlying asset on a certain specified future date for a certain price. The other party assumes a short position and agrees to sell the asset on the same date for same price.

forward contracts can be used to hedge foreign currency risk.

Future Contract :-

Like a forward contract, a future contract is an agreement between two parties to buy or sell an asset at certain time in the future for a certain price. Unlike forward contracts, future contracts are normally traded on exchange.

Options :- Options are traded both on exchanges and in the over-the-counter market. There are two types of options

A call option gives the holder the right to buy the underlying asset by a certain date for a certain price.

A put option gives the holder the right to sell the underlying asset by a certain date for a certain price.

The price in the contract is known as strike price or exercise price. The date of contract is known as maturity.

(An option is the right, but not the obligation)

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Ch-3 Hedging strategies Using futures

Hedging is the process of reducing the financial risks that arise in the course of normal business operations or are associated with investment.

A perfect hedge is the one that completely eliminate the risk.

Short Hedge:- A short hedge is a hedge, that involves a short position in future contracts.

A short hedge is appropriate when the hedger already owns an asset and expects to sell it at some time in the future.

Ex:- Suppose an oil producer has negotiated a contract to sell 1 million barrels of crude oil.

Gain $10,000 for each 1 cent increase in the price.

loss $10,000 for each 1 cent decrease in the price

Suppose spot price is $80 per barrel and future price is $79 per barrel.

long for Hedge:- Hedges that involves taking a long position in a future contract are known as long hedges. A long hedge is appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in a price now.

Ex:- Suppose a copper fabricator knows it will require 100,000 kgs of copper in next 6 months.
Spot price of copper is 340 cents per kg.
future price after six month is 320 cents per kg

Arguments for and against Hedging
Basis = spot price of asset – future price of asset

Cross hedging:- Hedging a cash commodity by buying a different but related future contract.
~~Cross hedging is employed where there is no~~

Ex:- Suppose someman has a long position in (wants to buy) some Gold Company. (say abc)
suppose future prices of abc are not there but price of abc is related with the price of Gold.

Basis Risk

Basis = Spot price of asset to be hedged
- future price of contract used.

Hedge ratio:- It is the ratio of the size of the position taken in future contracts to the size of exposure.

If the asset underlying the future contract is same as the asset being hedged, then hedge ratio is 1.

Calculating minimum - variance Hedge ratios

ΔS = Change in spot price

ΔF = Change in future price

then

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

where σ_S is the standard deviation of ΔS, σ_F is the standard deviation of ΔF, and ρ is the coefficient of correlation between two.

then, if $\rho = 1$ & $\sigma_S = \sigma_F$, then $h^* = 1$.

if $\rho = 1$ & $\sigma_F = 2\sigma_S$, then $h^* = 0.5$.

Optimal no. of Contracts

Q_A : Size of position being hedged (units)

Q_F : Size of one future contract (unit

N^* = Optimal no. of future contracts for hedging.

$$N^* = \frac{h^* Q_A}{Q_F}$$

Tailing the hedge:- When futures are used for hedging, tailing of the hedge can be allowed for the impact of daily settlement.

$$N^* = \frac{h^* V_A}{V_F}$$

where V_A = dollar value of position being hedged

V_F = dollar value of 1 future contract.

Ex:- An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging.

then $\sigma_F = 0.0313$ $\sigma_S = 0.0263$ $\rho = 0.928$

$$h^* = 0.928 \times \frac{0.0263}{0.0313} = 0.777.$$

Month	ΔF	ΔS
1	0.021	0.029
2	0.035	0.020
3	−0.046	−0.044
4	+0.001	0.008
5	0.044	0.026
6	−0.029	−0.019
7	−0.026	−0.010
8	−0.029	−0.007
9	0.048	0.043
10	−0.006	0.011
11	−0.036	−0.036
12	−0.11	−0.018
		0.009
13	0.019	
14	−0.027	−0.032
15	0.029	0.023

Let size of one future contract is 42,000 Gallon

and $N^* = \frac{0.777 \times 2,000,000}{42,000} = 37.03$

Nearest whole no. = 37.

By Tailing the Hedge,

Let spot price and future prices are \$1.94 & \$1.99 per gallon, then.

$V_A = 2,000,000 \times 1.94 = 3,880,000$

$V_F = 1.99 \times 42,000 = 83,580$

$N^* = \frac{0.7777 \times 3,880,000}{83,580} = 36.10$

No $\longrightarrow$ 36.

Stock Index futures:-

A stock index tracks changes in the value of a hypothetical portfolio of stocks.

Hedging an equity Portfolio:-

V_A = Current value of portfolio

V_F = Current value of one future contract

$$N^* = \beta \frac{V_A}{V_F}$$

where $\beta \rightarrow$ Beta of portfolio.

Exh ~~Value of S&P 500 index = 1,000~~

S&P 500 future price = 1,010

Value of portfolio = \$ 5,050,000

Beta of portfolio = 1.5

Suppose One future contract \$250 times index

then $N^* = 1.5 \times \frac{V_A}{V_F}$; V_A = 5,050,000

$\therefore$ V_F = 250 × 1,010 = 252,500

$N^* = 30$.

If $\beta = 1$, then $N^* = 20$.

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Stack and Roll

Sometimes the expiration date of Hedge is later than the delivery dates of all the future contracts that can be used. The Hedger must then roll the hedge forward by closing out one future contract and taking the same future position in a future contract with a later delivery date. The procedure is known as stack & roll.

Ex:- Suppose that in April 2011 a company realises that it will have 100,000 barrels of oil to sell in June 2012. (decides to hedge with ratio 1)

the current spot price is \$69.

April 2011 → \$68.20.

↓

Sept. 2011 → \$67.40

↓

March. 2012 \$66.50

↓

June 2012 \$65.90.

Spot price in June = \$66.00.

dollar gain per barrel,

(68.20 − 67.40) + (67 − 66.50) + (66.30 − 65.90)

= 1.70.

Comparison of forward and future contracts

Forward	Future
1.) Private contract between two parties.	1.) Traded on exchange
2.) Not standardized	2) Standardized contract
3) Usually one specified delivery date	3) Ranges of delivery date.
4.) Settled at end of contract	4.) Settled daily.
5.) Delivery or final cash settlement usually takes place	5) Contract is usually closed out prior to maturity.
6) Some Credit risk	6) Virtually no credit risk.

Forward Price for an investment asset :-

An investment asset is an asset that is held for investment purposes by significant no: of investors. Stocks & bonds are investment asset. Gold & silver are also investment asset.

A Consumption asset is an asset that is held primarily for consumption. e.g. copper, oil etc.

Assumptions :-

1.) The market participants are subject to no transaction cost when they trade.

2.) The market participants are subject to the same tax rate on all net trading profits.

3) The market participants can borrow money at the same risk-free rate of interest as they can lend money.

4) The market participants take advantage of arbitrage opportunities as they ~~co~~ occur.

Consider a forward contract on an investment asset with price S_0 that provides no income. Let T is the time to maturity, r is the risk-free rate, and F_0 is the forward price, then.

$$F_0 = S_0 e^{rT} \quad — \quad (*)$$

Ex:- ① Consider a long forward contract to purchase a non-divident-paying stock in 3 months. Assume $S_0 = \$40$, $r = 5\%$ per annum.

then $F_0 = 40 e^{0.05 \times 3/12} = \$40.50$

Suppose forward price is \$43. → borrow \$40 at risk free rate 5%.

then buy one share, and short a forward contract.
→ me also

Ex-② Consider a 4-month forward contract to buy a zero coupon bond that will mature in 1 year from now.

Let $S_0 = \$930$. 4 month-risk free rate is 6%. per ann

$r = 0.06 \quad T = 4/12$

$$F_0 = 930 e^{0.06 \times 4/12} = \$948.79.$$

If forward price is \$39, then short one share, invest that at 5% per annum for 3 months, and take a long position in 3-month forward contract.

Suppose that the underlying asset has no storage costs or income. If $F_0 > S_0 e^{rT}$, then

(i) Borrow S_0 dollars at an interest rate r for T years.

(ii) Buy 1-unit of ~~share~~ asset.

(iii) Short a forward contract on 1 unit of asset

$$Profit = F_0 - S_0 e^{rT}.$$

If $F_0 < S_0 e^{rT}$, then

(i) sell the asset for S_0.

(ii) Invest the proceeds at interest rate r for time T.

(iii) Take long position in a forward contract on 1 unit of the asset.

$$profit = S_0 e^{rT} - F_0$$

With income:

~~Sono~~ Suppose an investment asset will provide income with present value I during the life of a forward contract, then

$$F_0 = (S_0 - I) e^{rT}$$

Ex:- Consider a long forward contract to purchase a coupon-bearing bond whose current price is \$900. Suppose T = 9 months, suppose coupon payment of \$40 after 4 months.

4 month risk-free rate = 3% per annum

9 ———————— = 4%

then, $S_0 = \$900$, $I = 40e^{-0.03\times 4/12} = 39.60$ $r = 0.04$ $T = 9/12$

$F_0 = (900 - 39.60)e^{0.04\times 0.75} = \$886.60$.

Forward price = \$910

Action now:-

Borrow \$900; \$39.60 for 4 months and \$860.40 for 9 months

Buy 1 unit of asset

Enter into forward contract to sell asset in 9 months for \$910.

Action in 4-months:

Receive \$40 of income on asset

Use \$40 to repay first loan with interest.

Action in 9 months

sell asset for \$910

Use \$886.60 to repay 2nd loan

Profit :- \$23.40

Forward price = \$870

Action now:-

Short 1 unit of asset to realize \$900

Invest \$39.60 for 4 months and \$860.40 - 9 months.

Enter into forward contract to buy asset in 9 months for \$870.

Action in 4 months:-

Receive \$40 from 4-month investment

Pay income of \$40 on asset.

Action in 9 months

Receive \$886.60 from 9 months

Buy asset for \$870

Close out short position.

Profit:- \$16.60

Ex: Consider 10-month forward contract, $S_0 = \$50$
$0.75 are expected after 3 months, 6 months & 9 months
$r = 8\%$ per annum (for all)
then $I = 0.75\, e^{-0.08 \times 3/12} + 0.75\, e^{-0.08 \times 6/12} + 0.75\, e^{-0.08 \times 9/12}$

$= 2.162.$

$$F_0 = (50 - 2.162)\, e^{0.08 \times 10/12} = \$51.14.$$

With yield:-

Define q as the average yield per annum on an asset during the life of a forward contract with continuous compounding. then

$$F_0 = S_0\, e^{(r-q)T}.$$

Ex: Consider a 6 month contract on an asset expected to provide income 2% per annum of asset price once during 6-month period.

$r = 10\%$ per annum. $S_0 = \$25$

then $q = 0.0396$

$$F_0 = 25\, e^{(0.10 - 0.0396) \times 0.5} = \$25.77$$

Valuing forward contracts:

Long forward contract,

$$f = (F_0 - K)\, e^{-rT} \quad \text{—— } (*)$$

where f is value of forward contract today
i.e. forward price of today.

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K is delivery price of contract that was negotiated some time ago. delivery date is T years from today.
r is the T-year risk free rate.

If <u>short forward contract</u>

then $f = (K - F_0)e^{-rT}$

Sub ① long forward contract on a non-divided paying stock was entered into some time ago.

T = 6 month. r = 10%. $S_0 = \$25$, $K = \$24$.

$F_0 = 25e^{0.1 \times 0.5} = \$26.28$

$f = (26.28 - 24)e^{-0.1 \times 0.5} = \$2.17$.

(*) Can be written as. If no income,

$f = S_0 - Ke^{-rT}$

If I is present value of income then.

$f = S_0 - I - Ke^{-rT}$

If q is yield, then

$f = S_0 e^{-qT} - Ke^{-rT}$