PRODUCTION OF LOW TEMPERATURE BY ADIABATIC DEMAGNETIZATION

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THEORY

$$dW = -B\,dM \quad \text{— (1)}$$

$$dQ = dU + dW$$

$$T\,dS = dU - B\,dM \quad \text{— (2)}$$

Maxwell's third thermodynamic relation

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$-\left(\frac{\partial T}{\partial B}\right)_S = \left(\frac{\partial M}{\partial S}\right)_B = \frac{\left(\frac{\partial M}{\partial T}\right)_B}{\left(\frac{\partial S}{\partial T}\right)_B} \quad \text{— (3)}$$

$$T\left(\frac{\partial S}{\partial T}\right)_B = C_B$$, the specific heat at const B

$$\Rightarrow \left(\frac{\partial T}{\partial B}\right)_S = -\frac{T}{C_B}\left(\frac{\partial M}{\partial T}\right)_B \quad \text{--- (4)}$$

$$dT = \frac{-T}{C_B}\left(\frac{\partial M}{\partial T}\right)_B dB \quad \text{--- (5)}$$

Curie's law of Paramagnetism

$$\left\{\phi = \frac{C}{V}\right\}$$

$$M \propto \frac{B}{T}$$

$$\Rightarrow M = \phi \frac{B}{T} \quad \text{--- (6)}$$

$$\left(\frac{\partial M}{\partial T}\right)_B = -\frac{\phi B}{T^2} \quad \text{--- (7)}$$

$$dT = -\frac{T}{C_B}\left(-\frac{\phi B}{T^2}\right) dB$$

$$T\,dT = \left(\frac{\phi}{C_B}\right) B\,dB$$

$$\int T\,dT = \frac{\phi}{C_B}\int_B^0 B\,dB$$

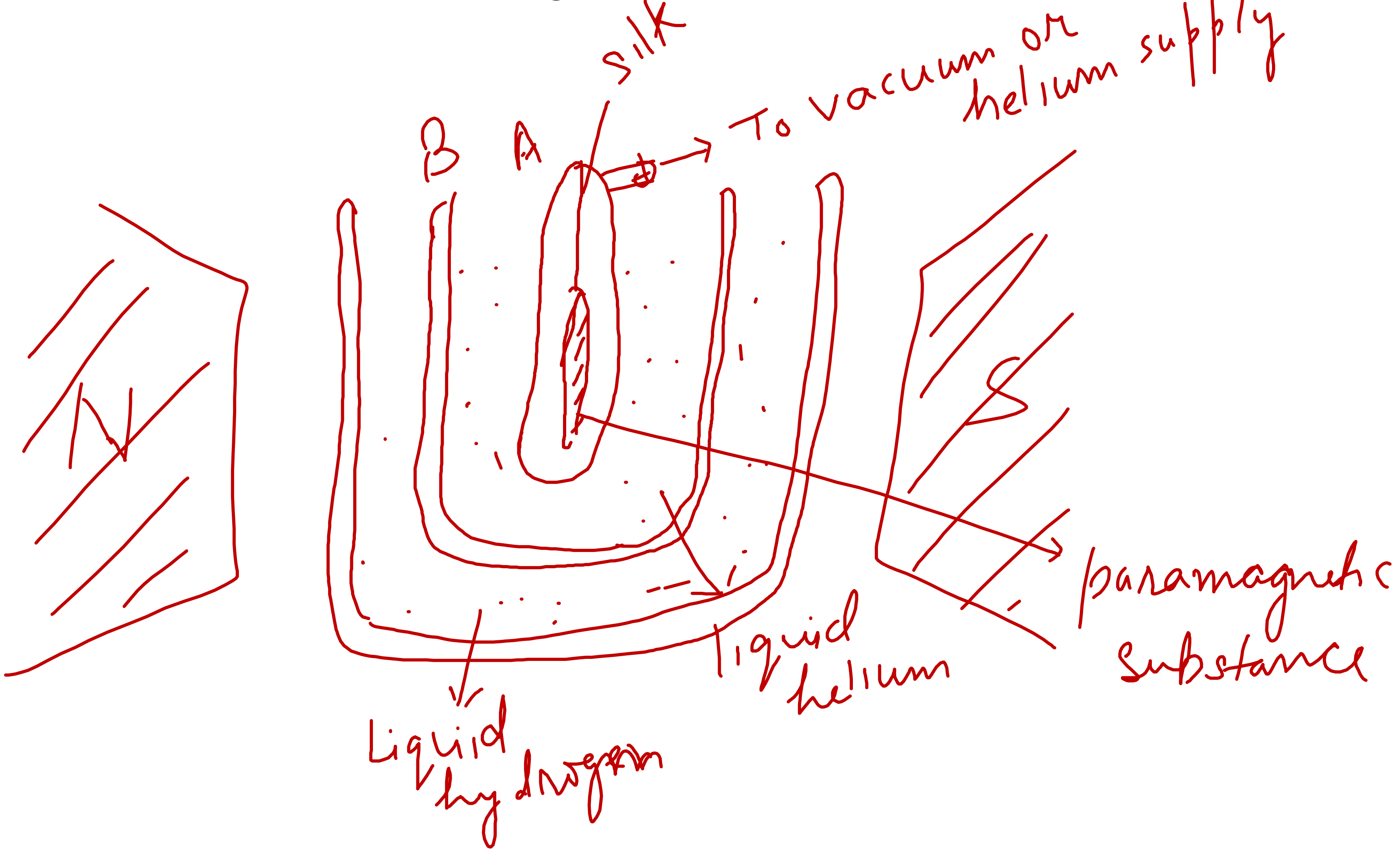
$$\frac{1}{2}\left(T_f^2 - T_i^2\right) = -\frac{\phi B^2}{2\,C_B}$$

$$\Rightarrow T_f^2 - T_i^2 = -\frac{\phi}{C_B} B^2$$

$$\Rightarrow \boxed{T_f - T_i = \Delta T = -\frac{\phi}{2 C_B T_{av}} B^2} \quad \text{--- (8)}$$

where $T_{av} = \dfrac{T_f + T_i}{2}$

EXPERIMENTAL METHOD



RELATION BETWEEN TWO SPECIFIC HEATS OF A GAS

$$C_p = \left(\frac{\partial Q}{\partial T}\right) = T\left(\frac{\partial S}{\partial T}\right)_p \quad \text{— (1)}$$

$$C_v = \left(\frac{\partial Q}{\partial T}\right)_v = T\left(\frac{\partial S}{\partial T}\right)_v \quad \text{— (2)}$$

$$dS = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial V}\right)_T dV$$

$$\Rightarrow \left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{\partial s}{\partial T}\right)_v + \left(\frac{\partial s}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \quad \text{— (3)}$$

$$\left(\frac{\partial s}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$$

$$\left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{\partial s}{\partial T}\right)_v + \left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p$$

$$T\left(\frac{\partial s}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_v + T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p$$

$$C_p = C_v + T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p$$

$$\Rightarrow \boxed{C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p} \quad — (4)$$

For a perfect gas $pv = RT$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v} \quad \& \quad \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p}$$

$$\Rightarrow C_p - C_v = \frac{TR^2}{pv} = \frac{TR^2}{RT} = R$$

$$\boxed{C_p - C_v = R} \quad — (5)$$

For a real gas

$$\left(P+\frac{a}{V^2}\right)=\frac{RT}{V-b}$$

$$\left(\frac{\partial P}{\partial T}\right)_V=\frac{R}{V-b} \quad — (6)$$

$$\left(\frac{\partial V}{\partial T}\right)_P=\frac{R/(V-b)}{\frac{RT}{(V-b)^2}-\frac{2a}{V^3}} \quad — (7)$$

$$C_p-C_v=\frac{T\left(\frac{R}{V-b}\right)\left(\frac{R}{V-b}\right)}{\frac{RT}{(V-b)^2}-\frac{2a}{V^3}}=\frac{R}{\left(1-\frac{2a}{V^3}\frac{(V-b)^2}{RT}\right)}=R\left[1-\frac{2a}{RTV}\right]^{-1}$$

$$\boxed{C_p-C_v=R\left[1+\frac{2a}{RTV}\right]} \quad — (8)$$

THANKYOU