

# Lecture 2

A dark blue diagonal gradient bar that starts from the bottom left corner and extends towards the top right corner, covering the lower half of the slide.

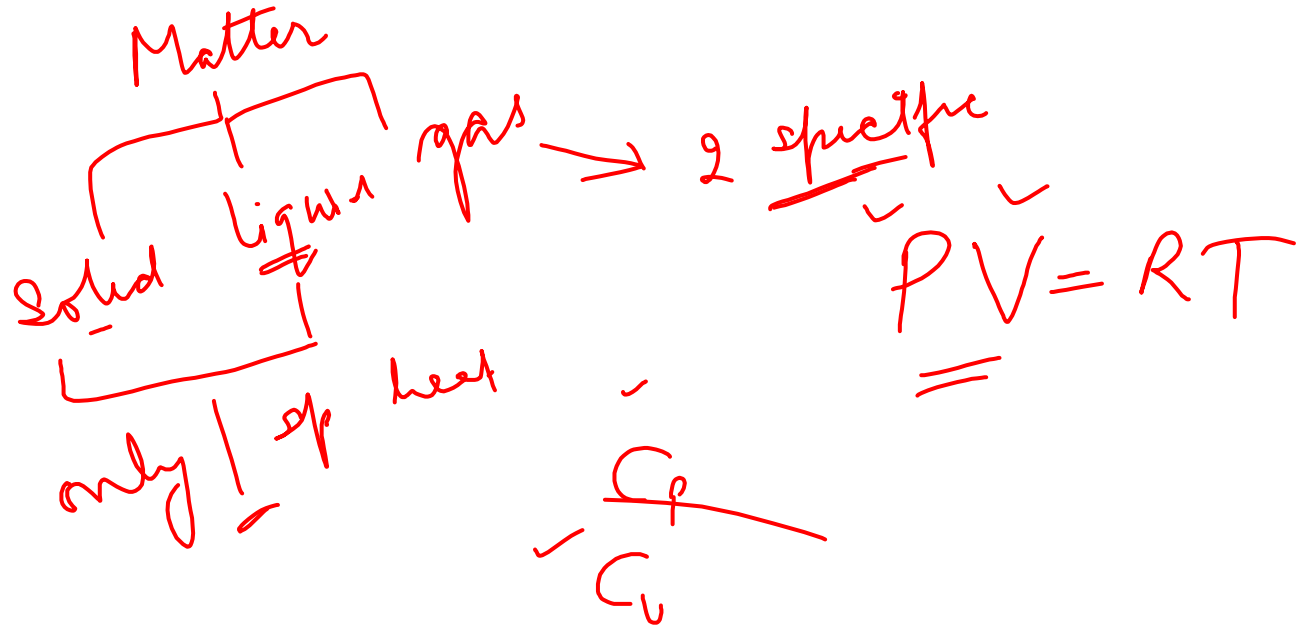
# Specific heat

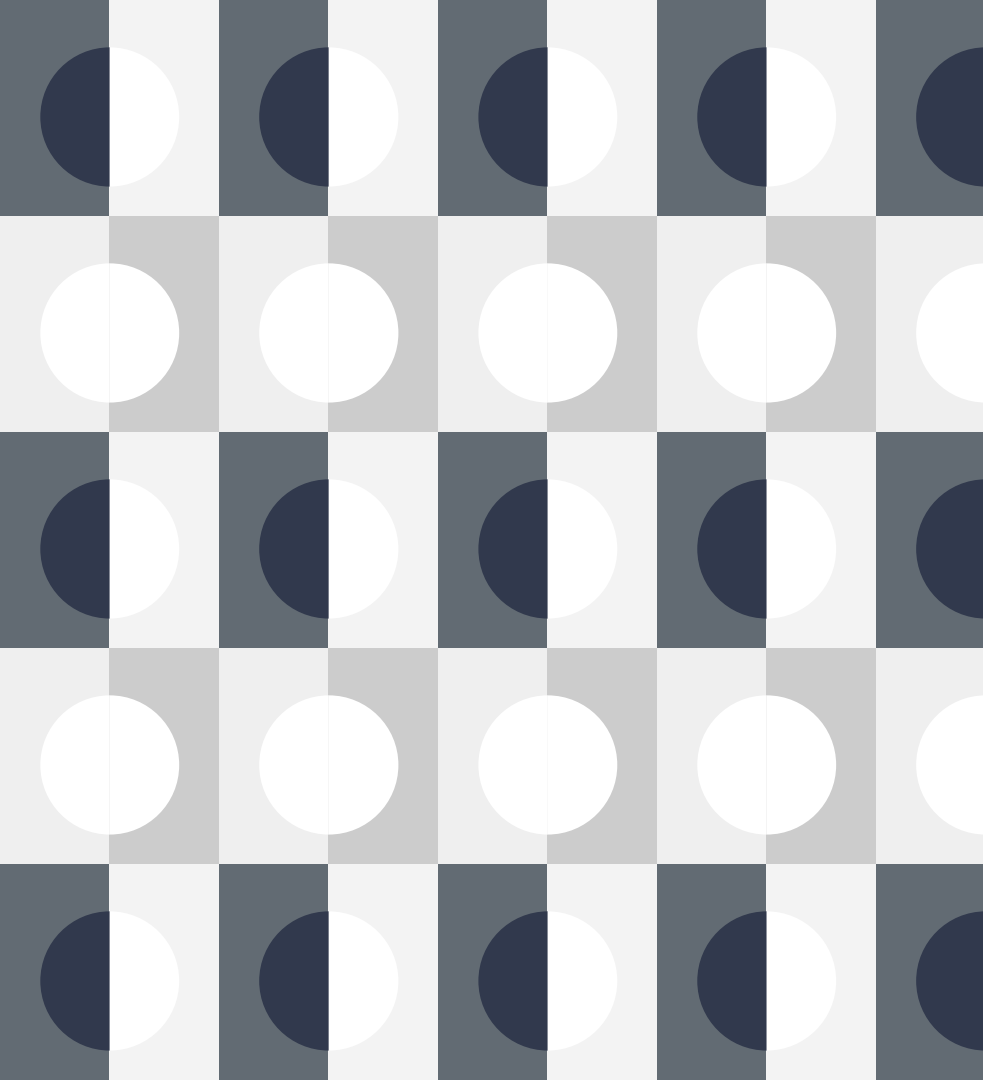
The **specific heat** is the amount of **heat** per unit mass required to raise the temperature by one degree Celsius. The relationship between **heat** and temperature change is usually expressed in the form shown below where  $c$  is the **specific heat**.

$$Q = cm\Delta T$$

heat added      specific heat      mass      change in temperature

# Why do gases have two specific heat capacities?





# Relation between $C_p$ and $C_v$ for ideal gases

$$u = u(v, T) \quad - (1)$$

$$du = \left( \frac{\partial u}{\partial T} \right)_v dT + \left( \frac{\partial u}{\partial v} \right)_T dv \quad - (2)$$

$$\boxed{\delta Q = du + \delta W} \quad - (3)$$

$$\delta W = P dv$$

$$\delta Q = \underline{du} + P dv$$

$$\delta Q = \left[ \left( \frac{\partial u}{\partial T} \right)_v dT + \left( \frac{\partial u}{\partial v} \right)_T dv \right] + P dv \quad - (4)$$

$dT$

$$\frac{\delta Q}{dT} = \left( \frac{\partial u}{\partial T} \right)_v + \left( \frac{\partial u}{\partial v} \right)_T \frac{dv}{dT} + \frac{P dv}{dT}$$

$$\Rightarrow \frac{\delta Q}{dT} = \left( \frac{\partial u}{\partial T} \right)_v + \left[ P + \left( \frac{\partial u}{\partial v} \right)_T \right] \frac{dv}{dT} \quad - (5)$$

$$\left( \frac{\partial Q}{\partial T} \right)_v = C_v$$

$$\frac{dV}{dT} = 0$$

$$\left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = C_V \quad \text{--- (6)}$$

$$\left(\frac{\partial Q}{\partial T}\right)_P = C_P$$

$$\text{(5)} \Rightarrow C_P = \left(\frac{\partial U}{\partial T}\right)_V + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right] \left(\frac{\partial V}{\partial T}\right)_P$$

$$C_P = C_V + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right] \left(\frac{\partial V}{\partial T}\right)_P$$

$$\Rightarrow C_P - C_V = \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right] \left(\frac{\partial V}{\partial T}\right)_P \quad \text{--- (7)}$$

$$du = 0$$

$$\left( \frac{\partial u}{\partial v} \right)_T = 0$$

$$\Rightarrow \left[ \begin{array}{l} pV = RT \\ p \left( \frac{\partial v}{\partial T} \right)_p = R \quad - (7) \\ \left( \frac{\partial v}{\partial T} \right)_p = \frac{R}{p} \end{array} \right]$$

$$(7) \Rightarrow C_p - C_v = p \left( \frac{\partial v}{\partial T} \right)_p + \left( \frac{\partial u}{\partial v} \right)_T \left( \frac{\partial v}{\partial T} \right)_p$$
$$= p \times \frac{R}{p}$$

$$\Rightarrow \boxed{C_p - C_v = R} \quad - (8)$$

# Relation between Adiabatic and Isothermal Elasticities

$$E = \frac{\text{stress}}{\text{strain}} = \frac{dP}{-dV/V}$$

(a) Isothermal Elasticity

$$PV = \text{Const.}$$

$$\Rightarrow P dV + V dP = 0$$

$$\Rightarrow \frac{V dP}{-dV} = P$$

$$\boxed{E_{\text{iso}} = P} \quad \text{--- (1)}$$



## ② Adiabatic Elasticity

$$pV^\gamma = \text{const}$$

$$\text{Diff} \rightarrow \gamma p V^{\gamma-1} dV + V^\gamma dp = 0$$

$$\Rightarrow \frac{V dp}{-dV} = \gamma p$$

$$\boxed{E_{adi} = \gamma p} \quad - \text{②}$$

$$\boxed{E_{adi} = \gamma E_{iso}} \quad - \text{③}$$