Lecture 2

Specific heat

The **specific heat** is the amount of **heat** per unit mass required to raise the temperature by one degree Celsius. The relationship between **heat** and temperature change is usually expressed in the form shown below where c is the **specific heat**.

$$\underbrace{Q}_{\text{heat added}} = \underbrace{c}_{\text{specific heat}}\ \underbrace{m}_{\text{mass}}\ \underbrace{\Delta T}_{\text{change in temperature}}$$

Why do gases have two specific heat capacities?

Matter

Solid liquid gas → 2 specific

only 1 sp heat

$PV = RT$

C_p

C_v

Relation between Cp and Cv for ideal gases

$$U = U(V, T) \quad — (1)$$

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \quad — (2)$$

$$\boxed{\delta Q = dU + \delta W} \quad — (3)$$

$$\delta W = P\,dV$$

$$\delta Q = dU + P\,dV$$

$$\delta Q = \left[\left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV\right] + P\,dV \quad — (4)$$

dT

$$\frac{\delta Q}{dT} = \left(\frac{\partial U}{\partial T}\right)_V + \left(\frac{\partial U}{\partial V}\right)_T \frac{dV}{dT} + \frac{P\,dV}{dT}$$

$$\Rightarrow \boxed{\frac{\delta Q}{\partial T} = \left(\frac{\partial U}{\partial T}\right)_V + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right] \frac{dV}{dT}} \quad — (5)$$

$$\left(\frac{\partial Q}{\partial T}\right)_V = C_V$$

$$\frac{dV}{dT} = 0$$

$$\boxed{\left(\frac{\delta Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = C_V} \quad \text{--- } (6)$$

$$\left(\frac{\delta Q}{\partial T}\right)_P = C_P$$

$$(5) \Rightarrow \quad C_P = \left(\frac{\partial U}{\partial T}\right)_V + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_P$$

$$C_P = C_V + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_P$$

$$\Rightarrow \boxed{C_P - C_V = \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_P} \quad \text{--- } (7)$$

$du = 0$

$$\boxed{\left(\frac{\partial u}{\partial v}\right)_T = 0}$$

$PV = RT$

$$\Rightarrow P\left(\frac{\partial v}{\partial T}\right)_P = R \quad \text{— (7)}$$

$$\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P}$$

$$(7) \Rightarrow C_p - C_v = P\left(\frac{\partial v}{\partial T}\right)_P + \underline{\underline{\left(\frac{\partial u}{\partial v}\right)_T\left(\frac{\partial v}{\partial T}\right)_P}}$$

$$= P \times \frac{R}{P}$$

$$\Rightarrow \boxed{C_p - C_v = R} \quad \text{— (8)}$$

Relation between Adiabatic and Isothermal Elasticities

$$E = \frac{\text{stress}}{\text{strain}} = \frac{dP}{-dV/V}$$

(a) Isothermal Elasticity

$$PV = \text{Const.}$$

$$\Rightarrow P\,dV + V\,dP = 0$$

$$\Rightarrow \frac{V\,dP}{-dV} = P$$

$$\boxed{E_{iso} = P} \quad \text{—} (1)$$

② <u>Adiabatic</u> <u>Elasticity</u>

$$PV^{\gamma} = const$$

Diff $\rightarrow \gamma P V^{\gamma-1} dV + V^{\gamma} dP = 0$

$$\Rightarrow \frac{V\,dP}{-dV} = \gamma P$$

$$\boxed{E_{adi} = \gamma P} \quad \text{— } ②$$

$$\boxed{E_{adi} = \gamma\, E_{iso}} \quad \text{— } ③$$