$$= \frac{(4^n)\sqrt{2}}{\sqrt{n}\,\sqrt{2\pi}} = \frac{4^n}{\sqrt{\pi n}}$$

Using it in eq^n (3), we have.

$$\sum_{n=1}^{\infty} P_{00}^{n} = \sum_{n=1}^{\infty} \frac{4^n}{\sqrt{\pi n}} \big(p(1-p)\big)^n$$

$$\Rightarrow \sum_{n=1}^{\infty} P_{00}^{n} = \sum_{n=1}^{\infty} \frac{\big(4p(1-p)\big)^n}{\sqrt{\pi n}} \quad \text{—— (4)}$$

<u>Case - II</u> If $p = 1/2$.

$$\sum_{n=1}^{\infty} P_{00}^{n} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi n}} \quad \text{—— (5)}$$

Ratio test.

$$\lim \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{\sqrt{\pi(n+1)}} \times \frac{\sqrt{\pi n}}{1}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{\sqrt{\pi(n+1)}} \times \frac{\sqrt{\pi n}}{1}$$

$$= \lim_{n\to\infty} \sqrt{\frac{n}{n+1}} = 1.$$

fails.

<u>nth root test:</u>

$$\lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \left(\frac{1}{\sqrt{\pi n}}\right)^{1/n}$$

$$\text{let } S = \lim_{n\to\infty} \left(\frac{1}{\sqrt{n\pi}}\right)^{1/n}$$

$$\Rightarrow \ln S = \lim_{n\to\infty} \frac{1}{n} \ln\left(\frac{1}{\sqrt{n\pi}}\right)$$

$$\Rightarrow \ln S = \lim_{n\to\infty} -\frac{1}{2n} \ln(n\pi)$$

$$= \lim_{n\to\infty} -\frac{1}{2n} \ln n$$

$$= \lim_{n\to\infty} -\frac{1}{2}\left(\frac{1/n}{1}\right) = 0$$

$$\Rightarrow \ln S = 0 \Rightarrow S = e^{0} = 1 .$$

$\therefore\ 4p(1-p) \leq 1$ with equality if and only if $p = 1/2$.

the series $\displaystyle\sum_{n=1}^{\infty} \frac{(4p(1-p))^{n}}{\sqrt{\pi n}}$ is convergent for $p \neq 1/2$

and divergent for $p = 1/2$.

$\sum \frac{1}{\sqrt{n\pi}} \to$ divergent

$\frac{1}{\sqrt{n}} \to$ divergent.

$\frac{1}{n^p} \to$ diverge $0 < p \le 1$

$\frac{1}{n^p} \to$ convergent for $p > 1$.

$$\therefore \sum_{n=1}^{\infty} P_{00}^{n} < \infty \text{ for } p \neq 1/2$$

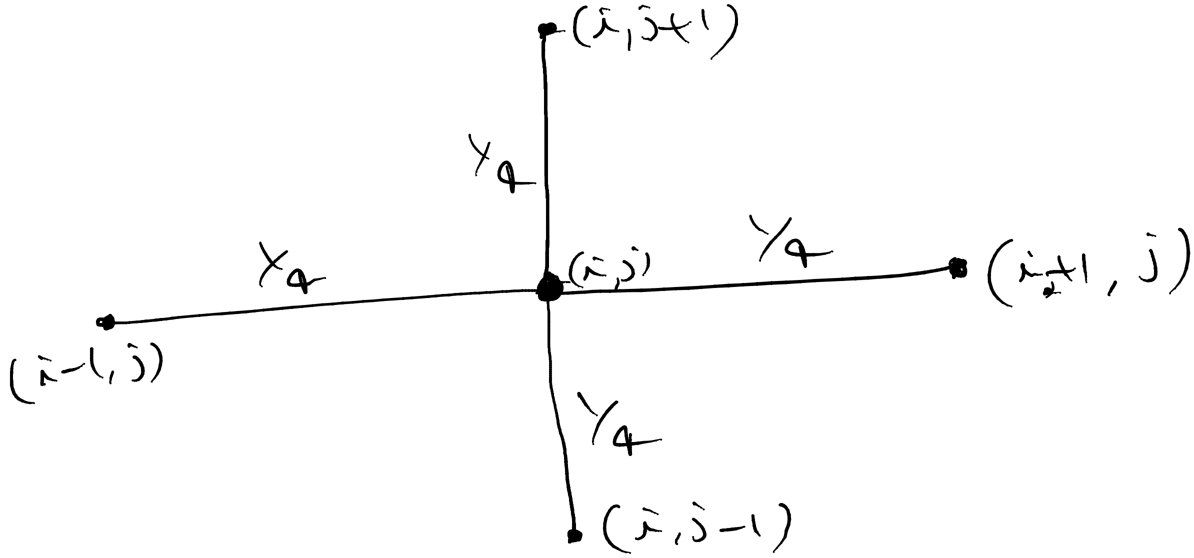
$$\text{and } \sum_{n=1}^{\infty} P_{00}^{n} = \infty \text{ for } p = 1/2$$

Thus, the all states of Markov chain indicating random walk is recurrent if $p = 1/2$ and transient if $p \neq 1/2$.

When $p = 1/2$, the random walk is

called symmetric random walk.

Two dimensional random walk:



$P_{00}^{2n-1} = 0$

$$P_{00}^{2n} = \sum_{i=0}^{n} \frac{2n!}{i!\, i!\, (n-i)!\,(n-i)!} \left(\frac{1}{4}\right)^{2n}$$

$$= \left(\frac{1}{4}\right)^{2n} \binom{2n}{n}\binom{2n}{n}$$

$$\sum_{n=1}^{\infty} P_{00}^{n} = \sum_{n=1}^{\infty} P_{00}^{2n} = \sum_{n=1}^{\infty} \binom{2n}{n}^2 \left(\frac{1}{4}\right)^{2n}$$

$$\simeq \sum_{n=1}^{\infty} \frac{4^{2n}}{n\pi} \cdot \left(\frac{1}{4}\right)^{2n}$$

Stirling's approximation

$$\binom{2n}{n} = \frac{4^n}{\sqrt{n\pi}}$$

$$\left\{\binom{2n}{n}\right\}^2 = \frac{4^{2n}}{n\pi}$$

$$\simeq \sum_{n=1}^{\infty} \frac{1}{n\pi}$$

$$= \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

$$\therefore \sum_{n=1}^{\infty} P_{00}^{n} = \infty$$

$\Rightarrow$ state 0 is recurrent

Thus all states of two dimensional symmetric random walk are recurrent.