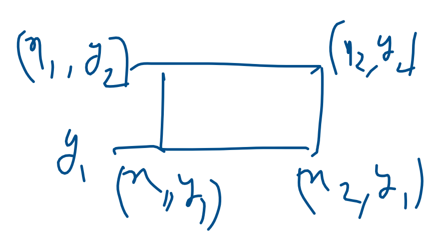
12.2

Wednesday, 7 October 2020 1:39 PM

(2+) $\int_{x_1}^{x_2}\int_{y_1}^{y_2} \frac{\partial^2 f}{\partial y \partial x}\, dy\, dx$



$= \int_{x_1}^{x_2} \frac{\partial f}{\partial x}\Big|_{y_1}^{y_2} dx$

$\int \frac{\partial^2 f}{\partial y \partial x} dy = \frac{\partial f}{\partial x}(x,y)\Big|_{y_1}^{y_2}$

$= \frac{\partial f}{\partial x}(x,y_2) - \frac{\partial f}{\partial x}(x,y_1)$

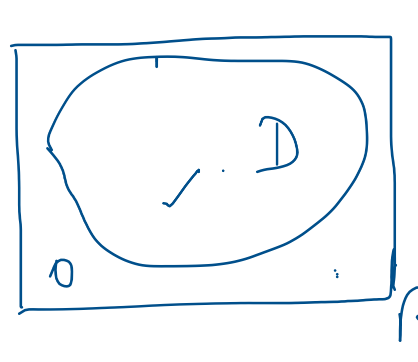
$= \int_{x_1}^{x_2}\left[\frac{\partial f}{\partial x}(x,y_2) - \frac{\partial f}{\partial x}(x,y_1)\right] dx$

$= f(x,y_2)\Big|_{x_1}^{x_2} - f(x,y_1)\Big|_{x_1}^{x_2}$

$= f(x_2,y_2) - f(x_1,y_2) - \left(f(x_2,y_1) - f(x_1,y_1)\right)$

Double Integration over non-Rectangular Regions

$\iint_D f(x,y)\, dA$



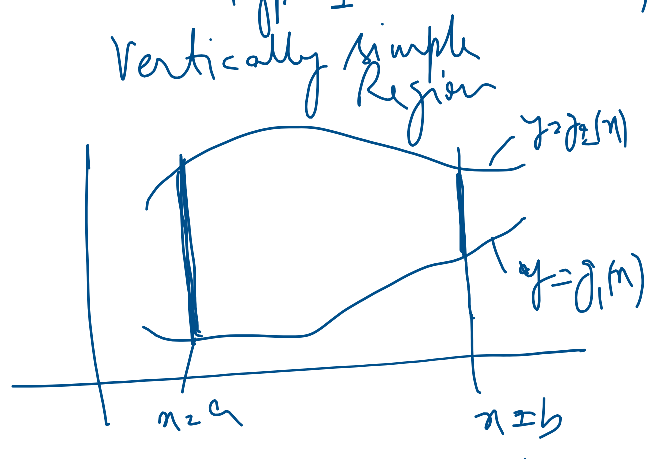
$F(x,y) = \begin{cases} f(x,y), & \text{if } (x,y) \in D \\ 0, & \text{if } (x,y) \in R \sim D \end{cases}$

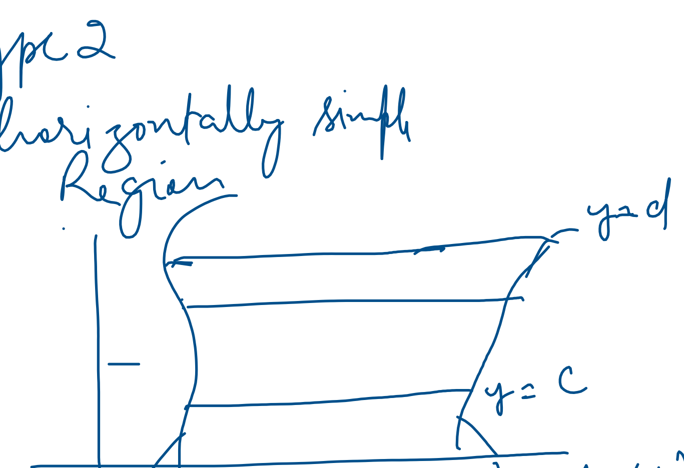
$\iint_R F(x,y)\, dA = \iint_D f(x,y)\, dA$

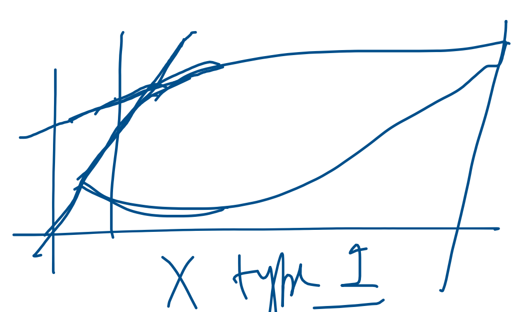
domain

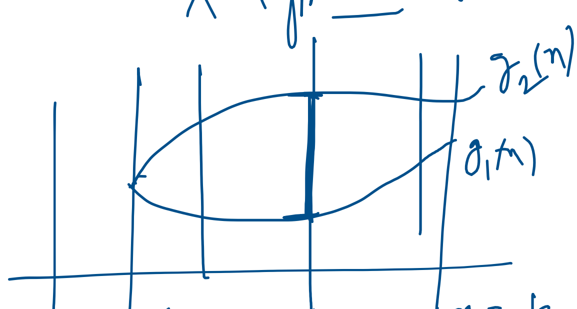
type 1 — Vertically simple Region

type 2 — horizontally simple Region

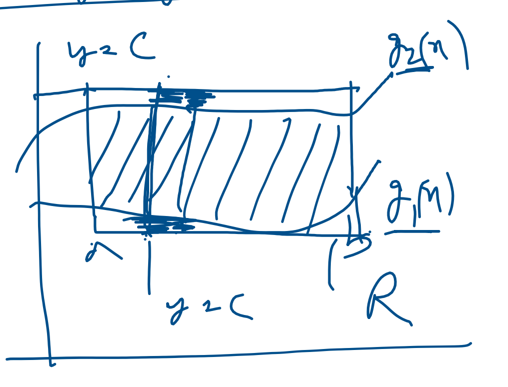








how to Integrate $f(x,y)$ over D (Region of type 1)



$F(x,y) = \begin{cases} f(x,y), & \text{if } (x,y) \in D \\ 0, & \text{if } (x,y) \in R \sim D \end{cases}$

$\iint_D f(x,y)\, dA$

$= \iint_R F(x,y)\, dA$

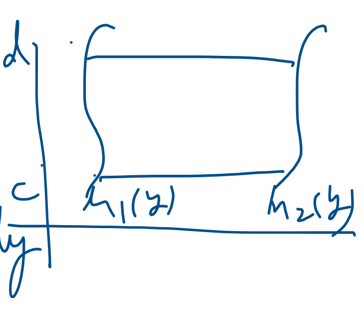
$= \int_a^b \int_c^d F(x,y)\, dy\, dx$

$\int_c^d F = \int_c^{g_1} 0 + \int_{g_1}^{g_2} f + \int_{g_2}^{d} 0$

$= \boxed{\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx}$

D: horizontally simple

$\iint_D f(x,y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dx\, dy$



Ex 1 $\int_0^1 \int_{x^2}^{\sqrt{x}} 160\, x y^3\, dy\, dx$

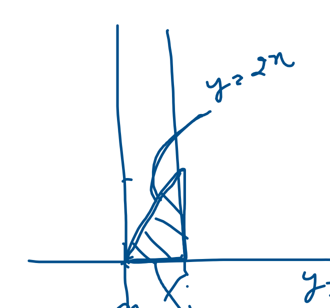
$= \int_0^1 160x \times \frac{y^4}{4}\Big|_{x^2}^{\sqrt{x}} dx$

$= \int_0^1 40x\left[x^2 - x^8\right] dx$

$= 40 \int_0^1 (x^3 - x^9)\, dx = 40\left|\frac{x^4}{4} - \frac{x^{10}}{10}\right|_0^1$

$= 40\left(\frac{1}{4} - \frac{1}{10}\right) = 40 \times \frac{5-2}{20} = 6$

Ex 2 $\iint_T (x+y)\, dA$ $y = 0,\ y = 2x,\ x = 1$



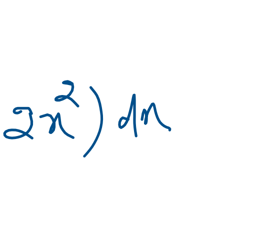
$\int_0^1 \int_0^{2x} (x+y)\, dy\, dx$

$= \int_0^1 \left(xy + \frac{y^2}{2}\Big|_0^{2x}\right) dx$

$= \int_0^1 \left(x(2x) + \frac{(2x)^2}{2}\right) dx = \int_0^1 (2x^2 + 2x^2)\, dx$

$= \frac{4x^3}{3}\Big|_0^1 = \frac{4}{3}$

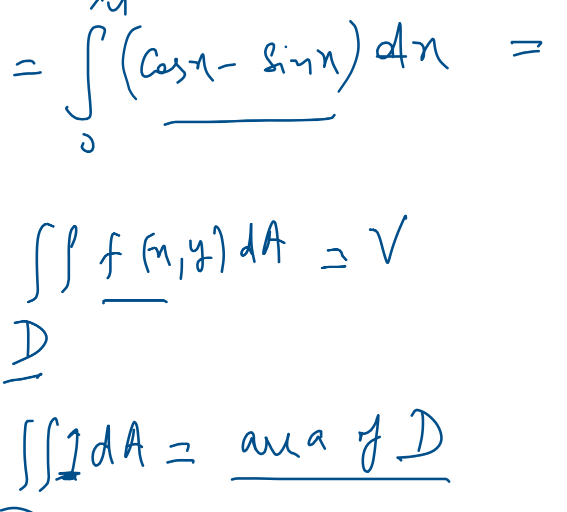
(b) By horizontally simple



$\int_0^2 \int_{y/2}^{1} (x+y)\, dx\, dy = \frac{4}{3}$

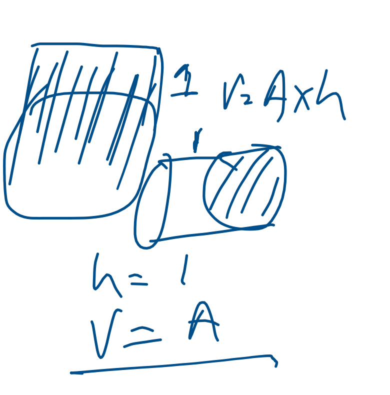
Ex 3 $y = \cos x,\ y = \sin x,\ 0 \le x \le \frac{\pi}{4}$

(a) single integration (b) double integration

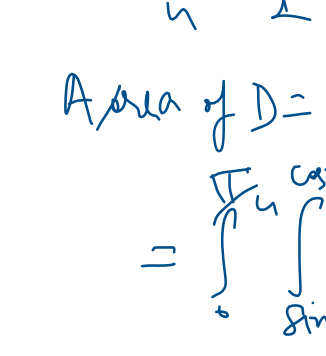


$A = \int_0^{\pi/4} (\cos x - \sin x)\, dx = \sqrt{2} - 1$

$\iint_D f(x,y)\, dA = V$



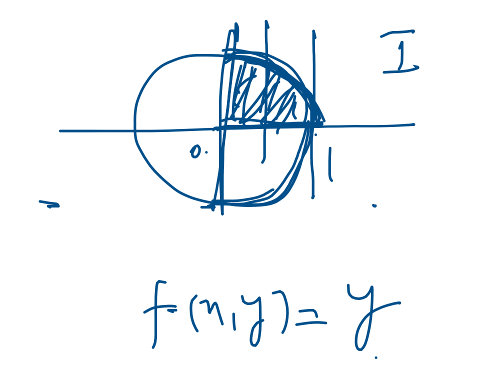
$\iint_D 1\, dA = \text{area of } D$

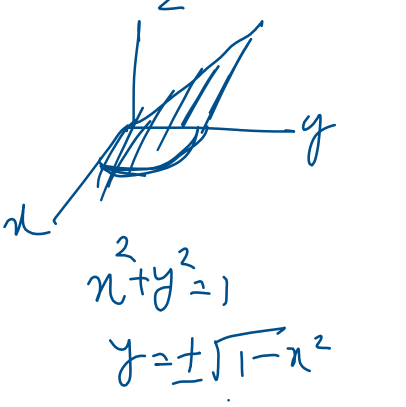


Area of D $= \iint_D dA$

$= \int_0^{\pi/4} \int_{\sin x}^{\cos x} dy\, dx = \sqrt{2} - 1$

Ex 4 $z = y$ $x^2 + y^2 \le 1$ in the 1^{st} quadrant



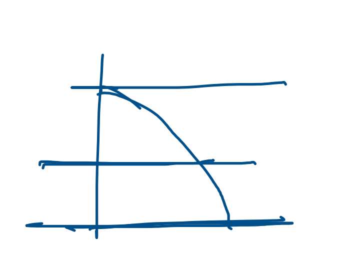


$f(x,y) = y$

$x^2 + y^2 = 1$

$y = \pm\sqrt{1 - x^2}$

$V = \int_0^1 \int_0^{\sqrt{1-x^2}} y\, dy\, dx$



horizontally simple.

$V = \int_0^1 \int_0^{\sqrt{1-y^2}} y\, dx\, dy$