Any instrument following this equation is a second order instrument. A practical example of this type is the spring balance. Linear devices range from mass spring arrangements, transducers, amplifiers and filters to indicators and recorders.

Most devices have first or second order responses, i.e. the equations of motion describing the devices are either first or second order linear differentials. For example, a search coil and mercury-in-glass thermometer have a first order response. Filters used at the output of a phase sensitive detector and amplifiers used in feedback measuring systems essentially have response due to a single time constant. First order systems involve only one kind of energy, e.g. thermal energy in the case of a thermometer, while a characteristic feature of second order system is an exchange between two types of energy, e.g. electrostatic and electromagnetic energy in electrical LC circuits, moving coil indicators and electromechanical recorders.

STATISTICAL ANALYSIS 1.8

The statistical analysis of measurement data is important because it allows an analytical determination of the uncertainty of the final test result. To make statistical analysis meaningful, a large number of measurements is usually required. Systematic errors should be small compared to random errors, because statistical analysis of data cannot remove a fixed bias contained in all measurements.

1.8.1 Arithmetic Mean

The most probable value of a measured variable is the arithmetic mean of the number of readings taken. The best approximation is possible when the number of readings of the same quantity is very large. The arithmetic mean of n measurements at a specific count of the variable x is given by the expression

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\sum_{n=1}^{n} x_n}{n}$$

where $\bar{x}$ = Arithmetic mean
x_n = nth reading taken
n = total number of readings

1.8.2 Deviation from the Mean

This is the departure of a given reading from the arithmetic mean of the group of readings. If the deviation of the first reading, x_1, is called d_1 and that of the second reading x_2 is called d_2, and so on,

The deviations from the mean can be expressed as

$$d_1 = x_1 - \bar{x},\ d_2 = x_2 - \bar{x} \ldots, \text{ similarly } d_n = x_n - \bar{x}$$

The deviation may be positive or negative. The algebraic sum of all the deviations must be zero.

Example 1.4 *For the following given data, calculate*
(i) Arithmetic mean; (ii) Deviation of each value; (iii) Algebraic sum of the deviations
Given

$$x_1 = 49.7;\ x_2 = 50.1;\ x_3 = 50.2;\ x_4 = 49.6;\ x_5 = 49.7$$

Solution

(i) The arithmetic mean is calculated as follows

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$= \frac{49.7 + 50.1 + 50.2 + 49.6 + 49.7}{5} = 49.86$$

(ii) The deviations from each value are given by

$$d_1 = x_1 - \bar{x} = 49.7 - 49.86 = -0.16$$
$$d_2 = x_2 - \bar{x} = 50.1 - 49.86 = +0.24$$
$$d_3 = x_3 - \bar{x} = 50.2 - 49.86 = +0.34$$
$$d_4 = x_4 - \bar{x} = 49.6 - 49.86 = -0.26$$
$$d_5 = x_5 - \bar{x} = 49.7 - 49.86 = -0.16$$

(iii) The algebraic sum of the deviation is

$$d_{\text{total}} = -0.16 + 0.24 + 0.34 - 0.26 - 0.16$$
$$= +0.58 - 0.58 = 0$$

1.8.3 Average Deviations

The average deviation is an indication of the precision of the instrument used in measurement. Average deviation is defined as the sum of the absolute values of the deviation divided by the number of readings. The absolute value of the deviation is the value without respect to the sign.

Average deviation may be expressed as

$$D_{\text{av}} = \frac{|d_1| + |d_2| + |d_3| + \cdots + |d_n|}{n}$$

or

$$D_{\text{av}} = \frac{\sum |d_n|}{n}$$

where D_{av} = average deviation

$|d_1|, |d_2|, \ldots, |d_n|$ = Absolute value of deviations

and n = total number of readings

Highly precise instruments yield a low average deviation between readings.

Example 1.5 *Calculate the average deviation for the data given in Example 1.4.*

Solution The average deviation is calculated as follows

$$D_{av} = \frac{|d_1| + |d_2| + |d_3| + \cdots + |d_n|}{n}$$

$$= \frac{|-0.16| + |0.24| + |0.34| + |-0.26| + |-0.16|}{5}$$

$$= \frac{1.16}{5} = 0.232$$

Therefore, the average deviation = 0.232.

1.8.4 Standard Deviation

The standard deviation of an infinite number of data is the Square root of the sum of all the individual deviations squared, divided by the number of readings. It may be expressed as

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}{n}} = \sqrt{\frac{d_n^2}{n}}$$

where σ = standard deviation

The standard deviation is also known as root mean square deviation, and is the most important factor in the statistical analysis of measurement data. Reduction in this quantity effectively means improvement in measurement.

For small readings (n < 30), the denominator is frequently expressed as (n – 1) to obtain a more accurate value for the standard deviation.

Example 1.6 *Calculate the standard deviation for the data given in Example 1.4.*

Solution

$$\text{Standard deviation} = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}{n-1}}$$

$$\sigma = \sqrt{\frac{(-0.16)^2 + (0.24)^2 + (0.34)^2 + (-0.26)^2 + (-0.16)^2}{5-1}}$$

$$\sigma = \sqrt{\frac{0.0256 + 0.0576 + 0.1156 + 0.0676 + 0.0256}{4}}$$

$$\sigma = \sqrt{\frac{0.292}{4}} = \sqrt{0.073} = 0.27$$

Therefore, the standard deviation is 0.27.

1.8.5 Limiting Errors

Most manufacturers of measuring instruments specify accuracy within a certain % of a full scale reading. For example, the manufacturer of a certain voltmeter may specify the instrument to be accurate within ± 2% with full scale deflection. This specification is called the limiting error. This means that a full scale deflection reading is guaranteed to be within the limits of 2% of a perfectly accurate reading; however, with a reading less than full scale, the limiting error increases.

Example 1.7 *A 600 V voltmeter is specified to be accurate within ± 2% at full scale. Calculate the limiting error when the instrument is used to measure a voltage of 250 V.*

Solution The magnitude of the limiting error is 0.02 × 600 = 12 V.

Therefore, the limiting error for 250 V is 12/250 × 100 = 4.8%

Example 1.8 (a) *A 500 mA voltmeter is specified to be accurate with ±2%. Calculate the limiting error when instrument is used to measure 300 mA.*

Solution Given accuracy of 0.02 = ±2%

Step 1: The magnitude of limiting error is = 500 mA × 0.02 = 10 mA

Step 2: Therefore the limiting error at 300 mA = $\frac{10 \text{ mA}}{300 \text{ mA}} \times 100\% = 3.33\%$

Example 1.8 (b) *A voltmeter reading 70 V on its 100 V range and an ammeter reading 80 mA on its 150 mA range are used to determine the power dissipated in a resistor. Both these instruments are guaranteed to be accurate within ±1.5% at full scale deflection. Determine the limiting error of the power.*

Solution The magnitude of the limiting error for the voltmeter is

$$0.015 \times 100 = 1.5 \text{ V}$$

The limiting error at 70 V is

$$\frac{1.5}{70} \times 100 = 2.143\ \%$$

The magnitude of limiting error of the ammeter is

$$0.015 \times 150 \text{ mA} = 2.25 \text{ mA}$$

The limiting error at 80 mA is

$$\frac{2.25 \text{ mA}}{80 \text{ mA}} \times 100 = 2.813\ \%$$

Therefore, the limiting error for the power calculation is the sum of the individual limiting errors involved.

Therefore, limiting error = 2.143 % + 2.813 % = 4.956 %

The induction principle is most generally used for Watt-hour meters. This principle is not preferred for use in ammeters and voltmeters because of the comparatively high cost and inaccuracy of the instrument.

BASIC METER MOVEMENT 2.2

The action of the most commonly dc meter is based on the fundamental principle of the motor. The motor action is produced by the flow of a small current through a moving coil, which is positioned in the field of a permanent magnet. This basic moving coil system is often called the D'Arsonval galvanometer.

The D'Arsonval movement shown in Fig. 2.1 employs a spring-loaded coil through which the measured current flows. The coil (rotor) is in a nearly homogeneous field of a permanent magnet and moves in a rotary fashion. The amount of rotation is proportional to the amount of current flowing through the coil. A pointer attached to the coil indicates the position of the coil on a scale calibrated in terms of current or voltage. It responds to dc current only, and has an almost linear calibration. The magnetic shunt that varies the field strength is used for calibration.

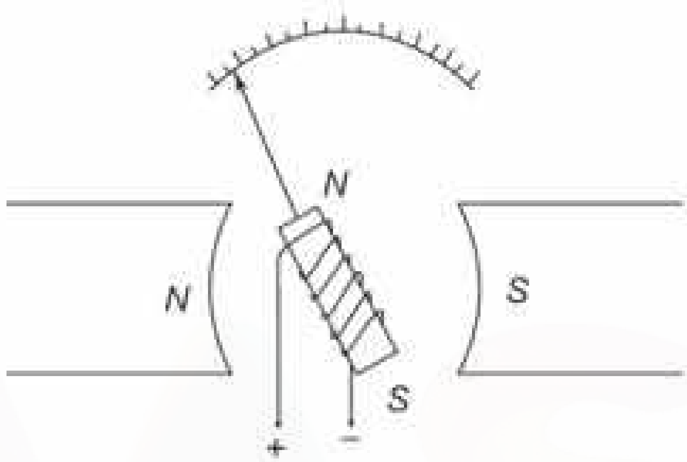

Fig. 2.1 D'Arsonval principle

2.2.1 Permanent Magnetic Moving Coil Movement

In this instrument, we have a coil suspended in the magnetic field of a permanent magnet in the shape of a horse-shoe. The coil is suspended so that it can rotate freely in the magnetic field. When current flows in the coil, the developed (electromagnetic) torque causes the coil to rotate. The electromagnetic (EM) torque is counterbalanced by a mechanical torque of control springs attached to the movable coil. The balance of torques, and therefore the angular position of the movable coil is indicated by a pointer against a fixed reference called a scale. The equation for the developed torque, derived from the basic law for electromagnetic torque is

$$\tau = B \times A \times I \times N$$

where τ = torque, Newton-meter
B = flux density in the air gap, Wb/m^2
A = effective coil area (m^2)
N = number of turns of wire of the coil
I = current in the movable coil (amperes)

The equation shows that the developed torque is proportional to the flux density of the field in which the coil rotates, the current coil constants (area and number of turns). Since both flux density and coil constants are fixed for a given instrument, the developed torque is a direct indication of the current in the coil. The pointer deflection can therefore be used to measure current.

Example 2.1 (a) *A moving coil instrument has the following data.*
Number of turns = 100
Width of the coil = 20 mm
Depth of the coil = 30 mm
Flux density in the gap = 0.1 Wb/m²
Calculate the deflecting torque when carrying a current of 10 mA. Also calculate the deflection, if the control spring constant is 2×10^{-6} Nm/degree.

Solution The deflecting torque is given by

$$\begin{aligned}\tau_d &= B \times A \times N \times I\\ &= 0.1 \times 30 \times 10^{-3} \times 20 \times 10^{-3} \times 100 \times 10 \times 10^{-3}\\ &= 600 \times 1000 \times 0.1 \times 10^{-9}\\ &= 600 \times 1000 \times 10^{-10}\\ &= 60 \times 10^{-6}\ \text{Nm}\end{aligned}$$

The spring control provides a restoring torque, i.e. $\tau_c = K\theta$, where K is the spring constant
As deflecting torque = restoring torque

$$\therefore \quad \tau_c = 6 \times 10^{-5}\ \text{Nm} = K\theta, \qquad \therefore \theta = \frac{6 \times 10^{-5}}{2 \times 10^{-6}} = 3 \times 10 = 30°$$

Therefore, the deflection is 30°.

Example 2.1 (b) *A moving coil instrument has the following data*
No. of turns = 100
Width of the coil = 20 mm
Depth of the coil = 30 mm
Flux density in the gap = 0.1 Wb/m²
The deflection torque = 30×10^{-6} Nm
Calculate the current through the moving coil.

Solution The deflecting torque is given by

$$\tau_d = B \times A \times N \times I$$

Therefore $$30 \times 10^{-6} = 0.1 \times 30 \times 10^{-3} \times 20 \times 10^{-3} \times 100 \times I$$

$$I = \frac{30 \times 10^{-6}}{0.1 \times 30 \times 10^{-3} \times 20 \times 10^{-3} \times 100}$$

$$I = \frac{30 \times 10^{-6}}{0.1 \times 600 \times 10^{-6} \times 100} = 5\ \text{mA}$$

2.2.2 Practical PMMC Movement

The basic PMMC movement (also called a D'Arsonval movement) offers the largest magnet in a given space, in the form of a horse-shoe, and is used when a large flux is required in the air gap. The D'Arsonval movement is based on the principle of a moving electromagnetic coil pivoted in a uniform air gap between the poles of a large fixed permanent magnet. This principle is illustrated in Fig. 2.1 With the polarities as shown, there is a repelling force between like poles, which exerts a torque on the pivoted coil. The torque is proportional to the magnitude of current being measured. This D'Arsonval movement provides an instrument with very low power consumption and low current required for full scale deflection (fsd).

Figure 2.2 shows a permanent horse-shoe magnet with soft iron pole pieces attached to it. Between the pole pieces is a cylinder of soft iron which serves to provide a uniform magnetic field in the air gap between the pole pieces and the cylindrical core.

The coil is wound on a light metal frame and is mounted so that it can rotate freely in the air gap. The pointer attached to the coil moves over a graduated scale and indicates the angular deflection of the coil, which is proportional to the current flowing through it.

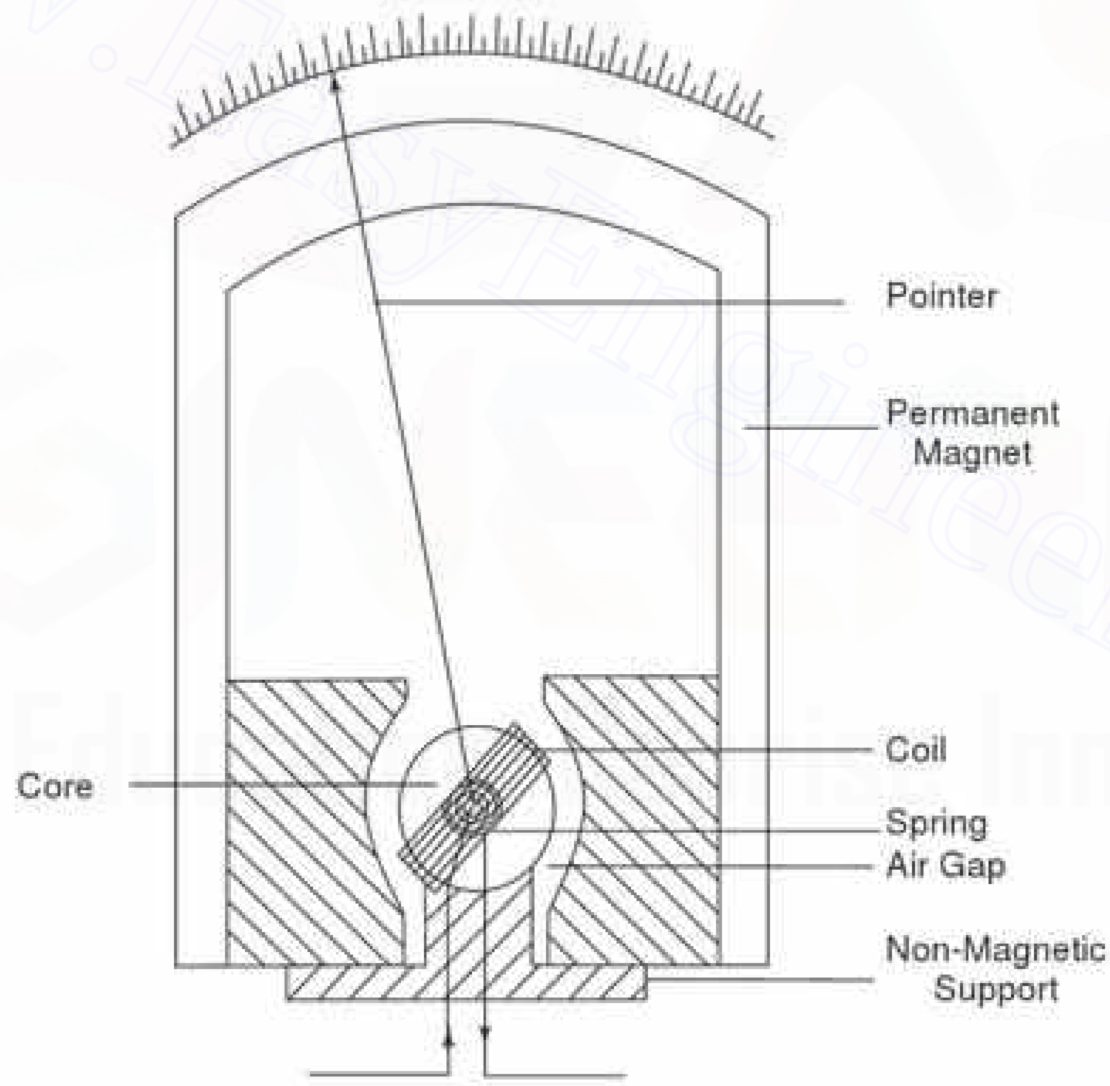

Fig. 2.2 Modern D'Arsonval movement

The Y-shaped member shown in Fig. 2.3 is the zero adjust control, and is connected to the fixed end of the front control spring. An eccentric pin through the instrument case engages the Y-shaped member so that the zero position of the pointer can be adjusted from outside. The calibrated force opposing the moving torque is provided by two phosphor-bronze conductive springs, normally equal in strength. (This also provides the necessary torque to bring the pointer back to its original position after the measurement is over.)

Chapter 3

Ammeters

DC AMMETER

3.1

The PMMC galvanometer constitutes the basic movement of a dc ammeter. Since the coil winding of a basic movement is small and light, it can carry only very small currents. When large currents are to be measured, it is necessary to bypass a major part of the current through a resistance called a shunt, as shown in Fig. 3.1. The resistance of shunt can be calculated using conventional circuit analysis.

Referring to Fig. 3.1

R_m = internal resistance of the movement.

I_{sh} = shunt current

I_m = full scale deflection current of the movement

I = full scale current of the ammeter + shunt (i.e. total current)

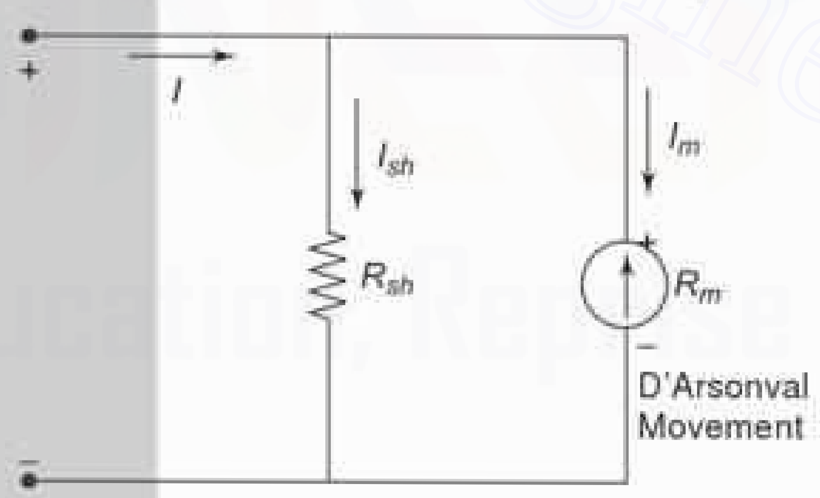

Fig. 3.1 Basic dc Ammeter

Since the shunt resistance is in parallel with the meter movement, the voltage drop across the shunt and movement must be the same.

Therefore $V_{sh} = V_m$

$\therefore \quad I_{sh} R_{sh} = I_m R_m, \qquad R_{sh} = \dfrac{I_m R_m}{I_{sh}}$

But $\quad I_{sh} = I - I_m$

hence $\quad R_{sh} = \dfrac{I_m R_m}{I - I_m}$

For each required value of full scale meter current, we can determine the value of shunt resistance.

Example 3.1 (a) *A 1 mA meter movement with an internal resistance of 100 Ω is to be converted into a 0 – 100 mA. Calculate the value of shunt resistance required.*

Solution Given $R_m = 100\ \Omega$, $I_m = 1$ mA, $I = 100$ mA

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{1\text{ mA} \times 100\ \Omega}{99\text{ mA}} = \frac{100\text{ mA}\Omega}{99\text{ mA}} = \frac{100\ \Omega}{99} = 1.01\ \Omega$$

The shunt resistance used with a basic movement may consist of a length of constant temperature resistance wire within the case of the instrument. Alternatively, there may be an external (manganin or constantan) shunt having very low resistance.

The general requirements of a shunt are as follows.

1. The temperature coefficients of the shunt and instrument should be low and nearly identical.
2. The resistance of the shunt should not vary with time.
3. It should carry the current without excessive temperature rise.
4. It should have a low thermal emf.

Manganin is usually used as a shunt for dc instruments, since it gives a low value of thermal emf with copper.

Constantan is a useful material for ac circuits, since it's comparatively high thermal emf, being unidirectional, is ineffective on the these circuits.

Shunt for low current are enclosed in the meter casing, while for currents above 200 A, they are mounted separately.

Example 3.1 (b) *A 100 μA meter movement with an internal resistance of 500 Ω is to be used in a 0 – 100 mA Ammeter. Find the value of the required shunt.*

Solution The shunt can also be determined by considering current I to be 'n' times larger than I_m. This is called a multiplying factor and relates the total current and meter current.

Therefore $I = n\,I_m$

Therefore the equation for

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{I_m R_m}{nI_m - I_m} = \frac{I_m R_m}{I_m(n-1)} = \frac{R_m}{(n-1)}$$

Given: $I_m = 100\ \mu$A and $R_m = 500\ \Omega$

Step 1: $$n = \frac{I}{I_m} = \frac{100\text{ mA}}{100\ \mu\text{A}} = 1000$$

Step 2: $$R_{sh} = \frac{R_m}{(n-1)} = \frac{500\ \Omega}{1000 - 1} = \frac{500}{999} = 0.50\ \Omega$$

MULTIRANGE AMMETERS 3.2

The current range of the dc ammeter may be further extended by a number of shunts, selected by a range switch. Such a meter is called a multirange ammeter, shown in Fig. 3.2.

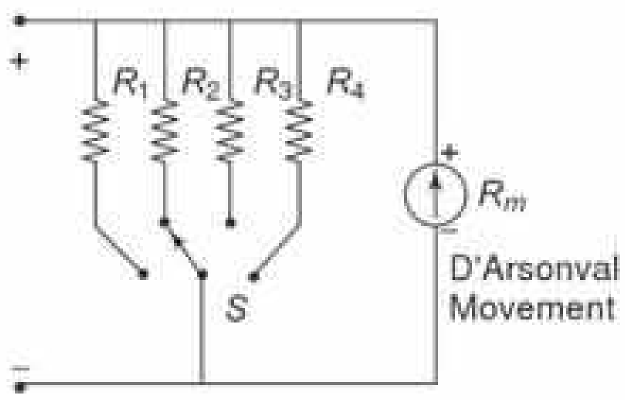

Fig. 3.2 Multirange ammeter

The circuit has four shunts R_1, R_2, R_3 and R_4, which can be placed in parallel with the movement to give four different current ranges. Switch S is a multiposition switch, (having low contact resistance and high current carrying capacity, since its contacts are in series with low resistance shunts). Make before break type switch is used for range changing. This switch protects the meter movement from being damaged without a shunt during range changing.

If we use an ordinary switch for range changing, the meter does not have any shunt in parallel while the range is being changed, and hence full current passes through the meter movement, damaging the movement. Hence a make before break type switch is used. The switch is so designed that when the switch position is changed, it makes contact with the next terminal (range) before breaking contact with the previous terminal. Therefore the meter movement is never left unprotected. Multirange ammeters are used for ranges up to 50A. When using a multirange ammeter, first use the highest current range, then decrease the range until good upscale reading is obtained. The resistance used for the various ranges are of very high precision values, hence the cost of the meter increases.

Example 3.2 *A 1 mA meter movement having an internal resistance of 100 Ω is used to convert into a multirange ammeter having the range 0–10 mA, 0–20 mA and 0–50 mA. Determine the value of the shunt resistance required.*

Solution Given $I_m = 1$ mA and $R_m = 100\ \Omega$

Case 1: For the range 0 – 10 mA

$$\text{Given } R_{sh1} = \frac{I_m \cdot R_m}{I - I_m} = \frac{1\text{ mA} \times 100}{10\text{ mA} - 1\text{ mA}} = \frac{100}{9} = 11.11\ \Omega$$

Case 2: For the range 0 – 20 mA

$$\text{Given } R_{sh2} = \frac{I_m \cdot R_m}{I - I_m} = \frac{1\text{ mA} \times 100}{20\text{ mA} - 1\text{ mA}} = \frac{100}{19} = 5.2\ \Omega$$

Case 3: For the range 0 – 50 mA

$$\text{Given } R_{sh3} = \frac{I_m \cdot R_m}{I - I_m} = \frac{1\text{ mA} \times 100}{50\text{ mA} - 1\text{ mA}} = \frac{100}{49} = 2.041\ \Omega$$