Q. Find last digit of 17^{29} ?

Sol^n If $\gcd(a, n) = 1$, then $a^{\phi(n)} \equiv 1 \pmod{n}$

last digit of 17^{29} $\Rightarrow$ we need to find $17^{29} \pmod{10}$.

$\gcd(17, 10) = 1 \Rightarrow$ and $\phi(10) = 4$.

$U(10) = \{1, 3, 7, 9\}$

$\phi(n) = |U(n)|$,

$\therefore 17^{\phi(10)} \equiv 1 \pmod{10} \Rightarrow 17^4 \equiv 1 \pmod{10}$

$\Rightarrow (17^4)^7 \equiv 1^7 \pmod{10}$

$\Rightarrow 17^{28} \equiv 1 \pmod{10}$

$\Rightarrow 17^{29} \equiv 17 \pmod{10}$

$\Rightarrow 17^{29} \equiv 7 \pmod{10}$

Last digit of 17^{29} is 7.

$17^{30} = (7)(17) \pmod{10}$

$\Rightarrow 17^{30} \equiv 119 \pmod{10} \Rightarrow 17^{30} \equiv 9 \pmod{10}$

Last digit of 17^{30} is 9.

Q. find $\phi(108)$?

If $n = p_1^{e_1} p_2^{e_2} \ldots p_k^{e_k}$

then $\phi(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_k}\right)$

$108 = 2^2 \times 3^3$

$\phi(108) = 108\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)$

$= 108\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = 36.$

find order of $U(108)$?

$|U(n)| = \phi(n) \Rightarrow |U(108)| = \phi(108)$
$= 36$

Q. Are $\mathbb{Z}_{10}$ & $\mathbb{Z}_{12}$ isomorphic ?

Sol $\mathbb{Z}_{10} = \{0, 1, 2, \ldots, 9\} \Rightarrow |\mathbb{Z}_{10}| = 10$

$\mathbb{Z}_{12} = \{0, 1, 2, \ldots, 9, 10, 11\} \Rightarrow |\mathbb{Z}_{12}| = 12.$

If the groups G & $\bar{G}$ isomorphic, then there exist one-to-one correspondence between G & $\bar{G}$. $\therefore |G| = |\bar{G}|$.

here $|R_{10}| \neq |R_{12}|$

$\therefore$ R_{10} and R_{12} are not isomorphic.