2014 (6MM)

Q. Check for convexity:

a) $U = xy$

b) $U = x^2 + y^2$

c) $U = \max(x, y)$

a) $MRS = -\frac{dx_2}{dx_1} = \frac{MU_1}{MU_2}$

$MU_1 = y$
$MU_2 = x$

→ $MRS = y/x = -dy/dx$

Also, $\bar{U} = xy \Rightarrow y = \frac{\bar{U}}{x} = \bar{U}x^{-1}$

So, $\frac{dy}{dx} = \frac{-\bar{U}}{x^2}$

→ Now, check for convexity

$\boxed{\frac{dMRS}{dx} < 0}$

⇒ use quotient rule

$\Rightarrow \frac{x\frac{dy}{dx} - y\frac{dx}{dx}}{x^2}$

$= \frac{x\left(\frac{-y}{x}\right) - y}{x^2}$

$= \frac{-2y}{x^2} < 0$

→ as $x\uparrow \Rightarrow MRS\downarrow$

⇒ MRS is diminishing

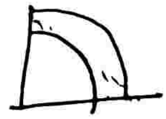

$\boxed{\frac{d^2y}{dx^2} > 0}$

$\Rightarrow \frac{d^2y}{dx^2} = -\bar{U}(-2x^{-3})$

$= \frac{2\bar{U}}{x^3} > 0$

⇒ convex func

b) $U = x^2 + y^2$ < diag + ve str ①  ①
MRS + $\frac{dMRS}{dx} < 0$ ① ①



$\rightarrow MRS = \frac{MU_1}{MU_2} = -\frac{dy}{dx}$

$= \frac{2x}{2y} = \frac{x}{y}$

$\rightarrow \frac{d(MRS)}{dx} = \frac{y\frac{dx}{dx} - x\frac{dy}{dx}}{y^2}$

$= \frac{y - x\left(\frac{-x}{y}\right)}{y^2}$

$= \left(\frac{y^2 + x^2}{y^3}\right) > 0$

So, ↑ing MRS.

$MRS = -\frac{dx_2}{dx_1} = -\frac{dy}{dx}$

Here, $\bar{U} = x^2 + y^2$

$\Rightarrow y^2 = \bar{U} - x^2$

$\Rightarrow y = \sqrt{\bar{U} - x^2}$

So, $\frac{dy}{dx} = \frac{1}{2\sqrt{\bar{U} - x^2}}(-2x)(1) = \frac{-x}{\sqrt{\bar{U} - x^2}}$

Now, to check $\frac{d^2y}{dx^2} > 0$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{(\sqrt{\bar{U} - x^2})(-1) - (-x)\left(\frac{1}{2\sqrt{\phantom{U}}}(-2x)\right)}{(\bar{U} - x^2)}$

$= \frac{-\sqrt{\bar{U} - x^2} - \frac{2x^2}{2\sqrt{\bar{U} - x^2}}}{(\bar{U} - x^2)^2}$

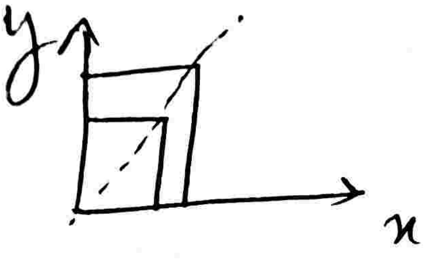
$= \frac{-(\bar{U} - x^2) - x^2}{(\bar{U} - x^2)^2(\sqrt{\bar{U} - x^2})}$

$= \frac{-\bar{U}}{\text{-do-}} < 0$

concave prefs

Not str convex prefs

c) $U = \max(x, y)$



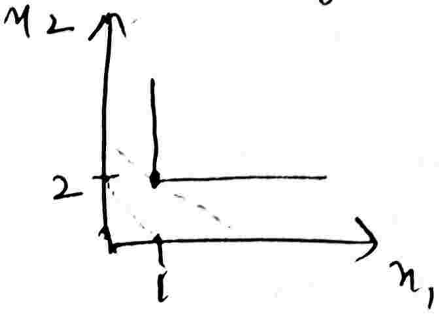

Inverted L-shaped ICs.

not str. convex prefs.

Pg - 62 **Perf comp**

→ Trick : To draw graph

$x_2$ S → 2
$x_1$ Tea → 1




① 1 $x_1$ plot
2 $x_2$ Pot

② find slope of line at kink

here : ⇒ $\frac{\Delta x_2}{\Delta x_1} = \frac{2}{1}$

$\frac{x_2}{x_1} = \frac{S}{T} = \frac{2}{T}$

$S = 2T$

$x_2 = 2x_1$

→ Trick to get ut. fnc

get $\frac{x_2}{x_1}$ = ratio = ?

Here, $\frac{x_2}{x_1} = \frac{2}{1}$

Instead of $\frac{\Delta x_2}{\Delta x_1}$ (rate) here, we are interested in ratio or proportion $\frac{x_1}{x_2}$ or $x_1 : x_2$

⇒ $x_2 = 2x_1$

⇒ ut fnc = U(x) = min $\{2x_1, x_2\}$

→ Trick to draw graph from ut fnc

$U = \min\{ax_1, bx_2\}$

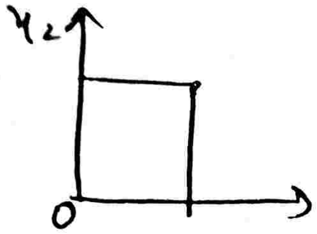
a ↓ Y-axis pt.
b ↓ X-axis pt.

$ax_1 = bx_2$

$\frac{x_1}{x_2} = \frac{b}{a}$

## Do Max prefs (concave prefs)

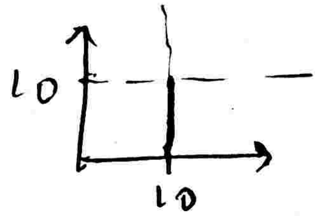
$U = \max\{x_1, x_2\}$




Say $U = 10$; $U = 10$

AND

if $x_1 = 10$ & $x_2 \leq 10$




if $x_2 = 10$ & $x_1 \leq 10$