Inadequacies of Classical physics and advent of Quantum theory

FAILURES OF CLASSICAL PHYSICS

UNIT - I
MODERN PHYSICS

(1) BLACK-BODY RADIATION:

Any object with a temp^r above "absolute zero" emits light of at all wavelength.

If the body is "perfectly black" (it doesn't reflect any light) then the light that comes from it is called black body radiation

The energy of a black body is not equally shared by all wavelength.

Some wavelengths get more energy than others

(i) Classical average energy per standing wave $\bar{\epsilon} = kT$ — classical picture

(ii) Rayleigh-Jeans formula — classical picture

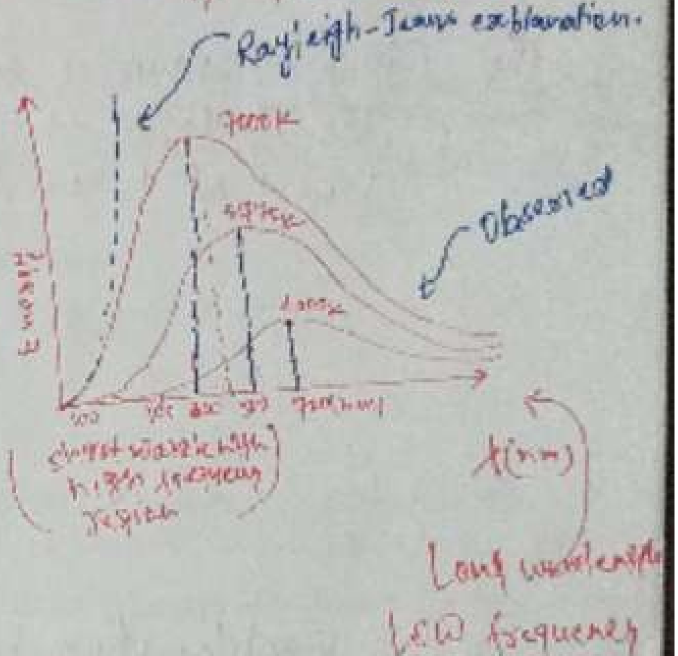
$$u(\nu)d\nu = \bar{\epsilon}\, G(\nu)\, d\nu = \frac{8\pi kT}{c^3}\nu^2 d\nu$$

(iii) Quantum picture:

$$u(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{\left(e^{h\nu/kT} - 1\right)}$$

at lower freq. $h\nu << kT$

$$\Rightarrow u(\nu)d\nu \approx \frac{8\pi kT}{c^3}\nu^2 d\nu$$



Some facts of Black body radiation:

(i) Spectrum depends on temp^r only and Independent of material (i.e iron, Charcoal, ceramics etc all shows the same spectra at fixed temp^r.)

(ii) as $T\uparrow \Rightarrow$ the Blackbody emits more radiation at all wavelength.

(iii) As $T\uparrow \Rightarrow$ Peak wavelength (λ_P) shifts towards shorter region

IV. BBR spectra becomes smaller at left hand side.

~~Explation~~ Explanation of Classical physics

(i) light is an EM wave which produces when charge oscilates or vibrates.

(ii) Now when heat increases $\Rightarrow$ K.E of e^-'s increases in random direction

$\Downarrow$

more light will be emitted by the hotter body.

$\Downarrow$

Classical physics could not explain the shape of the blackbody spectrum.

(iii) Classical physics said that each frequency of vibration should have the same energy. As there is no limit of higher frequency $\Rightarrow$ no upper limit of energy of the light produced by the e^-'s vibrates at high frequency, which is Wrong.

experimently Blackbody spectrum always becomes small at left hand side.

Explanation by Quantum mechanics

Energy of a vibrating e^- = (Any integer) $\times$ $h\nu$

At high frequency the amount of energy in Quantum of $h\nu$ is so ~~high~~ large that high frequency vibrations can never get going! This is why the blackbody spectrum always becomes smaller at the left hand side (high frequency side).

Planck's Quantum Theory

Light as wave — EM wave as Maxwell's showed this behaviour

In The Journey of progression of Science. Maxwell's suggestion about the wave nature of E.M ~~wave~~ Radiation was very helpful in explaining the phenomena ~~of~~ such as Interference, diffraction etc. But he failed to explain various other observations such as the nature of emission of radiation from hot bodies, photoelectric effect, The Independence of heat Capacity of Solid upon Temperature, line spectra of atoms.

Maxwell's eqn.

$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

$\nabla \times H = I_c + I_D$

where $I_D = \frac{\partial D}{\partial t}$

$\bar{D} = \epsilon_0 \vec{E}$

In case of Black body Radiation.

(A body which emits & absorbs all radiation of all frequency)

heat $\Longrightarrow$ Solid $\Longrightarrow$ emits radiation over wide range of wavelength.

$\Downarrow$

i.e colour changes continuously with further increase in Temp^r.

(Colour change is from lower freq. colour to higher freq. colour.) i.e from red to Blue.

i.e Variation of freq. for Black body radiation ~~is~~ depends on the Temperature.

At a given temp^r. the Intensity of radiation increases with the increase in wavelength of radiation and then decreases with increase in wavelength. These phenomena couldn't be explained by Maxwell's suggestions. And hence Planck proposed Planck's Q. Theory.

Planck's Quantum theory :—

(1) Different atoms and molecules can emit or absorbs energy in discrete Quantities only and the smallest amount of energy is called Quantum.

(2) The Quantum have the energy is directly proportional to the frequency of radiation.

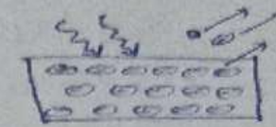
i.e $E = h\nu$ where h is called Planck's const.

and ~~Planck~~ concluded that these are the aspect of the process of absorption and emission only and they have no physical reality of radiation.

In 1905 Einstein reinterpreted Planck's law to explain photoelectric effect by assuming light as a Quanta of energy called photon

Photo electric effect

Observed by :- H. Hertz
Explained by :- Einstein
Verified by Experimentally by :- Millikan



K.E of electron :-

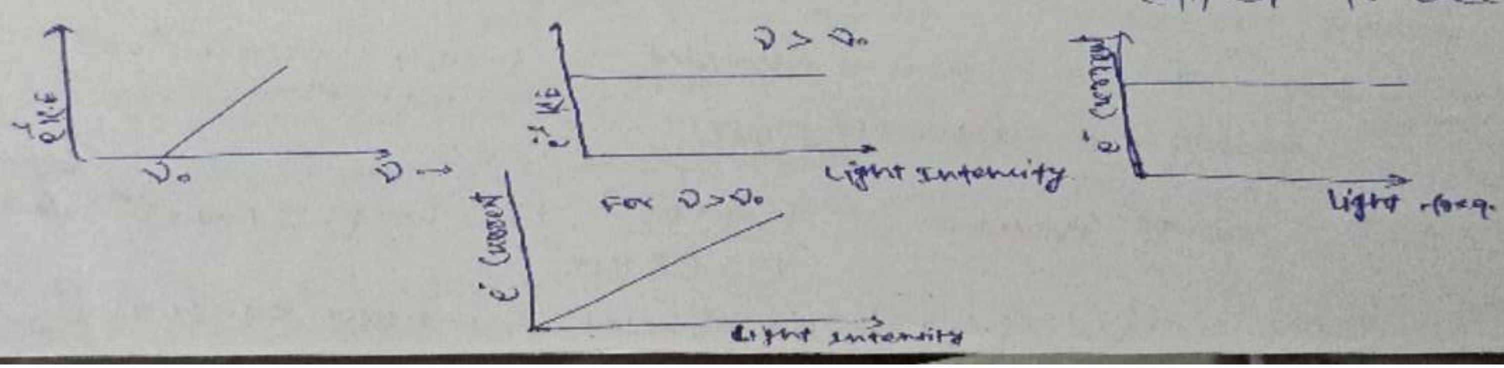
$$K.E_{max.} = h\nu - \phi$$

ϕ = work function of the metal = minimum energy required to remove a delocalised e^- from the surface of a metal.

$= h\nu_0$, where ν_0 = Threshold frequency.

$$\Rightarrow \quad K.E_{max} = h\nu - h\nu_0$$
$$= h(\nu - \nu_0)$$

As K.E is +ve $\Rightarrow$ $\nu > \nu_0$ for the photoelectric effect to occur.



Q: Ultraviolet light of wavelength 350 nm and Intensity 1.00 W/m² is directed at a potassium surface.

(a) Find the max^m K.E of photoelectrons.

(b) If 0.50 percent of the incident photons produces photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm² ?

Ans:- (a) Energy of photon in eV. is

$$E(eV) = \frac{1.24}{\lambda(\mu m)} = \frac{1.24}{(0.350\ \mu m)}$$

energy of 1 photon = $\boxed{E_p(eV) = 3.54\ eV}$ $= 3.5 \times 1.6 \times 10^{-19} = 5.68 \times 10^{-19}$ Jule.

Given work fun. of potassium = 2.2 eV.

$\Rightarrow$ K.E$_{max.}$ = 3.54 − 2.2 = 1.3 eV.

(b) Intensity = 1.00 W/m² ⇒ ~~Ene~~ associated power in Area 1.00 cm²

= power × Area

= Energy/time × Area

power associated = 1.00 × 1.00 × 10^{-4} m²
= 1 × 10^{-4}

Energy/time

$\Rightarrow$ no. of photons = $\frac{1 \times 10^{-4}}{5.68 \times 10^{-19}}$ = 1.76 × 10^{14} photons/sec.

No. of electrons ejected = 0.0050 × 1.76 × 10^{14} = 8.8 × 10^{11} elec/s.

Compton effect

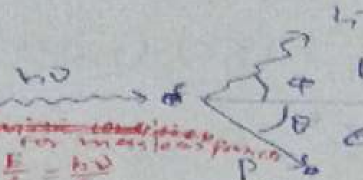
photon-electron interaction

7x16 12x16
19x28
19x7

Photon energy $E_p = h\nu$

7. Energy of photon(E) = pc = ~~Relativistic condition~~ for massless particle

momentum of photon(P) = $\frac{E}{c} = \frac{h\nu}{c}$



$P = \frac{E}{c} \Rightarrow P = \frac{h\nu}{c}$

along x-dir^n $\rightarrow$ $\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + P_e\cos\theta$ — (1)

P_e = momentum of e^-

$$\boxed{h\nu = h\nu'\cos\phi + P_e c\cos\theta} \quad \text{— (1)}$$

Along Y-dir^n $\rightarrow$ $0 = \frac{h\nu'}{c}\sin\phi - P_e\sin\theta$ — (2)

$$\boxed{h\nu'\sin\phi = P_e c\sin\theta} \quad \text{— (2)}$$

$$\left.\begin{aligned} P_e c\cos\theta &= h\nu - h\nu'\cos\phi \\ P_e c\sin\theta &= h\nu'\sin\phi \end{aligned}\right\} \begin{aligned} &(3) \text{ from eq}^n \text{ (1), eliminating } P_e c\cos\theta \\ &(4) \text{ from eq}^n \text{ (2) } - P_e c\sin\theta \end{aligned}$$

squaring (3) & (4) and adding :- we have

$$P_e^2c^2 = h^2\nu'^2\sin^2\phi + h^2\nu^2 + h^2\nu'^2\cos^2\phi - 2(h\nu)(h\nu')\cos\phi$$

$$P_e^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2(h\nu)(h\nu')\cos\phi \quad \text{— (A)}$$

Now

$E = K.E + m_oc^2$ ← Rest mass energy of particle of mass m_o

$E = \sqrt{m_o^2c^4 + p^2c^2}$ ← Hamiltonian of Relativistic particle = Total energy for all particle

$E = pc$ for massless particle

$\Rightarrow$ Both the energy values have to be same i.e the Rest mass energy of a particle = total Hamiltonian.

$$\Rightarrow (K.E + m_oc^2)^2 = m_o^2c^4 + p^2c^2$$

$$\Rightarrow p^2c^2 = KE^2 + 2m_oc^2 K.E$$

Now since $K.E = h\nu - h\nu'$

$$\Rightarrow P_e^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2m_oc^2(h\nu - h\nu') \quad \text{— (B)}$$

putting the value of $p_e^2 c^2$ from ③ in eq^n. Ⓐ, we have

$$2m_0c^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1-\cos\phi)$$

Divide this eq^n by $2h^2c^2$, we have

$$\frac{m_0c}{h}\left(\frac{\nu}{c} - \frac{\nu'}{c}\right) = \frac{\nu}{c}\frac{\nu'}{c}(1-\cos\phi)$$

and so, since $\frac{\nu}{c} = \frac{1}{\lambda}$ and $\frac{\nu'}{c} = \frac{1}{\lambda'}$

$$\frac{m_0c}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1-\cos\phi}{\lambda\lambda'}$$

or, $\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\phi)}$ is the Compton effect.

$\frac{h}{m_0c} = \lambda_c$ = Compton wavelength.

$\boxed{\lambda_c \simeq 2.426 \times 10^{-12}\,m \text{ for } e^-}$

$\Rightarrow \boxed{\lambda' - \lambda = \lambda_c(1-\cos\phi)}$

$(\lambda' - \lambda)_{max.}$ = max. Change is for $\phi = \pi = 180°$

$(\lambda' - \lambda)_{max} = 2\lambda_c$ = Twice the Compton wavelength

In the last class, we had

$$E = KE + m_0c^2$$

$$E = \sqrt{m_0^2c^4 + p^2c^2} \longrightarrow \text{total energy of any particle.}$$

$\Rightarrow$ Equating both we have

$$(KE + m_0c^2)^2 = m_0^2c^4 + p^2c^2$$

$$(KE)^2 + \cancel{(m_0c^2)^2} + 2(K.E)\,m_0c^2 = \cancel{m_0^2c^4} + p^2c^2$$

$$\Rightarrow p^2c^2 = (KE)^2 + 2m_0c^2(KE)$$

Now $KE = h\nu - h\nu'$

$$p^2c^2 = (h\nu - h\nu')^2 + 2m_0c^2(KE)$$

$$p^2c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu') + 2m_0c^2(h\nu - h\nu')$$

putting this value of p^2c^2 in (A), we have,

$$\cancel{(h\nu)^2} - 2(h\nu)(h\nu')\cos\phi + \cancel{(h\nu')^2}$$

$$= \cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2(h\nu)(h\nu') + 2m_0c^2(h\nu - h\nu')$$

dividing by $2h^2c^2$

$$\frac{\cancel{2}h\nu\, h\nu'(1-\cos\phi)}{\cancel{2}h^2c^2} = \frac{\cancel{2}m_0c^2(h\nu - h\nu')}{\cancel{2}h^2c^2}$$

$$\frac{\nu\nu'}{c^2}(1-\cos\phi) = \frac{m_0}{h}(\nu - \nu')$$

$c = \nu\lambda$

$\nu = \frac{c}{\lambda}$

$$= \frac{m_0c}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$\frac{c^2}{\lambda\lambda' c^2}(1-\cos\phi) = \frac{m_0c}{h}\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right)$$

$$\Rightarrow \boxed{(\lambda' - \lambda) = \frac{h}{m_0c}(1-\cos\phi)}$$

Compton effect.

$\lambda_c = \frac{h}{m_0 c}$ is called the Compton wavelength

putting the value of m_0 = e mass

$\lambda_c = 2.426 \times 10^{-12}$ m it is independent of wavelength.

$= 2.426$ pm. = Compton wavelength.

$\lambda - \lambda' =$ max^m when $1 - \cos\phi = 2$

$\Rightarrow \cos\phi = -1$

$\Rightarrow \phi = \pi$

i.e when scatt. angle is π, change will be max^m.

~~De Broglie wave~~

Wave particle duality

De Broglie wave / matter wave

De Broglie wave length,

Calculation of De Broglie wavelength / Numerical example.

Wave particle duality:

A moving body behaves in certain ways as though it has a wave nature.

A photon of frequency ν has the momentum

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \quad \text{as} \quad \nu\lambda = c$$

$\Rightarrow$ photon wavelength $\boxed{\lambda = \frac{h}{p}}$

\# De Broglie suggested that this eq^n. $\lambda = \frac{h}{p}$ is a completly general one that applies to a material particles as well as photon.

if the mass of the body $= m$

velocity $= v$

$p = mv$

$$\Rightarrow \boxed{\lambda_D = \frac{h}{mv}} \quad \text{where} \quad m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Ex.8 in phys | 1, 2, 3, 4, 5, 12, 13

Ex:- Find the De-Broglie wavelength of

(a) a mass 46 g golf ball with velocity of 30 m/s

(b) ~~A~~ an electron with velocity of 10^7 m/s

(a) $v << c \Rightarrow m = m_0$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(0.046\ kg)(30\ m/s)} = 4.8 \times 10^{-34}\ m.$$

(b) $v << c$. $m = m_0$

$$\lambda = \frac{h}{mv} = 7.3 \times 10^{-11}\ m.$$

Wave function:- (ψ)

- As we know in ripple water wave, height of the water surface varies.
- In E.M wave, it is, E & B varies
- In sound it is pressure

Similarly for matter wave or De-Broglie it is wave fun. (ψ)

Physical meaning of wave fun. (ψ):-

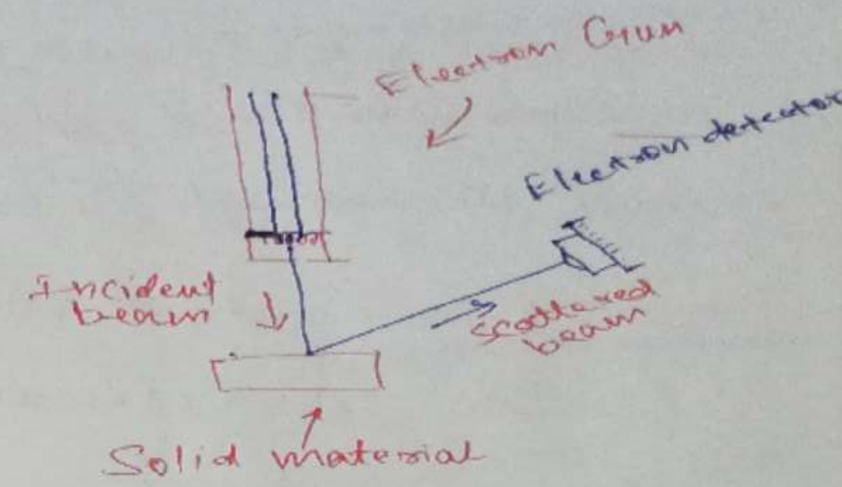
$\psi \rightarrow$ no physical meaning.

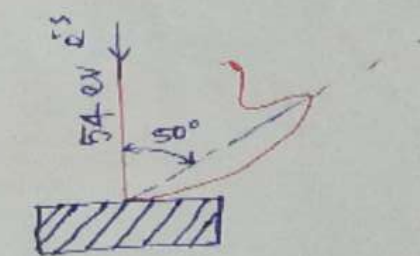
$|\psi|^2 \rightarrow$ represents the probability density.

~~Tunneling effect~~

atomic & molecular by S.P. Taneja

Wave Properties of Particles, or Davisson-Germer experiment





Instead of a continuous variation of scattered electron intensity with angle, distinct maxima & minima were observed whose positions depends on upon electron energy!

The Bragg equation for max^m in the diffraction pattern

$$n\lambda = 2d\sin\theta$$

for $d = 0.091\,nm$ and $\theta = 65°$, for $n=1$, the de Broglie wavelength λ of the diffracted ~~pattern~~ e's is

$$\lambda = 2d\sin\theta = 2 \times (0.091) \times \sin 65° = 0.165\,nm$$

$$\Rightarrow \boxed{\lambda = 0.165\,nm}$$

Now the De Broglie wave corresponding to e^-

$$\lambda = \frac{h}{mv}$$

$$KE = \frac{1}{2} mv^2$$

$$\Rightarrow mv = \sqrt{2m(KE)}$$

$$= \sqrt{2 \times (9.1 \times 10^{-31}\,kg) \times (54\,eV)\ 1.6 \times 10^{-19}\,(J/eV)}$$

$$= 4.0 \times 10^{-24}\ kg \cdot m/s$$

$\Rightarrow$ The electron wavelength

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{4.0 \times 10^{-24}\ kg\ m/s}$$

$$= 1.66 \times 10^{-10}\ m$$

$$\boxed{\lambda = 0.166\,nm}$$

Which agrees ~~with~~ well with the observed wavelength of 0.165 nm.

$\Rightarrow$ Davisson-Germer experiment directly verifies de Broglie's hypothesis of the wave nature of