SOLOW GROWTH MODEL

①

Endogeneity of Capital - Output Ratio

↓ Why?

Driven by the postulate of diminishing returns. [a departure from Harrod Domar Model, which assumes Constant Returns]

Outcome: Growth dies out in the long run and all the economy will converge to their steady state levels. of

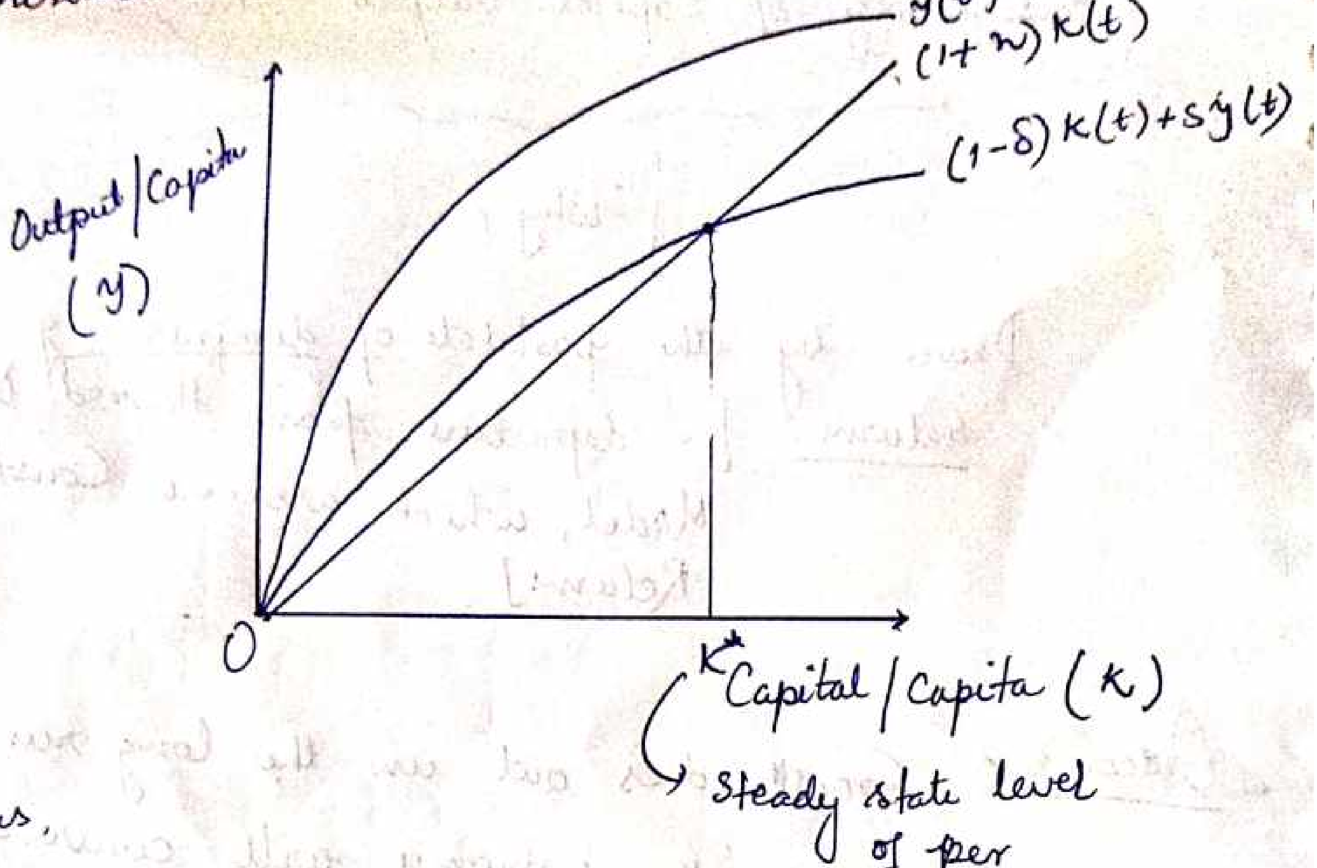
How it works?

$$\underbrace{K(t+1)}_{\text{Capital Stock at time } t+1} = (1-\underbrace{\delta}_{\text{depreciation rate}})K(t) + \underbrace{s}_{\text{savings rate}}\underbrace{Y(t)}_{\text{total income/output}} \quad \text{—— ①}$$

Dividing Eqn. ① by population @ $P(t)$, and assuming that popn. grows at a constant rate, n, i.e. $P(t+1) = (1+n)P(t)$. we get,

$$(1+n)\,k(t+1) = (1-\delta)\,k(t) + s\,y(t) \quad \text{—— ②}$$

Translating Eqn. 2 into the diagramatic representation.



Thus,

→ No Long Run growth of per capita output

→ Total output grows precisely at the rate of growth of population.

Determining the ~~state~~ steady state level:

Using 2,

$$(1+n)\, k(t+1) = (1-\delta)\, k(t) + s y(t).$$

Once $k(t)$ hits the steady state level,

$$k(t) = k(t+1) = k(t+2) \dots = k^*.$$

$\therefore \; (1+n)k^* = (1-\delta)k^* + sy^*$, where y^* = steady state output level.

$\Rightarrow (\delta+n)k^* = sy^*$.

$$\Rightarrow \boxed{\frac{k^*}{y^*} = \frac{s}{n+\delta}} \quad (3).$$

From (3), we see that the parameters s, n & δ have level effects on $\left(\frac{k^*}{y^*}\right)$.

Higher 's' or savings rate $\Rightarrow$ higher k^* and y^*. (i.e. the steady state levels are higher).

Similarly, higher 'n' or 'δ' $\Rightarrow$ lower k^* and y^*.

So, the Solow model infers that regardless of the initial per capita capital stock, 2 countries with similar savings rates, depreciation rates and population growth rates will converge to similar standards of living in the "LONG RUN".

ROLE OF TECHNICAL PROGRESS :-

SOLOW GROWTH MODEL

↓

NO LONG RUN GROWTH

↓

SOLOW MODEL ← Introduce Technical Progress

↓

YES LONG RUN GROWTH.

How ??

Technical Progress

↓

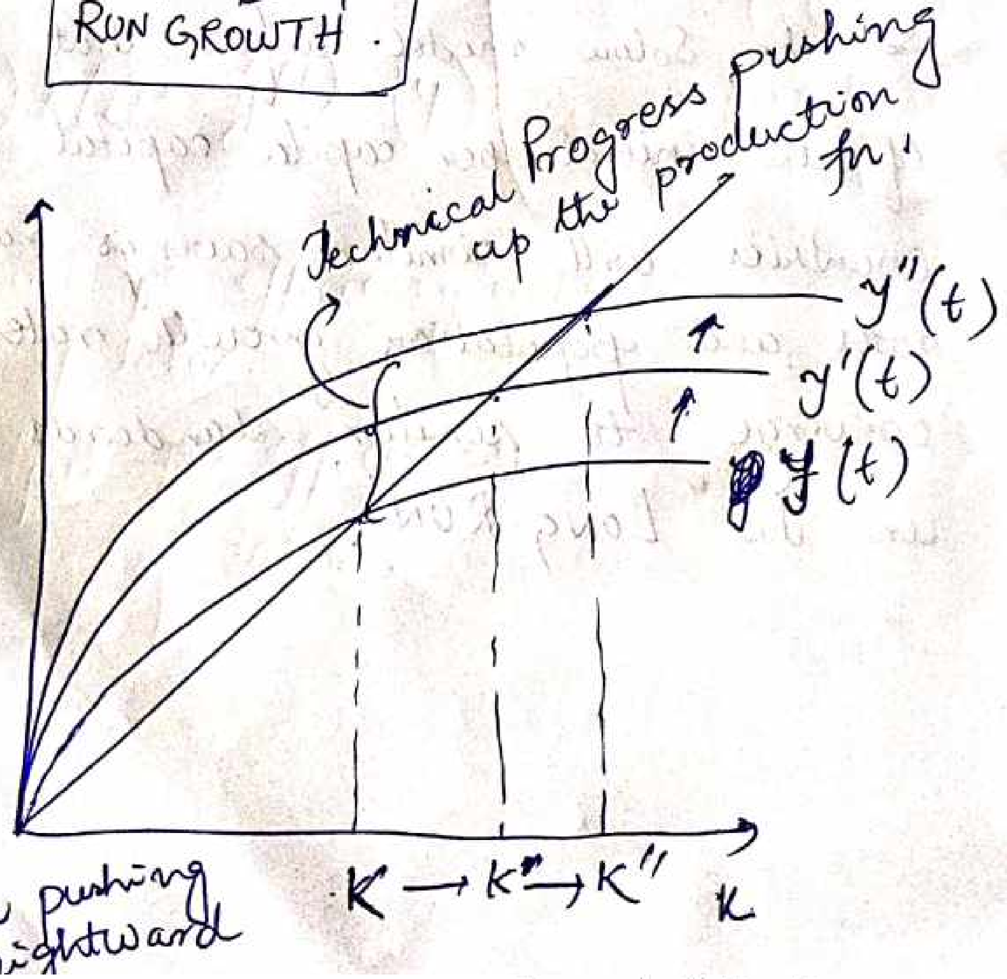
Production more advanced.

↓

Production fn. shifts upwards.

↓

Steady state keeps on pushing rightward



Introducing Technical Progress into the Original Solow Model Framework:

+ Technical Progress is captured by the efficiency of working Population ($P(t)$).

$\therefore$ Rate of Growth of Efficiency of Working Population = Rate of Growth of Technical Progress.

We have,

$$L(t) = E(t)\,P(t),$$

Where $L(t)$ = amount of labor in "efficiency units".

$E(t)$ = efficiency / productivity of individual at time 't'.

$$E(t+1) = (1+\pi)\,E(t).$$

i.e. π = rate of growth of efficiency of labor

= rate of technical progress.

Dividing Eqn. 1 by $L(t)$, we get.

$$\frac{K(t+1)}{L(t)} = (1-\delta)\frac{K(t)}{L(t)} + s\frac{Y(t)}{L(t)}.$$

$$\cancel{\Rightarrow \frac{K(t+1)}{E(t)P(t)} = (1-\delta)}$$

$$\Rightarrow \frac{(1+\pi)(1+n)K(t+1)}{L(t+1)} = (1-\delta)\hat{k}(t) + s\hat{y}(t)$$

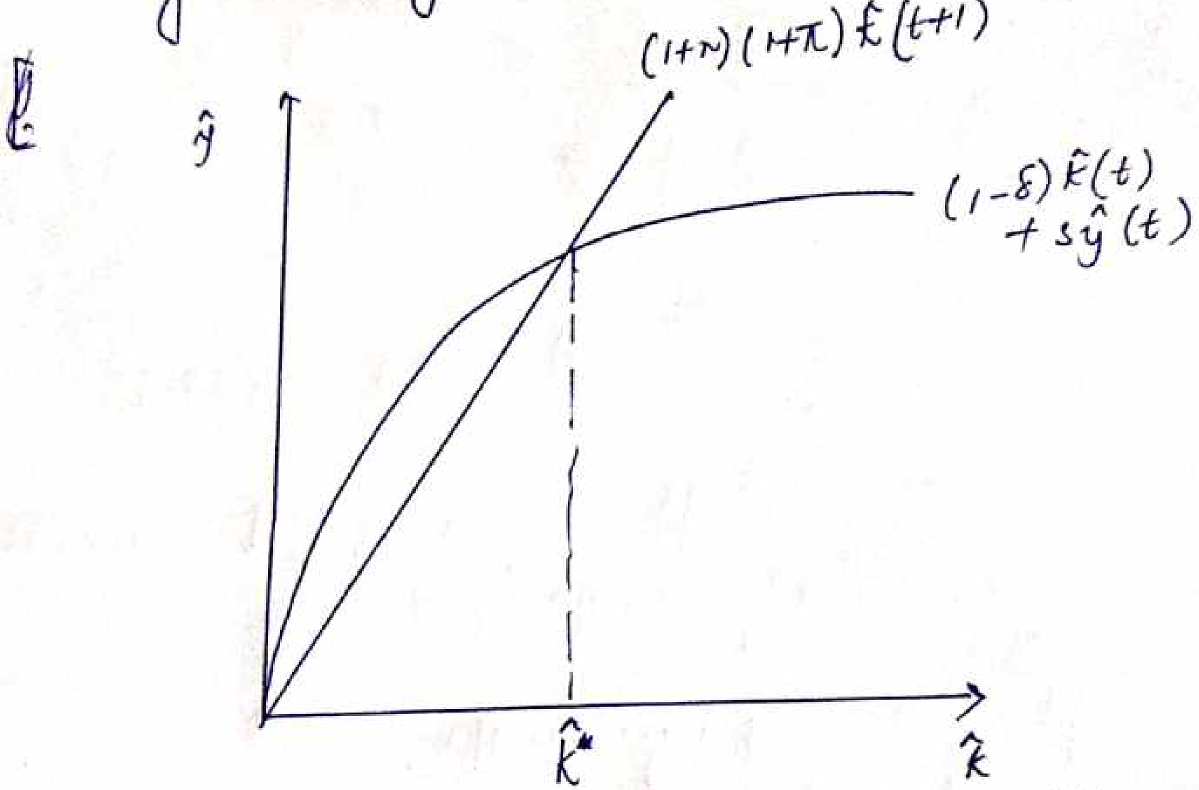
$$\Rightarrow (1+\pi)(1+n)\hat{k}(t+1) = (1-\delta)\hat{k}(t) + s\hat{y}(t) \quad (4).$$

$$; \boxed{\begin{aligned} L(t+1) &= E(t+1)P(t+1) \\ &= (1+n)(1+\pi)E(t)P(t) \\ &= (1+n)(1+\pi)L(t) \end{aligned}}$$

where, $\hat{k}$ = capital per efficiency unit of labor.

$\hat{y}$ = output per efficiency unit of labor.

Eqn. 4 is similar to eqn. 2, and if we follow our analysis of Solow, Eqn. 4 would also result into an outcome similar to that of 2, where eventually, the economy converges to the steady state.



All this lead to the steady state level $\hat{k}^*$. The novelty lies in the interpretation. Note that even though capital per efficiency unit converges to a stationary steady state, the amount of capital per ~~wo~~ member of the working population continues to increase. Indeed, the long run increase in per capita income takes place precisely at the rate of technical progress.

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