FRAUNHOFER DIFFRACTION

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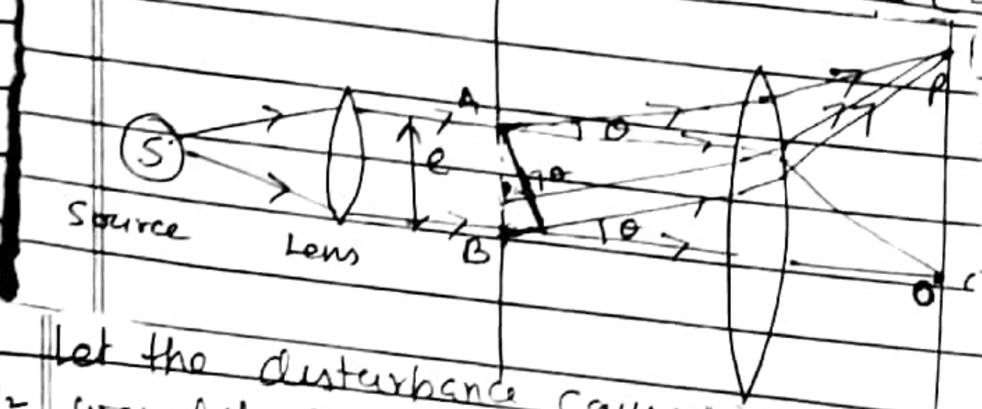
Single slit :- Let a parallel beam of light having wavelength λ incident normally on narrow slit of distance $e = AB$



Let the disturbance caused at c by wavelet from unit width of slit by

$$y_0 = A\cos\omega t \quad \text{--- (1)}$$

Then take small portion of the slit dx width at c. When it reaches at P amplitude Adx and phase will be $\omega t + \frac{2\pi}{\lambda}x\sin\theta$

Let the disturbance be dy we have

$$dy = Adx\cos\left(\omega t + \frac{2\pi}{\lambda}x\sin\theta\right)$$

for total disturbance

$$y = \int_{-e/2}^{+e/2} dy = \int_{-e/2}^{+e/2} A\cos\left(\omega t + \frac{2\pi}{\lambda}x\sin\theta\right)dx$$

$$y = A\cos\omega t\int_{-e/2}^{+e/2}\cos\frac{2\pi}{\lambda}x\sin\theta\, dx - A\sin\omega t\int_{-e/2}^{e/2}\sin\frac{2\pi}{\lambda}x\sin\theta\, dx$$

$$= A\cos\omega t\left[\frac{\sin\frac{2\pi}{\lambda}x\sin\theta}{\frac{2\pi}{\lambda}\sin\theta}\right]_{-e/2}^{e/2}$$

$$= A\cos\omega t \frac{\left(2\sin\frac{\pi}{\lambda}e\sin\theta\right)}{\frac{2\pi}{\lambda}\sin\theta}$$

$$y = A\cos\omega t \frac{\left(\sin\frac{\pi}{\lambda}e\sin\theta\right)}{\frac{\pi}{\lambda}\sin\theta}$$

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put $\frac{\pi}{\lambda}e\sin\theta = \alpha$

$y = A\cos\omega t \frac{\sin\alpha}{\alpha}$ — A_0 is the amplitude at $\theta = 0$

then $y = A_0 \cos\omega t \frac{\sin\alpha}{\alpha}$ $\quad A \to A_0$

Resultant amplitude $= y = R$

$$R = A_0 \frac{\sin\alpha}{\alpha}$$

Then Resultant Intensity at P is given by $I = R^2$

$$= A_0^2 \frac{\sin^2\alpha}{\alpha^2}$$

$\boxed{\cos\omega t = 1}$ $\theta = 0$

$\frac{\sin^2\alpha}{\alpha^2}$ gives the intensity at different point at diff value of θ

Position of central or Principal maxima for central point O on screen

$\alpha = 0$

$\lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$, $I = I_0 \frac{\sin^2\alpha}{\alpha^2} = I_0$

This is maximum as all waves reaches O in phase $\alpha = 0$

$$\frac{\pi}{\lambda} e\sin\theta = 0, \quad \theta = 0$$

This shows waves are travelling normal to the slit and O gives the position of central maximum.

Position of minima: The intensity is minimum (zero) when $\frac{\sin\alpha}{\alpha} = 0$

$\alpha \neq 0$,

$$\frac{\pi}{\lambda} e\sin\theta = \pm m\pi$$

$$e\sin\theta = \pm m\lambda \quad m = 1, 2, 3 \ldots$$

Secondary maxima: — The direction of mth secondary maximum is given by

$$e\sin\theta = \pm\left(m + \frac{1}{2}\right)\lambda \quad \text{for } m = 1, 2, 3$$

$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2},$

Now $I_1 = I_0 \left(\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = \frac{I_0}{22}$

$I_2 = I_0 \left(\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2 = \frac{I_0}{61}$

The intensity of secondary maxima falls off rapidly.

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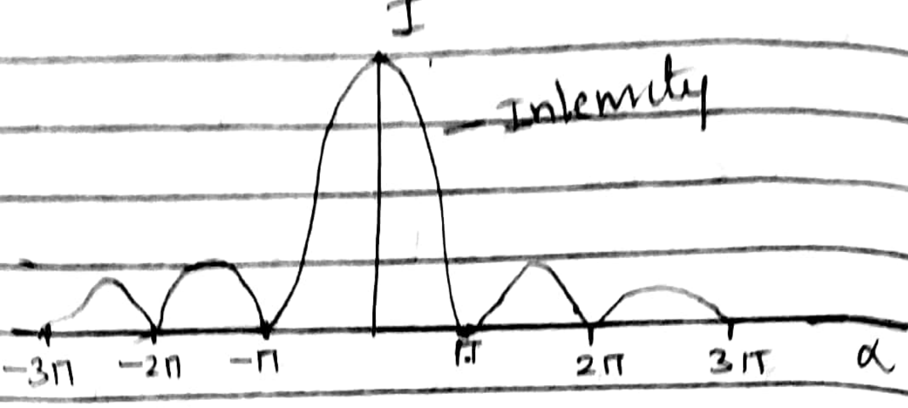
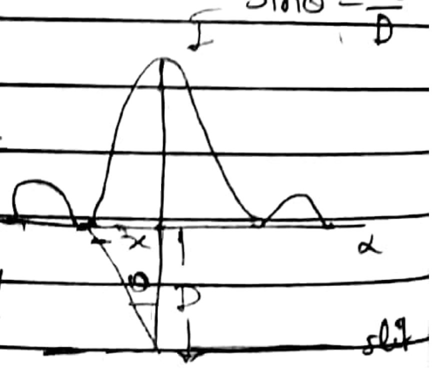


Thus the secondary maxima are displaced toward the centre of system.

Width of central maxima :- Let the distance of first ~~centre of~~ secondary minima from centre of principal maxima will be x the width of central maxima will be $= 2x$. Let the Distance between slit and screen be D then $\tan\theta =$

$\sin\theta = \frac{x}{D}$



For first minimum

$$\sin\theta = \pm\frac{\lambda}{e} = \frac{x}{D}$$

$$x = \frac{\lambda D}{e}$$

If $D = f$ focal length of lens

$$2x = \frac{2\lambda f}{e}$$

Effect of slit width →

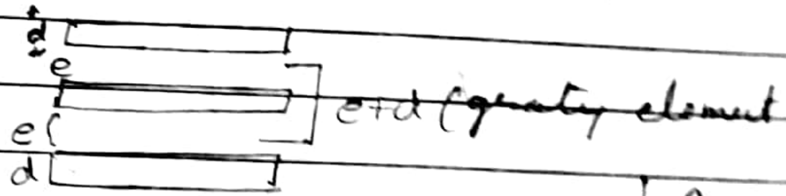
As we know $\sin\theta = \pm\frac{\lambda}{e}$ if slit width e is large θ will be small this mean that maxima and minima lie very close to central maxima. If e is very small then θ is large.

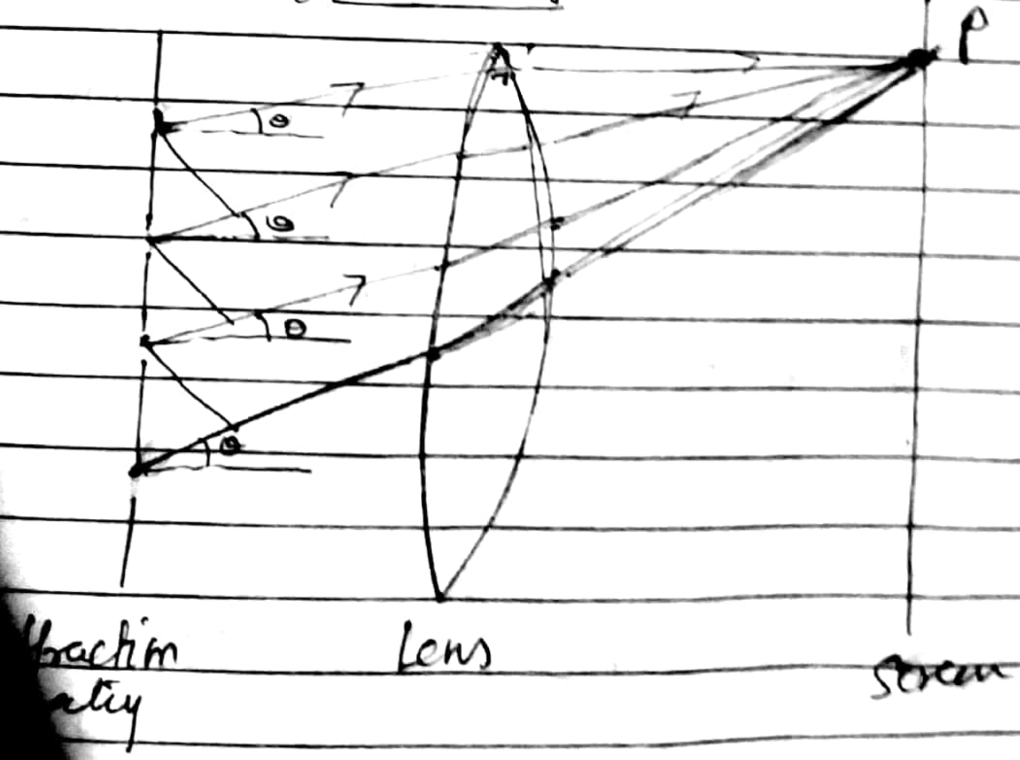
Then diffraction maxima and minima are

quit clear.

Fraunhofer diffraction due to N parallel slit (Diffraction grating)

Diffraction grating is an arrangement of N number of parallel slits of equal width are separated from one another by equal space. It is constructed by drawing several equidistant parallel lines on optically plane glass plate with with a pointed diamond. The distance b/w centre of two consecutive slit is called grating element i.e $e+d$
e → width of each slit, d = width of each opaque space.





Let a parallel beam of monochromatic light of wavelength λ be incident normally to the grating at angle θ each of amplitude

$$A_\theta = A_0 \frac{\sin\alpha}{\alpha}$$

and successive phase difference

$$\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta$$

The resultant disturbance y is given by

$$y = A_\theta\left[\cos\omega t + \cos(\omega t+\delta) + \cdots N \text{ term}\right]$$

$$y = \text{Real part of } A_\theta e^{i\omega t}\left[1 + e^{i\delta} + e^{2i\delta} + \cdots N\right]$$

$$= A_\theta e^{i\omega t}\left[\frac{1-e^{iN\delta}}{1-e^{i\delta}}\right]$$

Intensity $= yy^*$

$$= A_\theta e^{i\omega t}\left[\frac{1-e^{iN\delta}}{1-e^{i\delta}}\right]\left[\frac{1-e^{-iN\delta}}{1-e^{-i\delta}}\right] \times A_\theta e^{-i\omega t}$$

where $\cos N\delta = \frac{e^{iN\delta}+e^{-iN\delta}}{2}$

$$\cos\delta = \frac{e^{i\delta}+e^{-i\delta}}{2}$$

$$I = A_\theta^2\left[\frac{1-\cos N\delta}{1-\cos\delta}\right]$$

$$= A_\theta^2 \frac{2\sin^2\frac{N\delta}{2}}{2\sin^2\delta/2}$$

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$$= A_0^2 \frac{\sin^2\left[\frac{N\pi(e+d)\sin\theta}{\lambda}\right]}{\sin^2\left(\frac{\pi(e+d)\sin\theta}{\lambda}\right)}$$

we know that $A_0 = A_0 \frac{\sin\alpha}{\alpha}$

put this value in the ~~eq~~ equation above

then $\boxed{I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}}$

where $\beta = \frac{\pi(e+d)\sin\theta}{\lambda}$

Intensity distribution is the product of two term $A_0^2 \frac{\sin^2\alpha}{\alpha^2}$ represent the diffraction pattern due to single slit

$\frac{\sin^2 N\beta}{\sin^2\beta}$ represent the pattern due to N slits

If $N=1$, the $I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \cancel{\frac{\sin^2\beta}{\sin^2\beta}}$

$$= A_0^2 \frac{\sin^2\alpha}{\alpha^2}$$ (single slit diffraction)

$N=2$ the $I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$

$$= A_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 2\beta}{\sin^2\beta}$$

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$$= 4A_0^2 \frac{\sin^2\alpha}{\alpha^2} \cos^2\beta$$

(Double slit diffraction)

<u>Position of Principal Maxima</u>:– The intensity would be maximum when $\sin\beta = 0$ $\quad \beta = \pm n\pi$, $n = 0, 1, 2$.

But $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ using L Hospital rule $\frac{\sin N\beta}{\sin\beta} \rightarrow N$

then $I_p = A_0^2 \frac{\sin^2\alpha}{\alpha^2} N^2$

These maxima are most intense and called Principal maxima.

again $\beta = \pm n\pi$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$\boxed{(e+d)\sin\theta = \pm n\lambda}$$ this is known as grating equation for $n=0$ zero order maxima, $n = \pm 1$ first order maxima

$n = \pm 2$ second order maxima

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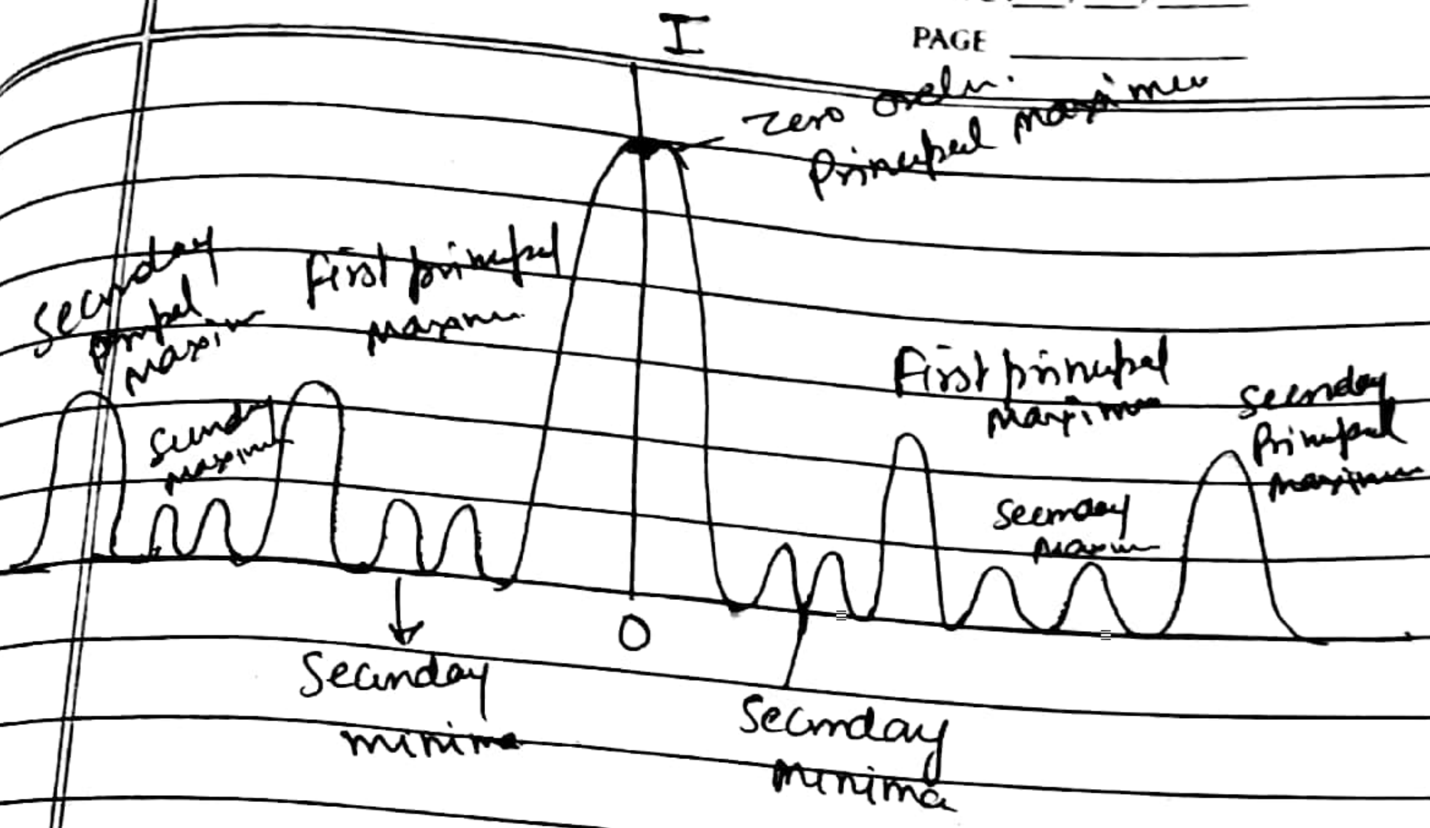
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zero order principal maxima
secondary principal maxima
first principal maxima
secondary maxima
first principal maxima
secondary principal maxima
secondary maxima
O
Secanday minima
Secanday minima