K- Map- Simplification / Minimisation of logic circuit:

Consider Logical equation $Y = (A+BC)\cdot(B+A\bar{C})$. — ①

This can be simplified into the form

$$Y = AB + BC + A\bar{C} \quad \text{— (2).}$$

This equation is called S.O.P. equation

$$Y = (A+B)\cdot(B+\bar{C})\cdot(A+C) \quad \text{— (3)}$$

This is product of sum equation POS.

In S.O.P and POS form of equations, all the individual terms do not involve all the three literals. If each term in S.O.P and POS forms contains all the literals and then these are known as standard (or canonical) SOP and POS, respectively. Each individual term in standard SOP form is called as minterm and in Standard POS form as maxterm.

SOP form can be converted into standard sum of product form (SSOP) by ANDing the terms in the expression with the terms formed by ORing the variable and its complement which are not present in that term.

convert eq (2) in SSOP

in first term C is missing, in second term A is missing and in last term B is missing. ~~term~~. OR the missing literal with its complement and AND it with that term

$$Y = AB[C+\bar{C}] + BC[A+\bar{A}] + A\bar{C}[B+\bar{B}]$$

$$Y = \underline{ABC} + \underline{\underline{AB\bar{C}}} + \underline{ABC} + \bar{A}BC + \underline{\underline{AB\bar{C}}} + A\bar{B}\bar{C}$$

$$Y = (ABC + ABC) + (AB\bar{C} + AB\bar{C}) + \bar{A}BC + A\bar{B}\bar{C}$$

$$Y = ABC + AB\bar{C} + \bar{A}BC + A\bar{B}\bar{C}. \quad \text{—— (4).}$$

This is SSOP form. Terms in eq (4) are called minterms.

Convert POS form of equation into standard POS form.

$$Y = (A+B)\cdot(A+C)\cdot(B+\bar{C}). \quad \text{—— (5).}$$

POS can be converted into standard POS by ORing the terms in the expression with the terms formed by ANDing the variable and its complement which are not present in that term.

From eq (5)

$$Y = (A+B+C\cdot\bar{C})\cdot(A+C+B\bar{B})\cdot(B+\bar{C}+A\cdot\bar{A})$$

using the property $[A+B\cdot C] = (A+B)\cdot(A+C)$.

$$= (A+B+C)\cdot(A+B+\bar{C})\cdot(A+C+B)\cdot(A+C+\bar{B}).$$
$$\cdot\,\cancel{(B+A+\bar{C})}\cdot\cancel{(\bar{B}+\bar{A}+\bar{C})}\cdot(B+\bar{C}+A)(B+\bar{C}+\bar{A}).$$

$$Y = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+C+\bar{B})\cdot(\bar{A}+B+\bar{C}). \quad \text{—— (6)}$$

Equation (6) is standard product of sum equation, and terms in eq. 6 are called Maxterms.

These standardization of Boolean expressions makes their evaluation, simplification and interpretation quite easy and systematic.

Minterms / Maxterms for 2, 3 & 4 variables

Two variables

Variable A	Variable B	Minterm m	Maxterm M
0	0	$\bar{A}\bar{B} = m_0$	$A+B = M_0$
0	1	$\bar{A}B = m_1$	$A+\bar{B} = M_1$
1	0	$A\bar{B} = m_2$	$\bar{A}+B = M_2$
1	1	$AB = m_3$	$\bar{A}+\bar{B} = M_3$

Three variables

A	B	C	Minterm m	Maxterm M
0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A+B+C = M_0$
0	0	1	$\bar{A}\bar{B}C = m_1$	$A+B+\bar{C} = M_1$
0	1	0	$\bar{A}B\bar{C} = m_2$	$A+\bar{B}+C = M_2$
0	1	1	$\bar{A}BC = m_3$	$A+\bar{B}+\bar{C} = M_3$
1	0	0	$A\bar{B}\bar{C} = m_4$	$\bar{A}+B+C = M_4$
1	0	1	$A\bar{B}C = m_5$	$\bar{A}+B+\bar{C} = M_5$
1	1	0	$AB\bar{C} = m_6$	$\bar{A}+\bar{B}+C = M_6$
1	1	1	$ABC = m_7$	$\bar{A}+\bar{B}+\bar{C} = M_7$

Four variables:

Variables	Minterms	Maxterms
A B C D	m_i	M_i
0 0 0 0	$\bar{A}\bar{B}\bar{C}\bar{D} = m_0$	$A+B+C+D = M_0$
0 0 0 1	$\bar{A}\bar{B}\bar{C}D = m_1$	$A+B+C+\bar{D} = M_1$
0 0 1 0	$\bar{A}\bar{B}C\bar{D} = m_2$	$A+B+\bar{C}+D = M_2$
0 0 1 1	$\bar{A}\bar{B}CD = m_3$	$A+B+\bar{C}+\bar{D} = M_3$
0 1 0 0	$\bar{A}B\bar{C}\bar{D} = m_4$	$A+\bar{B}+C+D = M_4$
0 1 0 1	$\bar{A}B\bar{C}D = m_5$	$A+\bar{B}+C+\bar{D} = M_5$
0 1 1 0	$\bar{A}BC\bar{D} = m_6$	$A+\bar{B}+\bar{C}+D = M_6$
0 1 1 1	$\bar{A}BCD = M_7$	$A+\bar{B}+\bar{C}+\bar{D} = M_7$
1 0 0 0	$A\bar{B}\bar{C}\bar{D} = m_8$	$\bar{A}+B+C+D = M_8$
1 0 0 1	$A\bar{B}\bar{C}D = m_9$	$\bar{A}+B+C+\bar{D} = M_9$
1 0 1 0	$A\bar{B}C\bar{D} = m_{10}$	$\bar{A}+B+\bar{C}+D = M_{10}$
1 0 1 1	$A\bar{B}CD = m_{11}$	$\bar{A}+B+\bar{C}+\bar{D} = M_{11}$
1 1 0 0	$AB\bar{C}\bar{D} = M_{12}$	$\bar{A}+\bar{B}+C+D = M_{12}$
1 1 0 1	$AB\bar{C}D = m_{13}$	$\bar{A}+\bar{B}+C+\bar{D} = M_{13}$
1 1 1 0	$ABC\bar{D} = m_{14}$	$\bar{A}+\bar{B}+\bar{C}+D = M_{14}$
1 1 1 1	$ABCD = m_{15}$	$A+B+C+D = M_{15}$

Eaxample: using these notation eq. 4 can written as (SOP)

$Y = m_3 + m_4 + m_6 + m_7$,

$= \Sigma m(3,4,6,7)$ — (7)

$\bar{A}BC = m_3$ $\quad ABC = M_7$

$A\bar{B}\bar{C} = m_4$

$AB\bar{C} = m_6$

and the standard POS eq (6) can be written as

$Y = (M_0 \cdot M_1 \cdot M_2 \cdot M_3) = \Pi M(0,1,2,5)$ — (8)

$= A+B+C = M_0$ $\quad \bar{A}+B+\bar{C} = M_5$

$A+B+\bar{C} = M_1$

$A+\bar{B}+C = M_2$

Karnaugh Map Representation of Logical Functions:

So far we have discussed the two standard forms of logical functions and their realizations using gate. Simplification using Boolean algebratic theorems has been discussed. Some times it is difficult to be sure that a logical function can be simplified.

There is another technique, which is graphical, known as Karnaugh map ~~tey~~ technique which provides a systematic method for simplifying and manipulating Boolean expression.

In this technique, the information contained in a truth table or available in POS or SOP form is represented on K-Map. This is perhaps the most extensively used tool for simplification of Boolean function. This may be used for any number of variables, It is generally used up to six variables beyound which it becomes very cumbersome.

K Map for Two variables:

No of cells = $2^n = 2^2 = 4$.

	$\bar{A}$	A
$\bar{B}$	$\bar{A}\bar{B} = m0$	$A\bar{B} = m_2$
B	$\bar{A}B$ m1.	$AB = m_3$

K Map for 3, variables:

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}$	$\bar{A}\bar{B}C$ m0	$\bar{A}B\bar{C}$ m2	$AB\bar{C}$ m6	$A\bar{B}\bar{C}$ m4
C	$\bar{A}\bar{B}\bar{C}$ m1	$\bar{A}BC$ m3	ABC m7	$A\bar{B}C$. m5

Eq. 7, and 8 are the standard forms of S.O.P and POS respectively. These two equations represents the same logical function ~~(a)~~ Eq.(1) therefore we notice that there is a complementary type of relation ship between a function expressed in terms of minterms and in terms of maxterms.
ie. the terms corresponding to decimal numbers, 3,4,6, and 7 are minterms and the terms corresponding to decimal numbers 0,1,2 and 5 are maxterms.

4 Variable K- Map.

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}\bar{D}$ m_0	$\bar{A}B\bar{C}\bar{D}$ m_4	$AB\bar{C}\bar{D}$ m_{12}	$A\bar{B}\bar{C}\bar{D}$ m_8
$\bar{C}D$	$\bar{A}\bar{B}\bar{C}D$ m_1	$\bar{A}B\bar{C}D$ m_5	$AB\bar{C}D$ m_{13}	$A\bar{B}\bar{C}D$ m_9
CD	$\bar{A}\bar{B}CD$ m_3	$\bar{A}BCD$ m_7	$ABCD$ m_{15}	$A\bar{B}CD$ m_{10}
$C\bar{D}$	$\bar{A}\bar{B}C\bar{D}$ m_2	$\bar{A}BC\bar{D}$ m_6	$ABC\bar{D}$ m_{14}.	$A\bar{B}C\bar{D}$ m_{11}.

Table:- 1

Representation of Truth Table on K- Map.

Inputs.

A	B	C	D	Y	
0	0	0	0	1	— m_0 $\bar{A}\bar{B}\bar{C}\bar{D}$
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	1	— m_5 — $\bar{A}B\bar{C}D$
0	1	1	0	0	
0	1	1	1	1	— m_7 — $\bar{A}BCD$
1	0	0	0	0	
1	0	0	1	1	— m_9 — $A\bar{B}\bar{C}D$
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	1	— m_{12} — $AB\bar{C}\bar{D}$
1	1	0	1	0	
1	1	1	0	1	— m_{14} — $ABC\bar{D}$
1	1	1	1	1	— m_{15} — $ABCD$

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	m_0 1	m_4 0	m_{12} 1	m_8 0
$\bar{C}D$	m_1 0	m_5 1	m_{13} 0	m_9 1
CD	m_3 0	m_7 1	m_{15} 1	m_{11} 0
$C\bar{D}$	m_2 0	m_6 0	m_{14} 1	m_{10} 0

The output Y is logical 1 corresponding to minterm $m_0, m_5, m_7, m_9, m_{12}, m_{14}, m_{15}$

$$Y = \Sigma m(0,5,7,9,12,14,15)$$

$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + AB\bar{C}\bar{D} + ABC\bar{D} + ABCD$$

Procedure: Note down the fundamental product corresponding to output high. Note down the minterm number. Then enter the value of output Y (0 or 1) in cell corresponding to its decimal or minterm or maxterm identification.

On the other hand, if a K-map is given we can make the truth table corresponding to this by following the reverse process. That is, the output Y is logical 1 corresponding to the decimal numbers/minterms represented by the cell with entries 1. In all other rows, the output Y is logical zero.

Representation of standard SOP from on K-Map.

A logical equation in standard SOP form can be represented on a K-map simply entering 1's in the cell of K Map corresponding to each minterm in the equation.

example: Represent $Y = \underset{m1}{\bar{A}\bar{B}C} + \underset{m2}{\bar{A}B\bar{C}} + \underset{m4}{A\bar{B}\bar{C}} + \underset{m7}{ABC}$.

Three variable, so No of cell in K-Map = 8, as shown below

sol: Corresponding to each minterm in equation there is cell in the K-Map and a 1 is entered in each one of these cells.

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}$	m0	m2 1	m6	1 m4
C	m1 1	m3	m7	1 m5

similarly from the K-Map, we can write the corresponding logical equation in standard SOP form by OR-ing the ... corresponding to each 1 entry in the K-Map.

Simplification of logical functions using K-Map.

Simplification of logical function with K-Map is based on the principle of combining terms in adjacent cells. Two cells are said to be adjacent if they differ in only one variable.

Example:- Table 1

Top row cell → $m_0 = \bar{A}\bar{B}\bar{C}\bar{D}$

$m_1 = \bar{A}\bar{B}\bar{C}D$

$m_0 = \bar{A}\bar{B}\bar{C}\bar{D}$

$m_4 = \bar{A}B\bar{C}\bar{D}$

Literal B changes from $\bar{B}$ to B.

Top row cell $m_0 = \bar{A}\bar{B}\bar{C}\bar{D}$

Bottom row cell → $m_2 = \bar{A}\bar{B}C\bar{D}$

Differ in Literal C
[C to $\bar{C}$]

$m_0 = \bar{A}\bar{B}\bar{C}\bar{D}$ – Extreme left cell

$m_8 = A\bar{B}\bar{C}\bar{D}$. – Extreme Right cell.

Differ in literal A
[$\bar{A}$ to A].

From these examples we can say that top row cells are adjacent with the bottom row cells.

(ii) Cells in rightmost column are adjacent with the cells in the ~~right~~ Left most cell.

(iii). Cells in adjoing rows and columns are adjacent.

Pairs, Quad, and octets:

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	0	0	0
CD	0	0	1	0
$C\bar{D}$	0	0	1	0

This map contains a pair of 1's that are vertical adjacent. The first 1 represents the product ABCD: the second 1 stands for the product $ABC\bar{D}$. As we move from the first 1 to the second 1, only one variable goes from uncomplemented to complemented form (D to $\bar{D}$); the other variables don't change form (A, B, and C remain uncomplemented). whenever this happens, you can eliminate the variable that changes form.

$$Y = ABC$$

<u>sol</u> by Algebraic method

$$Y = ABCD + ABC\bar{D}$$
$$= ABC[D + \bar{D}]$$
$$Y = ABC$$

simblify the K- Map:

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	0	1	1
$\bar{C}D$	0	1	0	0
CD	0	1	0	0
$C\bar{D}$	0	0	0	0

simplified product of 1's row ~~cta~~ pair = ~~$\bar{B}\bar{C}\bar{D}$~~ $A\bar{C}\bar{D}$

simplified product of 2nd column pair is = $\bar{A}BD$
(C changes to $\bar{C}$, so it can be elliminated.

The corresponding Boolean equation for this map is $Y = A\bar{C}\bar{D} + \bar{A}BD$.

<u>Simplify the K- Map</u>

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}$	1			1
C		1	1	

Simplified product of
1st row = $\bar{B}\bar{C}$, [A changes to $\bar{A}$]
2nd row pair = BC [A changes to $\bar{A}$]

Boolean expression = $BC + \bar{B}\bar{C}$

The SSOP form of equation can be written as

$$\cancel{Y = \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}BC + ABC}$$

$$\cancel{= \bar{\bar{A}}\bar{B}\ BC[\bar{A} + \bar{A}]}$$

$$Y = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + ABC.$$

$$= [\bar{A} + A]\bar{B}\bar{C} + [\bar{A} + A]BC$$

$$= \bar{B}\bar{C} + BC.$$

It can be seen from these examples, that when pair is formed, one variable is elliminated. Algebric method takes more time to simplify the the K-map.

Grouping Four Adjacent One's: Making Quad.

A quad is a group of four 1s that are horizontally or vertically adjacent.

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$				
$\bar{C}D$				
CD	1	1	1	1
$C\bar{D}$				

Fig (a)

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$				
$\bar{C}D$				
CD			1	1
$C\bar{D}$			1	1

Fig (b)

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$				
$\bar{C}D$				
CD	1	1	1	1
$C\bar{D}$				

Fig (c).

The 1s may be end-to-end as shown in (a) or in the form of a squre as in (b). When we see a quad, always encircle it because it leads to a simpler product. In fact, a quad means two variable and their complements drop out of the Boolean equation.

Ans (a) $Y = CD$,

(b) $Y = AC$.

For fig (b).

Pick any 1 as a starting point. When you move horizontally, D is the variable that changes form; when you move vertically, B changes form. Therefore the remaing variables (A and C) are the only ones appearing in the simplified product. Hence the

$$y = AC.$$

visualize the four 1's of fig a, as two pairs (see fig c). The first pair represents $CD\bar{A}$ ~~$A\bar{B}\bar{C}$~~, the second pair stands for ADC. The Boolean expression for these pairs is

$$\cancel{y = \bar{A}\bar{B}C + A\bar{B}C} \qquad \cancel{y = AC\bar{D} + ADC}$$

$$\cancel{= A\bar{B}(C+\bar{C})} \qquad \cancel{= AC[D+\bar{D}]}$$

$$\cancel{= AC.} \qquad \cancel{= AC}$$

$$y = ACD + \bar{A}CD$$

$$= CD[A + \bar{A}].$$

$$= CD.$$

Example:

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	1 m_0	m_4	m_{12}	1 m_8
$\bar{C}D$	1 m_1	m_5	m_{13}	1 m_9
CD	1 m_3	m_7 1	1 m_{15}	1 m_{11}
$C\bar{D}$	m_2	m_6	m_{14}	m_{10}

Here two Quads can be forms.

① In first quat. two variables appear as $\bar{B}$ and $\bar{C}$ in all the four terms.

K- Map Simplifications:

① From the examples it is learnt that a pair eliminates one variable and its complement, a quad ~~elimina~~ eliminates two variable and their complements, and an octet eliminates three variables and their complements.

Because of this octets must be encircled first, the quad second, and the pairs last. In this way, the greatest simplification results.

Example:

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	0	1	1
$\bar{C}D$	1	0		1
CD	1	0	0	0
$C\bar{D}$	1	1	1	1

Sol

No octet.

Two quad and one pair.

Pair represent the simplified product $\bar{A}\bar{B}D$

upper quad product is $A\bar{C}$

lower quad represents $C\bar{D}$.

Boolean expression is obtained by O-ring the simplified products.

$$Y = \bar{A}\bar{B}D + A\bar{C} + C\bar{D}.$$

(2). The variable A appears as A in two and $\bar{A}$ in the other two minterms.

3. The variable D appears as D in two and $\bar{D}$ in other two minterms.

4 The combination of these four minterms results in one term with two literals which are present in all the four terms.

similarly the second quad is simplified to CD. Therefore, the K- Map is simplified to

$$Y = \bar{B}\bar{C} + CD.$$

The Octet

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$				
$\bar{C}D$				
CD	1	1	1	1
$C\bar{D}$	1	1	1	1

This is the group of eight 1s. An octet like this eliminates three variables and their complements.

Here C is present in all the minterms.
D changes to $\bar{D}$, so eliminate D.
B changes to $\bar{B}$, A changes to $\bar{A}$, so B and A can be elliminated.

$$Y = C.$$

② Over lapping Groups.

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	0	1	1
$\bar{C}D$	0	1	1	1
CD	0	0	1	1
$C\bar{D}$	0	0	1	1

When you encircle groups, you are allowed to use the same 1 more than once. In other words, the same 1 can be common to two or more groups. The above K-Map illustrates the idea. The 1 representing the fundamental product $AB\bar{C}D$ (shaded one) is a part of pair and part of the octet.

octet simplification is A,
pair simplification is $B\bar{C}D$.

The simplified equation is

$$Y = A + B\bar{C}D.$$

③ Rolling the K-Map.

See the K-Map shown below.

Two pairs are shown here. These pairs results in the equation

$$Y = B\bar{C}\bar{D} + C\bar{D}B.$$

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	1	1	0
$\bar{C}D$	0	0	0	0
CD	0	0	0	0
$C\bar{D}$	0	1	1	0

Visualize the picking up the K-Map and rolling it so that top row touches the bottom side. These two pair forms the quad [because these are adjacent cell].

Quad leads the simplification $B\bar{D}$

$$Y = B\bar{D}.$$

Eliminating Redundant Groups:

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$			1	
$\bar{C}D$	1	1	1	
CD		1	1	1
$C\bar{D}$		1		

(a).

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$			1	
$\bar{C}D$	1	1	1	
CD		1		1
$C\bar{D}$		1		

(b).

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$			1	
$\bar{C}D$	1	1	1	
CD		1	1	1
$C\bar{D}$		1		

(C).

Given K-map (Fig-a), first you can do ~~as~~ encircle the quad in the center of Map.
Next you can group the remaining 1's into pairs by ~~ope~~ overlapping Fig (b). The final step is to review the encircled groups to see if any group is redundant (all of its 1s overlapped by other groups). In fig b each one 1 in the quad is overlapped by a pair. Because of this the quad is redundant and can be elliminated to get Fig (c). The equation for ~~Fig (c)~~ Fig (c) will contain one less product than the equation for fig (b). There for the most efficient way to group the 1s is fig c.

$$Y = AB\bar{C} + \bar{C}D\bar{A} + C\bar{A}B + CDA.$$