NUMBER SYSTEM.

Number is a collection of digits. For example, 193.25 signifies a ~~true~~ number with an integer part equal to 193 and a fractional part 0.25, seprated from the integer part with a radix point (•) also known as ~~radix~~ decimal point.

There are some other system also, used to represent numbers. Charateristics of commonly used number systems.

Number system	Symbols used.	Base or radix (b)	weight assigned to position		Example.
			i	$-f$	
Binary	0, 1	2	2^i	2^{-f}	1011.01
octal	0, 1, 2, 3, 4, 5, 6, 7	8	8^i	8^{-f}	365.7
Decimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	10	10^i	10^{-f}	365.249
Hexadecimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	16	16^i	16^{-f}	2F.15A.

A number can be represented as

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$$(N)_b = \underbrace{d_{n-1}\, d_{n-2} \cdots\cdots d_1 d_0}_{\text{Integer part}} \underset{\substack{\uparrow \\ \text{Radix point}}}{\bullet} \underbrace{d_{-1} d_{-2} d_{-3} \cdots\cdots d_{-m}}_{\text{Fractional part}},$$

where b is the radix or base of the number system and is equal to the number of symbols used in number system.

part

d_{n-1} :- is Most significant digit (MSD).

d_{m-1} :- is the least significant bit (L.S.B).

In general any number can be represented by the equation.

$$(N)_b = d_{n-1} \times b^{n-1} + d_{n-2} \times b^{n-2} + \dots + d_1 \times b^1 + d_0 \times b^0 + d_{-1} \times b^{-1}$$

$$+ d_{-2} \times b^{-2} + \dots + d_{-m} \times b^{-m}$$

Decimal Number System:-

0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Number of digits used are 10 therefore it has base 10. In each column the decimal system has weight of 10^n, where n is the column position number. For integer part n is positive (0, 1, 2 ...) and for fractional part n is negative (-1, -2, -3 ...). Consider the following example.

957.693

~~Position~~

Integer part.

Position column number for digit 7 is zeroth.

Position Column number for digit 5 is 1st

Position column number for digit 9 is 2nd

$$957 = 900 + 50 + 7$$

$$= 9 \times \underline{100} + 5 \times \underline{10} + 7 \times \underline{1}.$$

$$= 9 \times 10^2 + 5 \times 10^1 + 7 \times 10^0$$

10 → base.

weight weight weight.

weight = (Base)$^{\text{column position number}}$.

For fraction part

$0.693.$

Position column number for digit 6 is 1^{st}

Position column number for digit 9 is 2^{nd}

Position column number for the digit 6 is 3^{rd}.

$$.693$$

$$= 0.6 + 0.09 + .003$$

$$= 6 \times \frac{1}{10} + \frac{9}{100} + \frac{3}{1000}$$

$$= 6 \times 10^{-1} + 9 \times 10^{-2} + 3 \times 10^{-3}.$$

$$957.693$$

$$= 9 \times 10^{2} + 5 \times 10^{1} + 7 \times 10^{0} + 6 \times 10^{-1} + 9 \times 10^{-2} + 3 \times 10^{-3}$$

9	5	7	.	6	9	3
$\times 10^{2}$	$\times 10^{1}$	$\times 10^{0}$	.	$\times 10^{-1}$	$\times 10^{-2}$	$\times 10^{-3}$

Maximum weight (100). So this is Most significant bit. (MSB)

decimal.

Minimum weight So this is least significant bit (LSB).

957.693

MSB

LSB.

weight associated with digit 9 is 10^{2} (Maximum). So 9 is MSB. This digit is at the extreme right position. (MSB).

Weight associated with digit 3 is 10^{-3}, which is

As we move towards the left of radix point (decimal point) the weight of each digit is an increasing power of 10 and weight of each digit ~~decreses~~ ~~[illegible]~~ is an decreasing power of 10 as one move toward the right of decimal point.

BINARY NUMBER SYSTEM

In a digital computer, data and instructions are stored in bistable devices*. Bistable devices have two stable states viz ON or off High or low, conducting or nonconducting etc. These two states are represented by 1 or 0. 1 and 0 states are used in a number system known as Binary Number system. The binary system is less complicated then the decimal system because it has only two digits, hence has a base of two. The position of 1 or 0 in binary number indicates its weight, or value within the number, just as the position of a decimal digit determines the value of that digit. we can also say that each column weight is given by 2^n where n is the position number of the column. For integer part n is positive (0,1,2 ....) and for fractional part n is negative (-1,-2,-3 ---). This can be explained with the help of the following example.

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* some of common bistable devices are relays, swtiches, transistors etc.

$(1101 \cdot 010)_2$

Positional values in Binary Number System.

1	1	0	1	.	0	1	0
2^3	2^2	2^1	2^0	Binary point	2^{-1}	2^{-2}	2^{-3}

weights   weights

$$= (1\times 2^3) + (1\times 2^2) + (0\times 2^1) + (1\times 2^0) + \left(0\times \frac{1}{2^1}\right) + \left(1\times \frac{1}{2^2}\right) + \left(0\times \frac{1}{2^3}\right)$$

$$= (1\times 8) + (1\times 4) + (0\times 2) + (1\times 1) + (0\times 2^{-1}) + (1\times 2^{-2}) + (0\times 2^{-3})$$

$$= 8 + 4 + 0 + 1 + 0\times .5 + 1\times 0.25 + 0\times 0.125$$

$$= 13 + 0 + .25 + 0$$

$$= (13.25)_{10}$$

Counting in Binary:-

Decimal Number.	Binary Number.
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
10	1 0 1 0
11	1 0 1 1

Decimal Number.	Binary Number
15	1111

It can be seen from the table four bits are required to count from 0 to 15. In general with n bits we can count up to a number equal to 2^{n-1}

Largest Decimal Number = 2^{n-1}.

$n = 5$

$2^5 - 1 = 32 - 1 = 31.$

Binary - to - Decimal conversion.

convert the binary whole number 110110 to decimal

$(110110)_2 = (\quad)_{10}$

sol:- Determine the weight of each bit that is a 1 then find the sum of the weights to get the decimal number.

weight: 2^5 2^4 2^3 2^2 2^1 2^0

Binary Number:- 1 1 0 1 1 0

$2^5 + 2^4 + 2^2 + 2^1 + 2^0$

$= 32 + 16 + 4 + 2 + 1 = \cancel{57}\ 55$

$(110110)_2 = (55)_2$

convert the fractional binary number 0.1011 to decimal.

$(0.1011)_2 = (0.\quad)_{10}$

weight 2^{-1} 2^{-2} 2^{-3} 2^{-4}

$\times$ $\times$ $\times$ $\times$

1 0 1 1

$(1 \times 2^{-1}) + 0 + (1 \times 2^{-3}) + (1 \times 2^{-4})$

$= (1 \times 0.5) + (1 \times 0.125) + (1 \times 0.0625)$

$= 0.5 + 0.125 + 0.0625$

$= 0.6875$

Ans:- $(0.1011)_2 = (0.6875)_{10}$.

Decimal to Binary conversion.

(1) Repeated Division by 2- Method.

(2) sum of weights Method.

(1) Repeated devision Method.

In order to convert decimal numbers to binary divide the number ~~by~~ successively by (base) in this case it is 2, writing down (i) the remainders and

(ii) Divide each resulting quotient by 2 until there is a 0 whole number quotient.

The first remainder to be produced is LSB in the binary number, and the last remainder to be produced is the MSB.

Example: $(15)_{10} = (\quad)_2$.

		Remainder	
2	15		
2	7	1	LSB
2	3	1	
2	1	1	
	0	1	MSB.

$(15)_{10} = (1\,1\,1\,1)$.

$$2\overline{)15}$$ 7 ← Quotient, 14, 1 ← Remainder

	Quotient	Remainder	
$\frac{15}{2}$ =	7	1	LSB
$\frac{7}{2}$ =	3	1	
$\frac{3}{2}$ =	1	1	
$\frac{1}{2}$ =	0	1	MSB

(ii) $(25)_{10} = (\quad)_2$.

	Quotient(Q)	Remainder (R)
$\frac{25}{2}$ =	12	1
$\frac{12}{2}$ =	6	0
$\frac{6}{2}$ =	3	0
$\frac{3}{2}$ =	1	1

$(25)_{10} = (11001)_2$.

Converting Decimal Fraction to Binary:

Decimal Fractions can be converted to binary by repeated multiplication by 2, and a carry is recorded in integer position. Every time it is the resultant fraction multiplied by 2 again giving an integral carry that is either 0 or 1. These carries when read from top to bottom that is downwards gives the binary fraction. continue this process to desired number of decimal places or stop when ~~Example~~ the fractional part is all zero.

Examples:

$0.65 \times 2 = 1.3$, it is written as 0.3 with a carry of 1 (MSB)

$0.3 \times 2 = 0.6$ it is written as 0.6 with a carry of 0

$0.6 \times 2 = 1.2$ it is written as 0.2 with a carry of 1

$0.2 \times 2 = 0.4$ it is written as 0.4 with a carry of 0

$0.4 \times 2 = 0.8$ it is written as 0.8 with a carry of 0

and so on. In this example fractional part will never be equal to zero. So this multiplication by 2 continous until one gets the desired number of digits.

Terminating the process of multiplication depends on the size of word size processed by the computer and the accuracy method while solving the probl

Example 2:

$(0.65625)_{10} = (\quad)_2.$

Carry

$0.65625 \times 2 = 1.\overline{31250}$ 1 — MSB

$0.31250 \times 2 = 0.\overline{62500}$ 0

$0.6250 \times 2 = 1.\overline{2500}$ 1

$0.2500 \times 2 = 0.\overline{5000}$ 0

$0.5000 \times 2 = 1.\underline{0000}$ 1 — LSB

Fractional part becomes zero

~~0.652~~

$(0.65625)_{10} = (.10101)_2$

Example $(25.5)_{10} = (11001.1)_2$

(ii) $-(13.28)_{10} = (1101.010011)_2$

(iii) $(19)_{10} = (10011)_2$

(iv) $(45)_{10} = (101101)_2.$

convert fractional decimal number into binary and the number so obtained is converted back to decimal number. Find out the difference when conversion is upto (i) 4 decimal and (ii) 8 decimal places.

Sol:- $(0.7625)_{10} = (0. \quad)_2$.

$0.7625 \times 2 = 1.525$ 1 - MSB
$0.525 \times 2 = 1.05$ 1
$0.05 \times 2 = 0.10$ 0
$0.10 \times 2 = 0.20$ 0
$0.20 \times 2 = 0.40$ 0
$0.40 \times 2 = 0.80$ 0
$0.80 \times 2 = 1.60$ 1
$0.60 \times 2 = 1.20$ 1
$0.20 \times 2 = 0.40$ 0
$0.40 \times 2 = 0.80$ 0
$0.80 \times 2 = 1.60$ 1
$0.60 \times 2 = 1.20$ 1
$0.20 \times 2 = 0.40$ 0
$0.40 \times 2 = 0.80$ 0
$0.80 \times 2 = 1.60$ 1

(i) convert upto four decimal places

$(0.7625)_{10} = (0.1100)_2$

convert $(0.1100)_2$ to decimal number.

$(0.1100)_2 = (0.7500)_{10}$

Difference $= (0.0125)_{10}$

(ii) convert upto 8 decimal places.

$(0.7625)_{10} = (11000011)_2 = (0.76171875)_{10}$

Difference $(0.7625 - 0.7617) \approx 0.0008$

(iii) Sixteen Decimal place

$(0.7625)_{10} = (110000110011)_2 = (0.76245)_{10}$

Difference $(0.7625 - 0.76245) = (0.00005)_{10}$

(ii) Sum of weights Method.

$2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$ ———— (1)

$64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1$ Binary weights. —— (2)

Example: $(9)_{10} = (\quad)_2$

Represent the decimal number 9 into sum of binary weights.

$9 = 8 + 1$ ——— (3).

Placing 1's in the appropriate weight positions, 2^3 and 2^0, and zero's in the 2^2 and 2^1 positions.

2^3	2^2	2^1	2^0
1	0	0	1

$(9)_{10} = (1001)_2$.

(ii) $(25)_{10} = (\quad)_2$.

$25 = 16 + 8 + 1$

$\downarrow 2^4 \quad \downarrow 2^3 \quad \downarrow 2^0$

2^4	2^3	2^2	2^1	2^0
1	1	0	0	1

$(25)_{10} = (11001)_2$.

Converting Decimal fraction to binary.

$2^{-1} = 0.5 \quad 2^{-2} = .25 \quad 2^{-3} = 0.125, \quad 2^{-4} = 0.0625$

$(0.625)_{10} = (\quad)_2$

$(0.625) = 0.5 + 0.125 = 2^{-1} + 2^{-3} = 101$