Voltmeters and Multimeters

Chapter 4

INTRODUCTION 4.1

The most commonly used dc meter is based on the fundamental principle of the motor. The motor action is produced by the flow of a small amount of current through a moving coil which is positioned in a permanent magnetic field. This basic moving system, often called the D'Arsonval movement, is also referred to as the basic meter.

Different instrument forms may be obtained by starting with the basic meter movement and adding various elements, as follows.

1. The basic meter movement becomes a dc instrument, measuring
   (i) dc current, by adding a shunt resistance, forming a microammeter, a milliammeter or an ammeter.
   (ii) dc voltage, by adding a multiplier resistance, forming a milli voltmeter, voltmeter or kilovoltmeter.
   (iii) resistance, by adding a battery and resistive network, forming an ohmmeter.
2. The basic meter movement becomes an ac instrument, measuring
   (i) ac voltage or current, by adding a rectifier, forming a rectifier type meter for power and audio frequencies.
   (ii) RF voltage or current, by adding a thermocouple-type meter for RF.
   (iii) Expanded scale for power line voltage, by adding a thermistor in a resistive bridge network, forming an expanded scale (100 – 140 V) ac meter for power line monitoring.

BASIC METER AS A DC VOLTMETER 4.2

To use the basic meter as a dc voltmeter, it is necessary to know the amount of current required to deflect the basic meter to full scale. This current is known as full scale deflection current (I_{fsd}). For example, suppose a 50 μA current is required for full scale deflection.

This full scale value will produce a voltmeter with a sensitivity of 20,000 Ω per V.

The sensitivity is based on the fact that the full scale current of 50 μA results whenever 20,000 Ω of resistance is present in the meter circuit for each voltage applied.

$$\text{Sensitivity} = 1/I_{fsd} = 1/50\ \mu\text{A} = 20\ \text{k}\Omega/\text{V}$$

Hence, a 0 – 1 mA would have a sensitivity of 1 V/1 mA = 1 kΩ/V or 1000 Ω.

Example 4.1 *Calculate the sensitivity of a 200 μA meter movement which is to be used as a dc voltmeter.*

Solution The sensitivity

$$S = \frac{1}{(I_{fsd})} = \frac{1}{200\ \mu\text{A}}$$

Therefore $S = 5\ \text{k}\Omega/\text{V}$

DC VOLTMETER 4.3

A basic D'Arsonval movement can be converted into a dc voltmeter by adding a series resistor known as multiplier, as shown in Fig. 4.1. The function of the multiplier is to limit the current through the movement so that the current does not exceed the full scale deflection value. A dc voltmeter measures the potential difference between two points in a dc circuit or a circuit component.

To measure the potential difference between two points in a dc circuit or a circuit component, a dc voltmeter is always connected across them with the proper polarity.

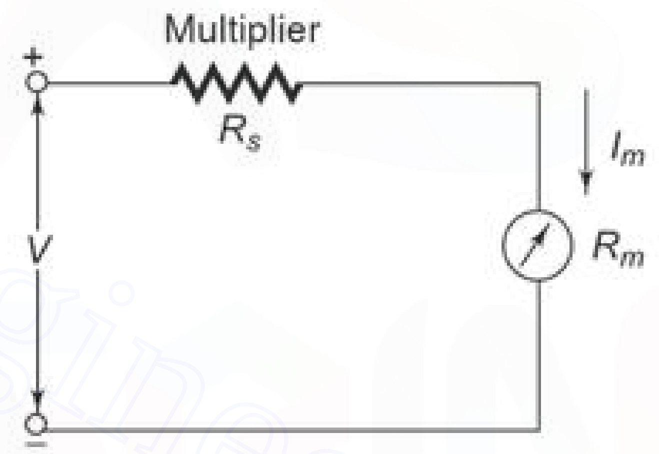

Fig. 4.1 Basic dc voltmeter

The value of the multiplier required is calculated as follows. Referring to Fig. 4.1,

I_m = full scale deflection current of the movement (I_{fsd})
R_m = internal resistance of movement
R_s = multiplier resistance
V = full range voltage of the instrument

From the circuit of Fig. 4.1

$$V = I_m (R_s + R_m)$$

$$R_s = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m$$

Therefore
$$R_s = \frac{V}{I_m} - R_m$$

The multiplier limits the current through the movement, so as to not exceed the value of the full scale deflection I_{fsd}.

The above equation is also used to further extend the range in DC voltmeter.

Example 4.2 (a) *A basic D'Arsonval movement with a full scale deflection of 50 μA and internal resistance of 500 Ω is used as a voltmeter. Determine the value of the multiplier resistance needed to measure a voltage range of 0 – 10 V.*

Solution Given

$$R_s = \frac{V}{I_m} - R_m = \frac{10}{50\ \mu\text{A}} - 500$$
$$= 0.2 \times 10^6 - 500 = 200\text{ k} - 500$$
$$= 199.5\text{ k}\Omega$$

Example 4.2 (b) *Calculate the value of multiplier resistance on the 50V range of a dc voltmeter that uses a 500 μA meter movement with an internal resistance of 1 kΩ.*

Solution

Step 1: The sensitivity of 500 μA meter movement is given by

$$S = 1/I_m = 1/500\ \mu\text{A} = 2\text{ k}\Omega/\text{V}.$$

Step 2: The value of the multiplier resistance can be calculated by

$$R_s = S \times \text{range} - R_m$$
$$R_s = 2\text{ k}\Omega/\text{V} \times 50\text{ V} - 1\text{ k}\Omega$$
$$= 100\text{ k}\Omega - 1\text{ k}\Omega = 99\text{ k}\Omega$$

MULTIRANGE VOLTMETER 4.4

As in the case of an ammeter, to obtain a multirange ammeter, a number of shunts are connected across the movement with a multi-position switch. Similarly, a dc voltmeter can be converted into a multirange voltmeter by connecting a number of resistors (multipliers) along with a range switch to provide a greater number of workable ranges.

Figure 4.2 shows a multirange voltmeter using a three position switch and three multipliers R_1, R_2, and R_3 for voltage values V_1, V_2, and V_3.

Figure 4.2 can be further modified to Fig. 4.3, which is a more practical arrangement of the multiplier resistors of a multirange voltmeter.

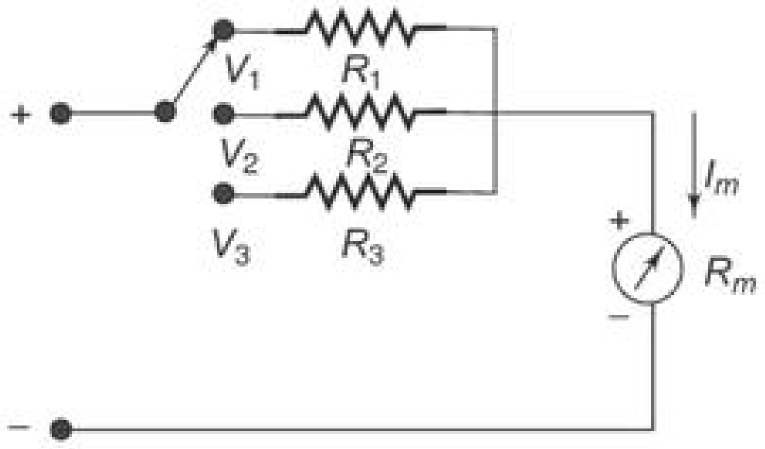

Fig. 4.2 Multirange voltmeter

In this arrangement, the multipliers are connected in a series string, and the range selector selects the appropriate amount of resistance required in series with the movement.

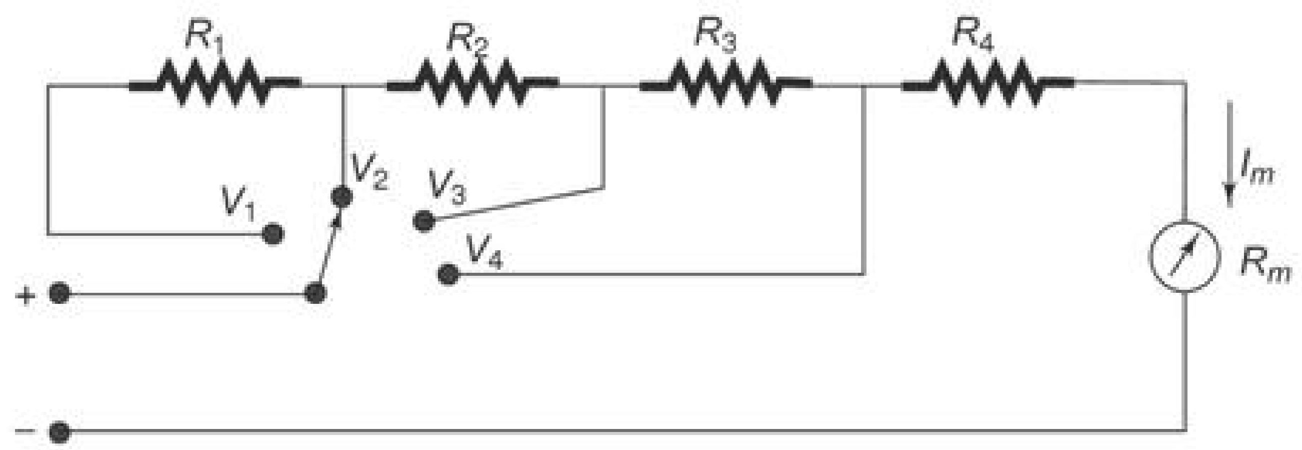

Fig. 4.3 Multipliers connected in series string

This arrangement is advantageous compared to the previous one, because all multiplier resistances except the first have the standard resistance value and are also easily available in precision tolerances.

The first resistor or low range multiplier, R_4, is the only special resistor which has to be specially manufactured to meet the circuit requirements.

Example 4.3 *A D'Arsonval movement with a full scale deflection current of 50 μA and internal resistance of 500 Ω is to be converted into a multirange voltmeter. Determine the value of multiplier required for 0–20 V, 0–50 V and 0–100 V.*

Solution Given $I_m = 50\ \mu A$ and $R_m = 500\ \Omega$

Case 1: For range 0 – 20V

$$R_s = \frac{V}{I_m} - R_m = \frac{20}{50 \times 10^{-6}} - 500 = 0.4 \times 10^6 - 500 = 400\text{ K} - 500 = 399.5\text{ k}\Omega$$

Case 2: For range 0 – 50 V

$$R_s = \frac{V}{I_m} - R_m = \frac{50}{50 \times 10^{-6}} - 500 = 1 \times 10^6 - 500 = 1000\text{ K} - 500 = 999.5\text{ k}\Omega$$

Case 3: For range 0 – 100 V

$$R_s = \frac{V}{I_m} - R_m = \frac{100}{50 \times 10^{-6}} - 500 = 2 \times 10^6 - 500 = 2000\text{ K} - 500 = 1999.5\text{ k}\Omega$$

Example 4.4 *A D'Arsonval movement with a full scale deflection current of 10 mA and internal resistance of 500 Ω is to be converted into a multirange voltmeter. Determine the value of multiplier required for 0–20 V, 0–50 V and 0–100 V.*

Solution Given $I_m = 10\text{ mA}$ and $R_m = 500\ \Omega$

Case 1: For range 0 – 20V

$$R_s = \frac{V}{I_m} - R_m = \frac{20}{10 \times 10^{-3}} - 500 = 2 \times 10^3 - 500 = 2000 - 500 = 1.5\text{ k}\Omega$$

Case 2: For range 0 – 50V

$$R_s = \frac{V}{I_m} - R_m = \frac{50}{10 \times 10^{-3}} - 500 = 5 \times 10^3 - 500 = 5000 - 500 = 4.5\ \text{k}\Omega$$

Case 3: For range 0 – 100V

$$R_s = \frac{V}{I_m} - R_m = \frac{100}{10 \times 10^{-3}} - 500 = 10 \times 10^3 - 500 = 10\text{K} - 500 = 9.5\ \text{k}\Omega$$

Example 4.5 *Convert a basic D'Arsonval movement with an internal resistance of 100 Ω and a full scale deflection of 10 mA into a multirange dc voltmeter with ranges from 0 – 5 V, 0 – 50 V and 0 – 100 V.*

Solution Given $I_m = 10$ mA, $R_m = 100\ \Omega$

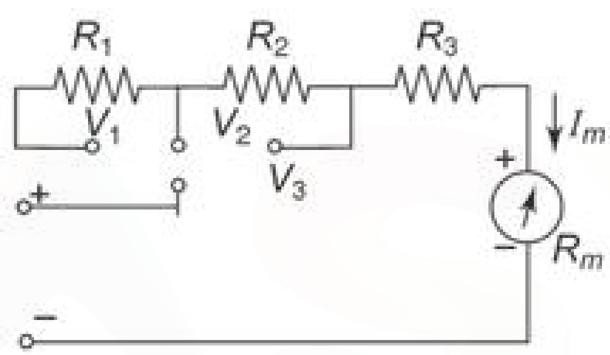

Fig. 4.3(a)

Step 1: For a 5 V (V_3) the total circuit resistance is

$$R_t = \frac{V}{I_{fsd}} = \frac{5}{10\ \text{mA}} = 0.5\ \text{k}\Omega$$

Therefore $R_3 = R_t - R_m = 500\ \Omega - 100\ \Omega$
$= 400\ \Omega$

Step 2: For a 50 V (V_2) position

$$R_t = \frac{V}{I_{fsd}} = \frac{50}{10\ \text{mA}} = 5\ \text{k}\Omega$$

Therefore $R_2 = R_t - (R_3 + R_m) = 5\ \text{k}\Omega - (400\ \Omega + 100\ \Omega)$
$= 5\ \text{k}\Omega - 500\ \Omega = 4.5\ \text{K}\Omega$

Step 3: For a 100 V range (V_1) position

$$R_t = \frac{V}{I_{fsd}} = \frac{100}{10\ \text{mA}} = 10\ \text{k}\Omega$$

Therefore $R_1 = R_t - (R_2 + R_3 + R_m) = 10\ \text{k}\Omega - (4.5\ \text{k}\Omega + 400\ \Omega + 100\ \Omega)$
$= 10\ \text{k}\Omega - 5\ \Omega = 5\ \text{k}\Omega$

Hence it can be seen that R_3 has a non-standard value.

Example 4.6 *Convert a basic D'Arsonval movement with an internal resistance of 50 Ω and a full scale deflection current of 2 mA into a multirange dc voltmeter with voltage ranges of 0 – 10 V, 0 – 50 V, 0 – 100 V and 0 – 250 V. Refer to Fig. 4.3.*

Solution For a 10 V range (V_4 position of switch), the total circuit resistance is

$$R_t = \frac{V}{I_{fsd}} = \frac{10}{2\,\text{mA}} = 5\ \text{k}\Omega$$

Therefore $R_4 = R_t - R_m = 5\ \text{k} - 50 = 4950\ \Omega.$

For 50 V range (V_3 position of switch), the total circuit resistance is

$$R_t = \frac{V}{I_{fsd}} = \frac{50}{2\,\text{mA}} = 25\ \text{k}\Omega$$

Therefore $\quad R_3 = R_t - (R_4 + R_m) = 25\ \text{k} - (4950 + 50) = 25\ \text{k} - 5\ \text{k}$

$\therefore \quad R_3 = 20\ \text{k}\Omega$

For 100 V range (V_2 position of switch), the total circuit resistance is

$$R_t = \frac{V}{I_{fsd}} = \frac{100}{2\,\text{mA}} = 50\ \text{k}\Omega$$

Therefore, $\quad R_2 = R_t - (R_3 + R_4 + R_m)$

$= 50\ \text{k} - (20\ \text{k} + 4950 + 50)$

$\therefore \quad R_2 = 50\ \text{k} - 25\ \text{k} = 25\ \text{k}\Omega$

For 250 V range, (V_1 position of switch), the total circuit resistance is

$$R_t = \frac{V}{I_{fsd}} = \frac{250}{2\,\text{mA}} = 125\ \text{k}\Omega$$

Therefore $\quad R_1 = R_t - (R_2 + R_3 + R_4 + R_m)$

$= 125\ \text{k} - (25\ \text{k} + 20\ \text{k} + 4950 + 50)$

$= 125\ \text{k} - 50\ \text{k}$

$= 75\ \text{k}\Omega$

Only the resistance R_4 (low range multiplier) has a non-standard value.

EXTENDING VOLTMETER RANGES 4.5

The range of a voltmeter can be extended to measure high voltages, by using a high voltage probe or by using an external multiplier resistor, as shown in Fig. 4.4. In most meters the basic movement is used on the lowest current range. Values for multipliers can be determined using the procedure of Section 4.4.

The basic meter movement can be used to measure very low voltages. However, great care must be used not to exceed the voltage drop required for full scale deflection of the basic movement.

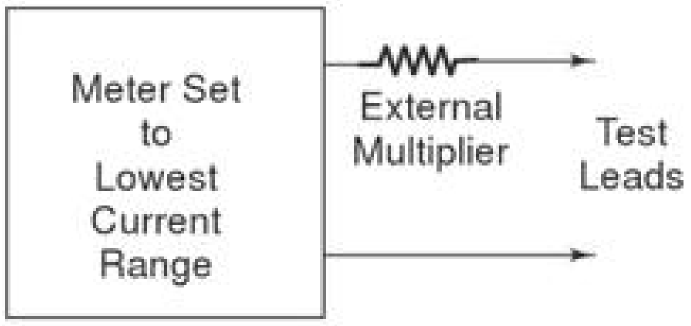

Fig. 4.4 Extending voltage range

Sensitivity The sensitivity or Ohms per Volt rating of a voltmeter is the ratio of the total circuit resistance R_t to the voltage range. Sensitivity is essentially the reciprocal of the full scale deflection current of the basic movement. Therefore, $S = 1/I_{fsd}\ \Omega/V$.

The sensitivity 'S' of the voltmeter has the advantage that it can be used to calculate the value of multiplier resistors in a dc voltmeter. As,

R_t = total circuit resistance [$R_t = R_s + R_m$]
S = sensitivity of voltmeter in ohms per volt
V = voltage range as set by range switch
R_m = internal resistance of the movement

Since $R_s = R_t - R_m$ and $R_t = S \times V$

∴ $R_s = (S \times V) - R_m$

Example 4.7 *Calculate the value of the multiplier resistance on the 50 V range of a dc voltmeter, that uses a 200 μA meter movement with an internal resistance of 100 Ω.*

Solution As $R_s = S \times$ Range – internal resistance, and $S = 1/I_{fsd}$.

∴ The sensitivity of the meter movement is

$$S = 1/I_{fsd} = 1/200\ \mu A = 5\ k\Omega/V.$$

The value of multiplier R_s is calculated as

$$R_s = S \times \text{Range} - \text{internal resistance} = S \times V - R_m$$
$$= 5\ k \times 50 - 100$$
$$= 250\ k - 100$$
$$= 249.9\ k\Omega$$

Example 4.8 *Calculate the value of multiplier resistance for the multiple range dc voltmeter circuit shown in Fig. 4.5 (a).*

Solution The sensitivity of the meter movement is given as follows.

$$S = 1/I_{fsd} = 1/50\ \mu A = 20\ k\Omega/V$$

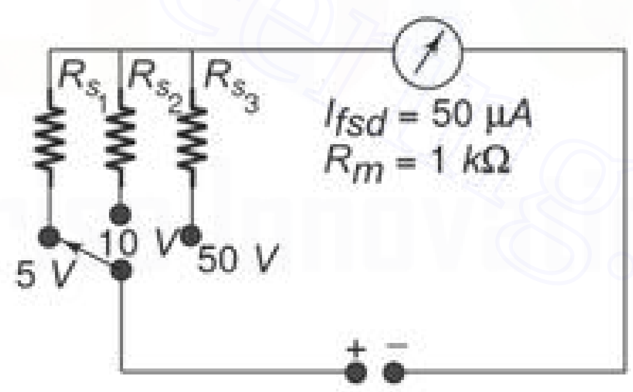

Fig. 4.5 (a)

The value of the multiplier resistance can be calculated as follows.

For 5 V range

$$R_{s_1} = S \times V - R_m$$
$$= 20\ k \times 5 - 1\ k$$
$$= 100\ k - 1\ k = 99\ k\Omega$$

For 10 V range

$$R_{s_2} = S \times V - R_m$$
$$= 20\ k \times 10 - 1\ k$$
$$= 200\ k - 1\ k = 199\ k\Omega$$

For 50 V range

$$R_{s_3} = S \times V - R_m$$
$$= 20\ k \times 50 - 1\ k$$
$$= 1000\ k - 1\ k = 999\ k\Omega$$

Example 4.9 *Calculate the value of multiplier resistance for the multirange dc voltmeter as shown in Fig 4.5(b).*

Solution

Step 1: The sensitivity of 50 μA meter movement is given by

$S = 1/I_m = 1/50\ \mu A = 20\ k\Omega/V$.

The value of the multiplier resistance can be calculated by

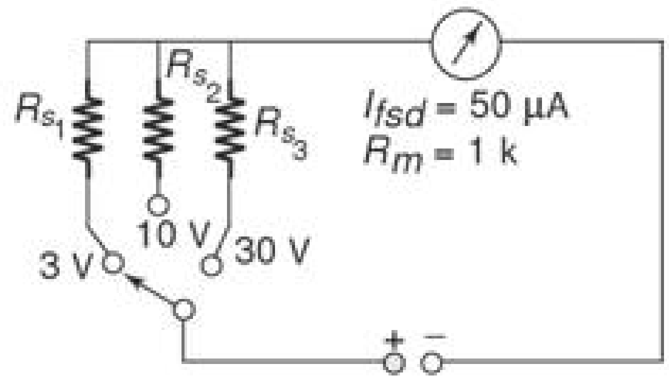

Fig. 4.5 (b)

Step 2: The value of the multiplier for 3 V range

$$R_s = S \times \text{range} - R_m$$
$$R_s = 20\ k\Omega/V \times 3V - 1\ k\Omega$$
$$= 60\ k\Omega - 1\ k\Omega = 59\ k\Omega.$$

Step 3: The value of the multiplier resistance For 10 V range can be calculated by

$$R_s = S \times \text{range} - R_m$$
$$R_s = 20\ k\Omega./V \times 10\ V - 1\ k\Omega$$
$$= 200\ k\Omega - 1\ k\Omega = 199\ k\Omega.$$

Step 4: The value of the multiplier resistance For 30V range can be calculated by

$$R_s = S \times \text{range} - R_m$$
$$R_s = 20\ k\Omega/V \times 30\ V - 1\ k\Omega$$
$$= 600\ k\Omega - 1\ k\Omega = 599\ k\Omega$$

Example 4.10 *A moving coil instrument gives a full scale deflection of 20 mA when the potential difference across its terminals is 100 mV. Calculate*

(a) Shunt resistance for a full scale deflection corresponding of 50 A.

(b) The series resistance for a full scale reading with 500 V. Also calculate the power dissipation in each case.

Solution Given meter current I_m = 20 mA and voltage = 100 mV

Step 1: Meter resistance $R_m = \dfrac{100\ mV}{20\ mA} = 5\ \Omega$

Step 2: Shunt resistance is given by

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{20\ mA \times 5\ \Omega}{50000\ mA - 20\ mA} = \frac{100\ mA}{49980\ mA} = .002\ \Omega$$

Step 3: Voltage Multiplier

$$R_{sh} = \frac{V}{I_m} - R_m = \frac{500\ V}{20\ mA} - 5\ \Omega = 25 \times 10^3 - 5\ \Omega$$
$$= 24995\ \Omega \approx 25\ k\Omega$$

Power = $V_m \cdot I_m$ = 500 × 20 mA = 10 W

LOADING 4.6

When selecting a meter for a certain voltage measurement, it is important to consider the sensitivity of a dc voltmeter. A low sensitivity meter may give a correct reading when measuring voltages in a low resistance circuit, but it is certain to produce unreliable readings in a high resistance circuit. A Voltmeter when connected across two points in a highly resistive circuits, acts as a shunt for that portion of the circuit, reducing the total equivalent resistance of that portion as shown in Fig. 4.6. The meter then indicates a lower reading than what existed before the meter was connected. This is called the loading effect of an instrument and is caused mainly by low sensitivity instruments.

Example 4.11 *Figure 4.6 shows a simple series circuit of R_1 and R_2 connected to a 100 V dc source. If the voltage across R_2 is to be measured by voltmeters having*

(a) a sensitivity of 1000 Ω/V, and

(b) a sensitivity of 20,000 Ω/V, find which voltmeter will read the accurate value of voltage across R_2. Both the meters are used on the 50 V range.

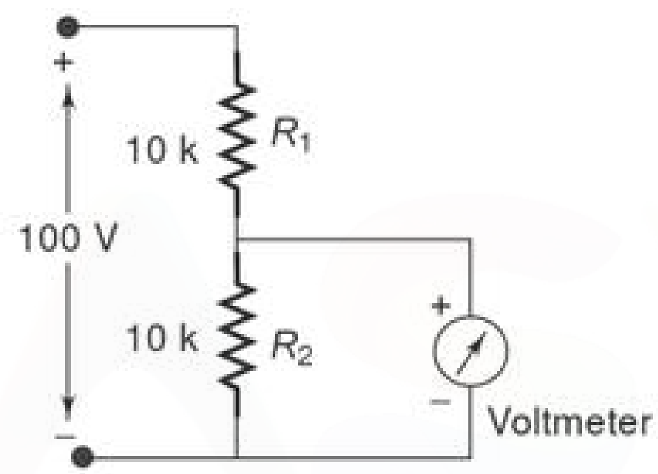

Fig. 4.6 Example on loading effect

Solution Inspection of the circuit indicates that the voltage across the R_2 resistance is

$$\frac{10\text{ k}}{10\text{ k} + 10\text{ k}} \times 100\text{ V} = 50\text{ V}$$

This is the true voltage across R_2.

Case 1

Using a voltmeter having a sensitivity of 1000 Ω/V.

It has a resistance of 1000 × 50 = 50 kΩ on its 50 V range.

Connecting the meter across R_2 causes an equivalent parallel resistance given by

$$R_{eq} = \frac{10\text{ k} \times 50\text{ k}}{10\text{ k} + 50\text{ k}} = \frac{500\text{ M}}{60\text{ k}} = 8.33\text{ k}\Omega$$

Now the voltage across the total combination is given by

$$V_1 = \frac{R_{eq}}{R_1 + R_{eq}} \times V$$

$$V_1 = \frac{8.33\text{ k}}{10\text{ k} + 8.33\text{ k}} \times 100\text{ V} = 45.43\text{ V}$$

Hence this voltmeter indicates 45.43 V.

Case 2

Using a voltmeter having a sensitivity of 20,000 Ω/V. Therefore it has a resistance of

$$20{,}000 \times 50 = 1000 \text{ k} = 1 \text{ M}\Omega$$

This voltmeter when connected across R_2 produces an equivalent parallel resistance given by

$$R_{eq} = \frac{10 \text{ k} \times 1 \text{ M}}{10 \text{ k} + 1 \text{ M}} = \frac{10^9}{1.01 \text{ M}} = \frac{10 \text{ k}}{1.01} = 9.9 \text{ k}\Omega$$

Now the voltage across the total combination is given by

$$V_2 = \frac{9.9 \text{ k}}{10 \text{ k} + 9.9 \text{ k}} \times 100 \text{ V} = 49.74 \text{ V}$$

Hence this voltmeter will read 49.74 V.

This example shows that a high sensitivity voltmeter should be used to get accurate readings.

Example 4.12 *Two different voltmeters are used to measure the voltage across R_b in the circuit of Fig. 4.7.*

The meters are as follows.

Meter 1: $S = 1$ kΩ/V, $R_m = 0.2$ k, range 10 V

Meter 2: $S = 20$ kΩ/V, $R_m = 1.5$ k, range 10 V

Calculate (i) voltage across R_b without any meter across it, (ii) voltage across R_b when the meter 1 is used (iii) voltage across R_b when the meter 2 is used, and (iv) error in the voltmeters.

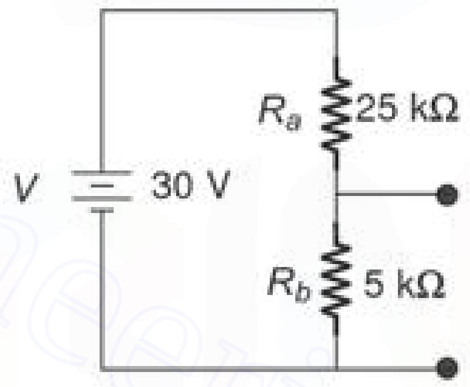

Fig. 4.7

Solution (i) The voltage across the resistance R_b, without either meter connected, is calculated using the voltage divider formula.

Therefore, $$VR_b = \frac{5 \text{ k}}{25 \text{ k} + 5 \text{ k}} \times 30 = \frac{150 \text{ k}}{30 \text{ k}} = 5 \text{ V}$$

(ii) Starting with meter 1, having sensitivity $S = 1$ kΩ/V

Therefore the total resistance it presents to the circuit

$$R_{m_1} = S \times \text{range} = 1 \text{ k}\Omega/\text{V} \times 10 = 10 \text{ k}\Omega$$

The total resistance across R_b is, R_b in parallel with meter resistance R_{m1}

$$R_{eq} = \frac{R_b \times R_{m_1}}{R_b + R_{m_1}} = \frac{5 \text{ k} \times 10 \text{ k}}{5 \text{ k} + 10 \text{ k}} = 3.33 \text{ k}\Omega$$

Therefore, the voltage reading obtained with meter 1 using the voltage divider equation is

$$VR_b = \frac{R_{eq}}{R_{eq} + R_a} \times V = \frac{3.33 \text{ k}}{3.33 \text{ k} + 25 \text{ k}} \times 30 = 3.53 \text{ V}$$

(iii) The total resistance that meter 2 presents to the circuit is

$$R_{m_2} = S \times \text{range} = 20 \text{ k}\Omega/\text{V} \times 10 \text{ V} = 200 \text{ k}\Omega$$

The parallel combination of R_b and meter 2 gives

$$R_{eq} = \frac{R_b \times R_{m_2}}{R_b + R_{m_2}} = \frac{5 \text{ k} \times 200 \text{ k}}{5 \text{ k} + 200 \text{ k}} = \frac{1000 \text{ k} \times 1 \text{ k}}{205 \text{ k}} = 4.88 \text{ k}\Omega$$

Therefore the voltage reading obtained with meter 2, using the voltage divider equation is

$$VR_b = \frac{4.88 \text{ k}}{25 \text{ k} + 4.88 \text{ k}} \times 30 = \frac{4.88 \text{ k}}{29.88 \text{ k}} \times 30 = 4.9 \text{ V}$$

(iv) The error in the reading of the voltmeter is given as:

$$\% \text{ Error} = \frac{\text{Actual voltage} - \text{Voltage reading observed in meter}}{\text{Actual voltage}} \times 100\%$$

$\therefore$ $$\text{voltmeter 1 error} = \frac{5 \text{ V} - 3.33 \text{ V}}{5 \text{ V}} \times 100\% = 33.4\%$$

Similarly $$\text{voltmeter 2 error} = \frac{5 \text{ V} - 4.9 \text{ V}}{5 \text{ V}} \times 100\% = 2\%$$

Example 4.13 *Find the voltage reading and % error of each reading obtained with a voltmeter on (i) 5 V range, (ii) 10 V range and (iii) 30 V range, if the instrument has a 20 kΩ/V sensitivity and is connected across R_b of Fig. 4.8 (a).*

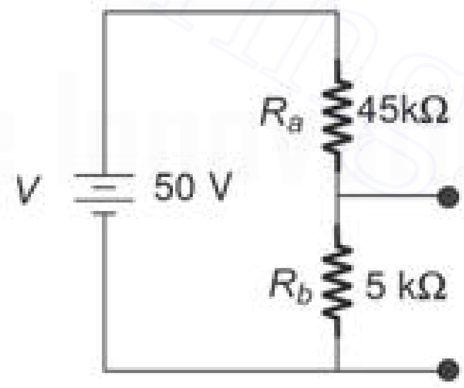

Fig. 4.8 (a)

Solution The voltage drop across R_b without the voltmeter connected is calculated using the voltage equation

$$VR_b = \frac{R_b}{R_a + R_b} \times V = \frac{5 \text{ k}}{45 \text{ k} + 5 \text{ k}} \times 50 = \frac{50 \times 5 \text{ k}}{50 \text{ k}} = 5 \text{ V}$$

On the 5 V range

$$R_m = S \times \text{range} = 20 \text{ k}\Omega \times 5 \text{ V} = 100 \text{ k}\Omega$$

$\therefore$ $$R_{eq} = \frac{R_m \times R_b}{R_m + R_b} = \frac{100 \text{ k} \times 5 \text{ k}}{100 \text{ k} + 5 \text{ k}} = \frac{500 \text{ k}}{105 \text{ k}} = 4.76 \text{ k}\Omega$$

The voltmeter reading is

$$VR_b = \frac{R_{eq}}{R_a + R_{eq}} \times V = \frac{4.76\text{ k}}{45\text{ k} + 4.76\text{ k}} \times 50 = 4.782\text{ V}$$

The % error on the 5 V range is

$$\%\text{ Error} = \frac{\text{Actual voltage} - \text{Voltage reading in meter}}{\text{Actual voltage}}$$

$$= \frac{5\text{ V} - 4.782\text{ V}}{5\text{ V}} \times 100 = \frac{0.217\text{ V}}{5\text{ V}} \times 100 = 4.34\ \%$$

On 10 V range

$$R_m = S \times \text{range} = 20\text{ k}\Omega/\text{V} \times 10\text{ V} = 200\text{ k}\Omega$$

$$\therefore \quad R_{eq} = \frac{R_m \times R_b}{R_m + R_b} = \frac{200\text{ k} \times 5\text{ k}}{200\text{ k} + 5\text{ k}} = 4.87\text{ k}\Omega$$

The voltmeter reading is

$$VR_b = \frac{R_{eq}}{R_{eq} + R_a} \times V = \frac{4.87\text{ k}}{4.87\text{ k} + 45\text{ k}} \times 50 = 4.88\text{ V}$$

The % error on the 10 V range $= \dfrac{5\text{ V} - 4.88\text{ V}}{5\text{ V}} \times 100 = 2.34\%$

On 30 V range

$$R_m = S \times \text{range} = 20\text{ k}\Omega/\text{V} \times 30\text{ V} = 600\text{ k}$$

$$\therefore \quad R_{eq} = \frac{R_m \times R_b}{R_m + R_b} = \frac{600\text{ k} \times 5\text{ k}}{600\text{ k} + 5\text{ k}} = \frac{3000\text{ k} \times 1\text{ k}}{605\text{ k}} = 4.95\text{ k}$$

The voltmeter reading on the 30 V range

$$VR_b = \frac{R_{eq}}{R_{eq} + R_a} \times V = \frac{4.95\text{ k}}{45\text{ k} + 4.95\text{ k}} \times 50 = 4.95\text{ V}$$

The % error on the 30 V range

$$= \frac{5\text{ V} - 4.95\text{ V}}{5\text{ V}} \times 100 = \frac{0.05}{5\text{ V}} \times 100 = 1\ \%$$

In the above example, the 30 V range introduces the least error due to loading. However, the voltage being measured causes only a 10% full scale deflection, whereas on the 10 V range the applied voltage causes approximately a one third of the full scale deflection with less than 3% error.

Example 4.14 *A current meter that has an internal resistance of 100 Ω is used to measure the current thro resistor R_3 in Fig 4.8(b) given below. Determine the % of the reading due to ammeter loading.*