11 November 2021 13:14

Ring:

$(R, +, \cdot)$ is a ring if

1. $(R, +)$ is an abelian group
2. $(R, \cdot)$ is a semigroup → closure → Associative
3. Distributive laws are satisfied.

Commutative ring with unity:

$a \cdot b = b \cdot a \ \forall a, b \in R$

$1 \in R$ (multiplicative identity)

$S = \{0, 2, 4\}, +_6, \cdot_6$ ring

unity → 4.

Unit:

In a ring R, an element $a \in R$ is called unit, if a^{-1} (multiplicative called unit, if $\bar{a}^{1}$ (multiplicative inverse) exists.

$\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$

units are 1, 5 only.

$(1)^{-1} = 1$
$(5)^{-1} = 5.$
other inverse not exist.

Q Find all units in ring $\mathbb{Z}_{18}$ and $\mathbb{Z}_{30}$?

Imp. question

Soln $\mathbb{Z}_{18} \to$ units are
1, 5, 7, 11, 13, 17

Inverses $\to$ 1, 11, 13, 5, 7, 17.

$(\bar{a}^{1})$
$= a\bar{a}^{1} = 1$

$\mathbb{Z}_{30} \to$ units are.
1, 7, 11, 13, 17, 19, 23, 29.

23×17
391

Inverses $\to$ 1, 13, 11, 7, 23, 19, 17, 29.

Factor

In a ring R, a is a factor of b

- there exists an element c in R

such that $b = ac$.

Notation $a | b$ (a divides b)

Example: In $\mathbb{Z}_{18}$, 3 is the factor of 15

$\because 15 = 3 \cdot_{18} 5$

Chapter 13

Zero-Divisors:

In a commutative ring R, a nonzero element a is called a zero divisor, if there exist a nonzero element b in R such that

$a \cdot b = 0$.

Example: In a ring $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$.

2, 3 and 4 are zero divisors.

$2 \cdot_6 3 = 6 \pmod 6) = 0$ $\quad 3 \cdot 4 = 12 \pmod 6) =$

$2 \cdot_6 3 = 6 \pmod 6 = 0$, $3 \cdot_6 4 = 12 \pmod 6 = 0$.

$4 \cdot_6 3 = 12 \pmod 6 = 0$.

Integral Domain

Notation I or D $\rightarrow$ Gallian.

A commutative ring with unity is called an integral domain if it has no zero divisors.

Or.

A ring D is called an integral Domain if (i) D is commutative
(ii) D contains unity
(iii) D has no zero divisors.

Examples:

1. $\mathbb{Z}$ is an integral domain.
2. Ring of Gaussian integers. $\mathbb{Z}[i] = \{a+bi \mid a,b \in \mathbb{Z}\}$ is an I. D.
3. The Ring $\mathbb{Z}[\sqrt{2}] = \{a+b\sqrt{2} \mid a,b \in \mathbb{Z}\}$ is an I.D.
4. $2\mathbb{Z}$ is not an integral domain ($\because 1 \notin 2\mathbb{Z}$)
5. $\mathbb{Z}_6$ is not an I.D. ($\because$ 2, 3 & 4 are zero divisors).