Working rule for reducing a hyperbolic eqn. to its canonical form:

step 1. Let the given eqn. $Rr + Ss + Tt + f(x,y,z,p,q) = 0$ —(1.)

be hyperbolic so that $S^2 - 4RT > 0$ —— (2.)

step 2. Write λ- quadratic eqn. $R\lambda^2 + S\lambda + T = 0$ —— (3.)

Let λ_1 and λ_2 be its two distinct roots.

step 3. Then corresponding characteristic eqns. are

$$\frac{dy}{dx} + \lambda_1 = 0 \quad \text{and} \quad \frac{dy}{dx} + \lambda_2 = 0$$

Solving these, we get

$$f_1(x,y) = c_1 \quad \text{and} \quad f_2(x,y) = c_2 \quad \text{—— (4.)}$$

step 4. We select u, v such that

$$u = f_1(x,y) \quad \text{and} \quad v = f_2(x,y) \quad \text{—— (5.)}$$

step 5. Using relations (5.), find p, q, r, s and t in terms of u and v.

Step 6. Substituting the values of p, q, r, s, t in (1.) and simplifying we shall get the required canonical form.

Classification of PDEqns. of Second order:

Consider a general PDE of second order for a fun. of two independent variables x and y in the form:

$$Rr + Ss + Tt + f(x, y, z, p, q) = 0 \qquad \text{------} \quad (1.)$$

where R, S and T are continuous functions of x and y only possessing partial derivatives defined in some domain D on the xy-plane. Then (1.) is s.t.b.

(i) Hyperbolic at a point (x, y) in domain D if

$$S^2 - 4RT > 0$$

(ii) Parabolic at a point (x, y) in domain D if

$$S^2 - 4RT = 0$$

(iii) Elliptic at a point (x, y) in domain D if

$$S^2 - 4RT < 0$$

where r, s and t have their usual meaning i.e.

$$r = \frac{\partial^2 z}{\partial x^2}, \quad s = \frac{\partial^2 z}{\partial x \partial y}, \quad t = \frac{\partial^2 z}{\partial y^2}$$