B.SC(H) Mathematics
II Sem, Sec B

Assignment

All questions are compulsory.

1. Use cylindrical shells to find the volume of the solid generated when the region enclosed by the given curves is revolved about the y axis : $y = x^3$, $x = 1, y = 0$

2. Find the arc length of the curve over the interval :- $y = x^{2/3}$ from $x = 1$ to $x = 8$.

3. Find the area of the surface generated by revolving the given curve about the x axis :- $y = 7x$, $0 \leq x \leq 1$

4. Find the reduction formula for
$\int \cos^n x \, dx$, $n \in \mathbb{N}$.

Last date to submit the Assignment is 1 Dec 2023

Vandana, 19/11/2023
Dr. Vandana
Associate Prof.,
Deptt. of Mathematics
SHIVAJI COLLEGE

Test

B. Sc(H) Mathematics
III Sem, Sec B
24/10/2023

All questions are compulsory & carry Equal Marks

① Prove that a bounded function f on $[a,b]$ is integrable if and only if for $\epsilon > 0$ $\exists$ a partition P of $[a,b]$ such that

$$U(P,f) - L(P,f) < \epsilon.$$

② Show that a bounded function f on $[a,b]$ is Riemann Integrable iff f is Darboux integrable.

③ Let
$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}, \quad x \in [a,b]$$

Then find $U(P,f)$ and $L(P,f)$, and show that f is not Riemann ~~I~~ Integrable.

Vandana, 19/12/2023
Dr. Vandana
Associate Prof.,
Deptt. of Mathematics
Shivaji College

INTERNAL EXAM (CLASS TEST)

4th Dec' 2023

BSc(H) Mathematics Sem V Section A

M.M. 20

TIME - 1 HOUR

Do any 4. All questions carry equal Marks.

Q1 Define a continuous mapping from a M.S (X, d_X) to a M.S (Y, d_Y). For metric space $X = Y = C[0,1]$ with uniform metric show that $\phi : X \to Y$ def. as

$$\phi(x)(t) = \int_0^t x(s)\, ds, \quad x \in C[0,1],$$ is continuous on X.

Q2 Show that if a mapping $f : X \to Y$ is continuous on X, then $f^{-1}(G)$ is open in X for all open subsets G of Y. Show that for A, B disjoint closed subsets of a M.S (X, d) there exist open sets G, H such that $A \subseteq G$, $B \subseteq H$ and $G \cap H = \phi$

Q3 Let (X, d_X), (Y, d_Y) be metric spaces and let $f : X \to Y$. Show that following statements are equivalent:

(i) f is continuous on X

(ii) $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$ for all subsets B of Y

iii) $f(\overline{A}) \subseteq \overline{f(A)}$ for all subsets A of X.

Q4 Show that the metrics d_1, d_2, d_∞ on $\mathbb{R}^n$ defined as follows are equivalent:

$$d_1(x,y) = \sum_{i=1}^{n} |x_i - y_i|$$

$$d_2(x,y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2\right)^{1/2}$$

$$d_\infty(x,y) = \max\{|x_i - y_i| \mid 1 \le i \le n\}$$

Q5 Let $(\mathbb{R}, d)$ be the space of all real numbers with the usual metric. Show that ~~A subset I~~ an interval I is a connected subset of $\mathbb{R}$.

4/12/2023

31st October, 2023 (Dr Neetu Rani) SET-A

Class-Test, Numerical Analysis, Max. Time: $1\frac{1}{2}$ hrs.

Sem - V, B.Sc (H) Mathematics Max. Marks: 20

Note: 1. Attempt any five questions.

2. All questions carry equal marks.

Q 1: Consider the function $f(x) = \ln x$. Construct the Lagrange form of the interpolating polynomial for f passing through the points $(1, \ln 1)$, $(2, \ln 2)$, and $(3, \ln 3)$. Use this polynomial to estimate both $\ln(1.5)$ and $\ln(2.4)$. What is the error in each approximation?

Q.2: Set up the SOR method with $\omega = 0.6$ to solve the system of equations: $4x_1 + 2x_2 - x_3 = 1$, $2x_1 + 4x_2 + x_3 = -1$, $-x_1 + x_2 + 4x_3 = 1$. Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations.

Q.3: Solve the given system using scaled partial pivoting,
$2x_1 + 7x_2 + 5x_3 = 14$, $6x_1 + 20x_2 + 10x_3 = 36$,
$4x_1 + 3x_2 = 7$.

Q.4: For the function, $f(x) = e^x + x^2 - x - 4$, use Newton's method to approximate a real root. Use an absolute tolerance of 10^{-6} as a stopping condition.

Q.5: Define the order of convergence of an iterative method for finding an approximation to the root of $g(x) = 0$. Find the order of convergence of Secant method.

Q.6: For the function, $f(x) = e^x + x^2 - x - 4$, use the bisection method to approximate a real zero. Use an absolute tolerance of 10^{-5} as a stopping criterion.

Q.7: Construct an algorithm to approximate a real root of an equation using method of false position.

N.R.

31st October, 2023 (Dr. Neetu Rani) SET-B

Class Test, Numerical Analysis, Max. Time: $1\frac{1}{2}$ hrs

Sem-V, B.Sc.(H) Mathematics, Max. Marks: 20

Note: 1. Attempt any five questions.

2. All questions carry equal marks.

Q.1: Write out the Newton form of the interpolating polynomial for $f(x) = \sin x$ that passes through the points $(0, \sin 0)$, $(\pi/4, \sin \pi/4)$, and $(\pi/2, \sin \pi/2)$. Use the polynomial to estimate both $\sin \pi/3$ and $\sin \pi/6$. What is the error in each approximation?

Q.2: Set up the Gauss-Seidel iteration scheme to solve the system of equations:

$$4x_1 - x_2 = 0, \quad 2x_1 + 4x_2 - x_3 = 2,$$
$$-2x_2 + 4x_3 - x_4 = -3, \quad -2x_3 + 4x_4 = 1.$$

Take the initial approximation as $X^{(0)} = (0,0,0)$ and do three iterations.

Q.3: Using scaled partial pivoting, find matrices L, U and P such that $LU = PA$, where

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 2 & 3 \\ 1 & -1 & 1 & 2 \\ -1 & 1 & -1 & 5 \end{bmatrix}.$$

Q.4: Apply the secant method to the given function using the indicated values for p_0 and p_1. Iterate until $|p_n - p_{n-1}| < 5\times10^{-5}$. Given,

$f(x) = \tan(\pi x) - x - 6$, $p_0 = 0.4$, $p_1 = 0.48$

Q.5: Consider the function $g(x) = e^{-x^2}$. Prove that g has a unique fixed point on the interval $[0,1]$. With a starting approximation of $p_0 = 0$, use the fixed point iteration scheme $p_n = e^{-p_{n-1}^2}$ to approximate the fixed point on $[0,1]$ to within 5×10^{-5}.

Q.6: Define the order of convergence of an iterative method for finding an approximation to the root of $g(x)=0$. Find the order of convergence of Newton's iterative formula.

Q.7: Construct an algorithm to approximate a real root of an equation using the bisection method.

N.R.

30th November, 2023 (Dr. Neetu Rani)

Class Test, Numerical Analysis, Max. Time: $1\frac{1}{2}$ hrs.

Sem - V, B.Sc.(H) Mathematics, Max. Marks: 20

Note. 1. Attempt any five questions.
2. All questions carry equal marks.

Q1: Verify that the second-order central difference approximation for the second derivative provides the exact value of the second derivative, regardless of the value of h, for the functions $f(x) = 1$, $f(x) = x$, $f(x) = x^2$, and $f(x) = x^3$, but not for the function $f(x) = x^4$.

Q2: Apply Euler's method to approximate the solution of the given initial value problem over the indicated interval in t using the indicated number of time steps:
$x' = tx^3 - x \quad (0 \le t \le 1)$, $x(0) = 1$, $N = 4$.

Q3: Write out the Newton form of the interpolating polynomial for $f(x) = e^x$ that passes through the points $(-1, e^{-1})$, $(0, e^0)$, and $(1, e^1)$.

Q4: Solve the given system by ~~Gauss~~ Seidel method taking $x^{(0)} = 0$ and terminate iterations when $\|x^{(k+1)} - x^{(k)}\|_\infty$ falls below 5×10^{-6}.
$4x_1 - x_2 = 2$, $-x_1 + 4x_2 - x_3 = 4$, $-x_2 + 4x_3 = 10$.

Q5: Using scaled partial pivoting during the factor step, find matrices L, U and P such that $LU = PA$.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 2 & 3 \\ 1 & -1 & 1 & 2 \\ -1 & 1 & -1 & 5 \end{bmatrix}.$$

Q6: Write an algorithm for secant method and find out a simple zero of the function $f(x) = x^3 + 2x^2 - 3x - 1$ on the interval $(-1, 0)$ within an absolute tolerance of 5×10^{-5}.

Q6: Define order of convergence and determine the order of convergence for the secant method.

Q.7: Compute each of the following limits and determine the corresponding rate of convergence:
(i) $\lim_{n \to \infty} (\sqrt{n+1} - \sqrt{n})$ (ii) $\lim_{x \to 0} \dfrac{\cos x - 1 + x^2/2 - x^4/24}{x^6}$

N.R.

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Practical Mock Test (Set B)
23rd November, 2023
Department of Mathematics, Shivaji College, University of Delhi

Name of the Course	: B.Sc. (H) Mathematics
Name of the Paper	: **Numerical Analysis**
Semester	: V
Duration	: 1 Hour
Maximum Marks	: 10

Instruction for Candidates

1) Attempt any four questions.
2) Each question is of 2.5 marks.

(1) Perform 5 iterations of the Newton-Raphson method to find out the smallest positive root of the equation $x^3 - 5x + 1 = 0$.

(2) Write a program to solve the following system of linear equations using Gauss-Jacobi method

$7x_1 - 2x_2 + x_3 + 2x_4 = 3$

$2x_1 + 8x_2 + 3x_3 + x_4 = -2$

$-x_1 + 5x_3 + 2x_4 = 5$

$2x_2 - x_3 + 4x_4 = 4$

(3) Find Lagrange Interpolating Polynomial for the function log(x) , using the points x=1, 3, 5, 7, 9. Hence Plot the resulting polynomial and compare with the actual graph of log(x) in the interval [1,10].

(4) Approximate the integral $\int_0^1 \left(\frac{1}{1+x^2}\right) dx$ using Simpson's rule. Also find out the absolute error in the solution obtained.

(5) Use Euler's Method to solve the initial value problem $\frac{dy}{dx} = \frac{y-x}{x+y}$, $y(0) = 1, 0 \leq x \leq 0.1$

N.R.

Practical Mock Test (Set A)
23rd November, 2023
Department of Mathematics, Shivaji College, University of Delhi

Name of the Course	: B.Sc. (H) Mathematics
Name of the Paper	: **Numerical Analysis**
Semester	: V
Duration	: 1 Hour
Maximum Marks	: 10

Instruction for Candidates

1) Attempt any four questions.
2) Each question is of 2.5 marks.

(1) Perform 10 iterations of the Regula-Falsi method to find out a root of the equation $\cos(x) - x\,e^x = 0$ in the interval [0,1].

(2) Write a program to solve the following system of linear equations using Gauss-Seidel method

$7x_1 - 2x_2 + x_3 + 2x_4 = 3$
$2x_1 + 8x_2 + 3x_3 + x_4 = -2$
$-x_1 + 5x_3 + 2x_4 = 5$
$2x_2 - x_3 + 4x_4 = 4$

(3) For the following set of points, find out Newton interpolating polynomial: x_1=0.5, x_2=1.5, x_3=3.0, x_4=5.0, x_5=6.5, x_6=8.0 and $f(x_1)$=1.625, $f(x_2)$=5.875, $f(x_3)$=31.0, $f(x_4)$=131.0, $f(x_5)$=282.125, $f(x_6)$=521.0. Further approximate the value of f at x=7, using the resulted polynomial.

(4) Approximate the integral $\int_{0.2}^{1.4}(sin(x) - log(x) + e^x)\,dx$ using Trapezoidal rule. Also find out the absolute error in the solution obtained.

(5) Use Runge-Kutta Method to solve the initial value problem $\frac{dy}{dx} = x^2 + y - 4,\ y(0) = 1, 0 \le x \le 0.6$ with step size of 0.2.

N.R

Assignment (Assigned on 25.10.2023) **Dr. Neetu Rani**

B.Sc.(H) Mathematics, Semester-V, July-December 2023, Numerical Analysis

Reference Book: Bradie, Brian (2006), A Friendly Introduction to Numerical Analysis, DSE-I (i)

S. No.	Roll No.	Name	Exercise	Questions
1	21/17002	Jay Jhala	1.1	1-5, 7, 9, 10, 11, 15-18
2	21/17005	Annu Yadav	1.2	1-13, 16-21
3	21/17019	Deepanshu	2.1	1-5, 10-12, 14-16
4	21/17047	Nikita	3.5	3-16
5	21/17051	Kshama Pandey	2.3	1-3, 5-16
6	21/17091	Naysha	3.2	1-20
7	21/17108	Hitesh	2.5	1-18
8	21/17109	Kalim Akhtar	2.4	1-14
9	21/17120	Akrist Ojha	2.2	1, 2, 4-11
10	21/17131	Manoj Kumar	3.8	1-10, 13, 14, 16
11	21/17132	Dhananjay Madan	5.1	1-17
12	21/17140	Hariom Kumar Sharma	5.5	1-12
13	21/17153	Madhuri	5.3	2-19
14	21/17169	Gaurabh Singh	6.2	1-16
15	21/17173	Disha Sharma	6.4	1-15, 17
16	21/17008	Pranjali Aggarwal	7.2	1-15
17	21/17011	Pritesh Kumar	7.2	16-24, 26
18	21/17024	Prachi Bohra	1.2	1-13, 16-21
19	21/17049	Vikash Singh		
20	21/17057	Prayag Raj	2.1	1-5, 10-12, 14-16
21	21/17076	Suraj Yadav	2.2	1, 2, 4-11
22	21/17080	Vanshika Chaudhary	6.4	1-15, 17
23	21/17081	Vishal	6.4	1-15, 17
24	21/17090	Sahil Kumar	2.4	1-14
25	21/17092	Sahil Choudhary	1.2	1-13, 16-21
26	21/17100	Prabhat Rastogi	5.5	1-12
27	21/17107	Yuvraj	5.1	1-17
28	21/17111	Vikrant Gusain	1.1	1-5, 7, 9, 10, 11, 15-18
29	21/17114	Shalini Kumari Kashyap	5.1	1-17
30	21/17115	Tanya	7.2	16-24, 26

N.R.

S. No.	Roll No.	Name	Exercise	Questions
31	21/17116	Rohit	3.5	3-16
32	21/17118	Rajneesh Kumar	2.3	1-3, 5-16
33	17122	Shivam Yadav	3.5	3-16
34	21/17126	Sachin Kumar	7.2	16-24, 26
35	21/17127	Pratham Debbarma	3.2	1-20
36	21/17129	Shivagya Singh Saini	3.8	1-10, 13, 14, 16
37	21/17137	Suraj Singh	6.2	1-16
38	21/17143	Vaibhav Bhatt	2.5	1-18
39	21/17145	Tarang Sharma	5.3	2-19
40	21/17150	Sumit Kumar	2.5	1-18
41	21/17156	Sudhanshu	7.2	1-15
42	21/17157	Savik Rana	5.1	1-17
43	21/17162	Sakshi	2.5	1-18
44	21/17177	Ishant Rawat	2.5	1-18
45				

Note: 1. Use only A-4 size paper for solving the assignment questions.

2. Submit your assignment within one week of completion of topic assigned.

N.R.

Test II   Dr. APARNA JAIN

B.Sc.(H) Mathematics, Sem V

Dated: 30th Nov 2023

Time: 1 hr.   MM: 20

Discrete Mathematics (DSE II)

SET B.

Do any 4 out of 5 questions.

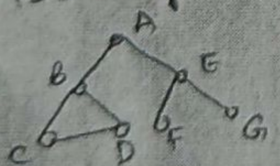
Q1 Prove that lattices isomorphic as algebras are also isomorphic as posets.

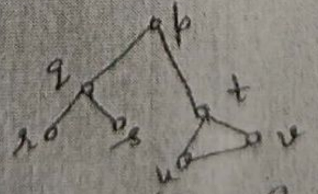
Q2 Find the DNF of the Boolean polynomial

$$p = [(x_2 + x_1 x_3)(x_1 + x_3) x_2]'$$

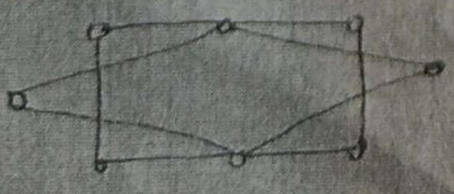
Q3 Prove that if B is a Boolean Algebra, $(x \wedge y)' = x' \vee y'$ $\forall x, y \in B$.
Give example of a lattice which is not a Boolean Algebra.

Q4 For the following graphs, if the graphs are not isomorphic, give reasons. If they are isomorphic, exhibit an isomorphism.





Q5 Is the following graph Eulerian? If yes, provide an Eulerian Circuit. If not, justify.



Aparna

WhatsApp

Test 3rd oct 2023 (Set B)

Test — Dr. Aparna Jain

B.Sc. (H) Mathematics, Sem V

Dated: 3rd oct. 2023

Time: 1hr — MM: 20

Discrete Mathematics (DSE II)

SET B

Do any 4 out of 5 questions.

Q1 Let P & Q be finite ordered sets & let $\phi : P \longrightarrow Q$ be a bijection. Let it be given that $x < y$ in P iff $\phi(x) < \phi(y)$ in Q. Prove that $x \prec y$ in P iff $\phi(x) \prec \phi(y)$ in Q.

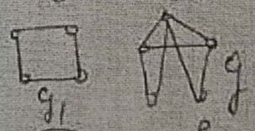
Q2 Let L be a lattice as an algebra. Define a relation $\leq$ on L as $a \leq b$ iff $a \wedge b = a$. Prove that $(\leq)$ is a partial order relation on L.

Q3 (i) Give example of a chain and an example of an antichain.

(ii) Draw Hasse Diagrams of all Posets with 3 elements. Which of these are lattices.

Q4 (i) Does there exist a graph with degree sequence 4, 4, 4, 3, 2 ? Justify

(ii) Is the graph G_1 a subgraph of the graph G? Justify your answer.



Q5 State & prove Euler's Theorem for pseudographs.

Aparna

Aparna

Test 3rd oct 2023 WhatsApp (Set A)

Dr. Aparna Jain

Test

B.Sc.(H) Mathematics, Semester V

Dated: 3rd oct 2023

Time 1 hr. MM: 20

Discrete Mathematics (DSE II)

SET A

Do any 4 out of 5 questions.

Q1 Let P & Q be finite Posets & let $\phi : P \to Q$ be a bijection. If ϕ is an order isomorphism, prove that $x < y$ in P iff $\phi(x) < \phi(y)$ in Q.

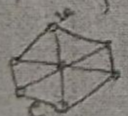
Q2 (i) Give example of a poset with 4 elements which is not a lattice.

(ii) Give an example of a mapping ϕ between Posets P & Q such that ϕ is not order preserving.

Q3 Let L be a lattice as an ordered set. Define 2 operations meet & join in L as $x \vee y = \sup\{x,y\}$ & $x \wedge y = \inf\{x,y\}$. Prove that $\wedge$ & $\vee$ are associative in L.

Q4 (i) A graph has 5 vertices of degree 4 and 2 vertices of degree 2. How many edges does it have?

(ii) for the following graph, draw $G-\{e\}$ & $G-\{v\}$



Q5 Prove that the number of odd vertices in a pseudograph is even.

Aparna

Aparna

B.Sc. (H) Maths.
Sem-V, Sec-A

Class Test

Max. Marks: 10

Answer any two questions each carry 5 marks.

1. For every positive integer $n (\geq 2)$, show that

$$Aut(Z_n) \simeq U(n).$$

2. If m divides the order of a finite Abelian group G, then show that G has a subgroup of order m.

3. Show that the number of conjugates of a subset S in a group G is the index of the normalizer of S, $|G : N_G(S)|$.

4. Find the number of cyclic subgroups of order 10 in $Z_{100} \oplus Z_{25}$.

Uttam Kumar Sinha

B.Sc.(H) Maths.
Sem-II, Sec-B

ASSIGNMENT GROUP THEORY – II

Max. Marks : 10

1. For every positive integer n, show that

$$Aut(Z_n) \approx U(n).$$

2. Let G and H be finite cyclic groups. Then show that $G \oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime.

3. Find the number of cyclic subgroups of order 10 in $Z_{100} \oplus Z_{25}$.

4. If a group G is the internal direct product of a finite number of subgroups $H_1, H_2, ----, H_n$, then show that G is isomorphic to the external direct product of $H_1, H_2, ---, H_n$; i.e.,

$$H_1 \times H_2 \times \ldots \times H_n \approx H_1 \oplus H_2 \oplus \ldots \oplus H_n .$$

5. If m divides the order of a finite Abelian group G, then show that G has a subgroup of order m.
(Hints: Use Fundamental Theorem for finite Abelian groups)

6. Let G be a group and $H \leq G$. Let G act by left multiplication on the set A of left cosets of H in G and π_H be the associated permutation representation afforded by this action. Then show that the kernel of the action (i.e., the $\ker \pi_H$) is $\bigcap_{x \in G} xHx^{-1}$, and $\ker \pi_H$ is the largest normal subgroup of G contained in H.

7. Show that the number of conjugates of a subset S in a group G is the index of the normalizer of S, $|G : N_G(S)|$.

Uttam Kumar Sinha

B. Sc. (H) Maths. Class Test Max. Marks: 10

Sem - V, Section - B

Answer any two questions each carry 5 marks.

1. Let G and H be finite cyclic groups. Then show that $G \oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime.

2. Find the number of cyclic subgroups of order 10 in $Z_{100} \oplus Z_{25}$.

3. If a group G is the internal direct product of a finite number of subgroups $H_1, H_2, \ldots, H_n$, then show that G is isomorphic to the external direct product of $H_1, H_2, \ldots, H_n$; i.e.,

$$H_1 \times H_2 \times \ldots \times H_n \approx H_1 \oplus H_2 \oplus \cdots \oplus H_n.$$

4. Let G be a group and $H \leq G$. Let G act by left multiplication on the set A of left cosets of H in G and π_H be the associated permutation representation afforded by this action. Then show that the kernel of the action, $\ker \pi_H$ is $\bigcap_{x \in G} xHx^{-1}$ and $\ker \pi_H$ is the largest normal subgroup of G contained in H.

Uttam Kumar Sinha

B.Sc. (H) Maths. Sem-V, Sec.-A

ASSIGNMENT GROUP THEORY – II

Max. Marks: 10

1. For every positive integer n, show that
$$\mathrm{Aut}(\mathbb{Z}_n) \approx U(n).$$

2. Let G and H be finite cyclic groups. Then show that $G \oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime.

3. Find the number of cyclic subgroups of order 10 in $\mathbb{Z}_{100} \oplus \mathbb{Z}_{25}$.

4. If a group G is the internal direct product of a finite number of subgroups $H_1, H_2, \ldots, H_n$, then show that G is isomorphic to the external direct product of $H_1, H_2, \ldots, H_n$; i.e.,
$$H_1 \times H_2 \times \ldots \times H_n \approx H_1 \oplus H_2 \oplus \ldots \oplus H_n.$$

5. If m divides the order of a finite Abelian group G, then show that G has a subgroup of order m.
(Hints: Use Fundamental Theorem for finite Abelian groups)

6. Let G be a group and $H \leq G$. Let G act by left multiplication on the set A of left cosets of H in G and π_H be the associated permutation representation afforded by this action. Then show that the kernel of the action (i.e., the $\ker \pi_H$) is $\bigcap_{x \in G} xHx^{-1}$, and $\ker \pi_H$ is the largest normal subgroup of G contained in H.

7. Show that the number of conjugates of a subset S in a group G is the index of the normalizer of S, $|G : N_G(S)|$.

Uttam Kumar Sinha

B.Sc (H) Maths Vth Sem
Deptt of Maths
Paper – Probability & Statistics

T.A – 45 min

M.M – 30

Attempt any three questions

1. For Binomial Distribution, prove that mean $= n\theta$ & variance $= n\theta(1-\theta)$
2. State & prove lack of memory property for exponential distribution
3. Prove that Poisson is a limiting case of Binomial Distribution
4. Prove that moments about mean of a normal distribution is given by $\mu_{2n+1} = 0$, $\mu_{2n} = 1 \cdot 3 \cdot 5 \cdots (2n-1)\sigma^{2n}$
5. Find the mgf of Gamma Distribution & find its mean
6. For –ve binomial distribution, prove that mean $= \frac{k}{\theta}$ & Variance $= \frac{k}{\theta}\left(\frac{1}{\theta}-1\right)$

Dr. Deepti
Associate Professor
Deptt of Mathematics

B.Sc (H) Maths Vth Sem
Shivaji College
Paper Name – Probability & Statistics

MM – 20 Time – 45 min.

Attempt Any two questions

1. State & Prove Boole's Inequality

2. Let X be a continuous r.v. having pdf

$$f(x) = \begin{cases} 1/2 & -1 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Define $Y = X^2$. Find the probability distribution of Y

3. Find the mean & variance for the distribution having pdf

$$f(x) = \begin{cases} 6x(1-x) & ; 0 < x < 1 \\ 0 & ; \text{elsewhere} \end{cases}$$

Hence find $P[\mu - 2\sigma < X < \mu + 2\sigma]$

Dr. Deepti
Associate Professor

Assignment (Metric Spaces) Dated: 27.11.2023

B.Sc (H) Maths III Year (2023-24)

Q1. Let $X = \mathbb{R} \cup \{\infty\} \cup \{-\infty\}$. Define $f : X \to \mathbb{R}$ by the rule

$$f(x) = \begin{cases} \dfrac{x}{1+|x|} & \text{if } -\infty < x < \infty \\ 1 & \text{if } x = \infty \\ -1 & \text{if } x = -\infty \end{cases}$$

Define d on X as follows:

$$d(x, y) = |f(x) - f(y)|, \quad x, y \in X$$

Prove that d is a metric on X.

Q2 Let $X = \mathbb{R} \cup \{\infty\} \cup \{-\infty\}$

Define d on X as follows:

$$d(x, y) = |\tan^{-1}x - \tan^{-1}y|, \quad x, y \in X$$

Prove that d is a metric on X.

Q3. Let $X = \mathbb{R}$. For $x, y \in X$, define

$$d(x, y) = \begin{cases} |x| + |x-y| + |y| & \text{when } x \neq y \\ 0 & \text{when } x = y \end{cases}$$

Show that d is a metric on X.

Q4 Let (X, d) be a metric space. Let $X = U \cup V$ where $U \cap V = \phi$.

Define D on X as follows:

$$D(x, y) = \begin{cases} d(x,y) + 1, & \text{if exactly one among } x \text{ and } y \text{ belongs to } U \\ d(x, y), & \text{otherwise} \end{cases}$$

Prove that D is metric on X.

[signature]

Q5. Let $X = \{ (x_1, x_2) \in \mathbb{R}^2 : |x_1| < 2$ and $|x_2| < 1 \}$ be equipped with the metric from $\mathbb{R}^2$. For any $x = (x_1, x_2) \in X$ and $r > 0$

Show that

$$S_X(x, r) = X.$$

Q6. Let $A = \{ z \in \mathbb{C} : |z+1|^2 \leq 1 \}$ and $B = \{ z \in \mathbb{C} : |z-1|^2 < 1 \}$.

Let $A \cup B$ be equipped with metric induced from $\mathbb{C}$.

Identify $Cl_{A \cup B}(B)$.

Q7. Let $(\mathbb{R}, d_1)$ be the metric space where

$$d_1(x, y) = \begin{cases} |x| + |x-y| + |y| & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$$

Show that ε-ball about 0 with metric d_1 is same as the $(\varepsilon/2)$-ball about 0 with the usual metric.
Find (i) $S(0, 2)$ (ii) $S(2, 3)$.

Q8. Show that the metrics d_1, d_2 and d_∞ defined on $\mathbb{R}^n$ by

$$d_1(x, y) = \sum_{i=1}^{n} |x_i - y_i|$$

$$d_2(x, y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2 \right)^{1/2}$$

$$d_\infty(x, y) = \max \{ |x_i - y_i| : 1 \leq i \leq n \}$$

are equivalent.

Q9. Suppose f is continuous real-valued function defined on $\mathbb{R}$ such that $\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 0$. Prove that f is uniformly continuous.

Q10. Show that the function $f : \mathbb{R} \to (-1, 1)$ defined by

$$f(x) = \frac{x}{1+|x|}$$

is ... that f is uniformly cts.

Q11. Let $X = [0, 2\pi] \subseteq \mathbb{R}$. and $Y = \{z \in \mathbb{C} : |z| = 1\}$.

Consider the map $f : X \to Y$ defined by $f(t) = e^{it}$, $0 \le t \le$

Show that f is both one-one and onto. It is continu

but not homeomorphism.

Q12 (a) Given an example of a function that is continuous and open, but is not a homeomorphism.

(b) Give an example of a function that is continuous and open, but is not closed.

Q13 (a) Given an example of two closed sets F_1 and F_2 in the metric space $(\mathbb{R}, u)$ which are disjoint but $d(F_1, F_2) = 0$

(b) Let $A = \{\frac{1}{n} : n \in \mathbb{N}\}$. Find $d(\bar{A})$, when

(i) d is usual metric on $\mathbb{R}$

(ii) d is ~~metric~~ discrete metric on $\mathbb{R}$.

Q14 Let (X_1, d_1) and (X_2, d_2) be metric spaces. Let $f : X_1 \to X_2$ be constant map. Show that f is continuous. Use this to show that a continuous mapping need not have the property that image of every open sets is open.

Q15. Let $X = C[0,1]$. Consider d and ρ defined on $C[0,1]$

$$d(f,g) = \sup\{|f(x) - g(x)| : x \in [0,1]\}$$

$$\rho(f,g) = \int_0^1 |f(x) - g(x)|\,dx$$

Is identity map $i : (X, d) \to (X, \rho)$ a homeomorphism? Justify your answer.

Dated : 16.10.23.

Test – Metric Spaces

me : 1 hr Max Marks : 30

B.Sc. (H) Mathematics III Year (2023-24)

Attempt any five Questions

Q1. Let $X = \mathbb{R}^n = \{ x = (x_1, x_2, \ldots, x_n) : x_i \in \mathbb{R} \ 1 \le i \le n \}$.

Define $d(x, y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2 \right)^{1/2}$ where
$x = (x_1, x_2, \ldots, x_n)$
$y = (y_1, y_2, \ldots, y_n)$

Verify that d is a metric on X.

OR

Let $S \neq \phi$ and $\mathcal{B}(S)$ denote the set of all bounded real or complex valued fns on S.

Define $d(f, g) = \sup_{x \in S} |f(x) - g(x)|$ where $f, g \in \mathcal{B}($

Verify that d is a metric on $\mathcal{B}(S)$.

Q2. Let d be a metric on a nonempty set X. Show that

$$e(x, y) = \min \{1, d(x, y)\} \quad \text{where } x, y \in X$$

is also metric on X.

OR

Let d be a metric on a nonempty set X. Define $d' : X \times X \to \mathbb{R}$ by $d'(x, y) = \dfrac{d(x, y)}{1 + d(x, y)}$

Verify that d' is metric on X. Is d' is bounded? Justify your answer.

Q3. Let $X = C[a, b]$ and $d(f, g) = \sup \{ |f(x) - g(x)| : x \in [a, b$
Show that convergence of a sequence of functions in $C[a, b]$ in the uniform metric is the same as uniform convergence.

Q4. Let $X = C[a, b]$ and $d(f, g) = \sup \{ |f(x) - g(x)| ; a \le x \le b \}$
be associated metric. Show that (X, d) is a complete metric space.

Time: 1:15 m

Max Marks: 30

Q5. (a). Let (X, d) be metric space and f be a self map that is one to one. Define $D(x,y) = d(f(x), f(y))$ where $x, y \in X$. Prove that D is a metric on X.

(b) Let $X = C[0,1]$. Defined $d(x, y) = \sup\{|x(t) - y(t)| : 0 \le t \le 1\}$ where $x, y \in X$. Calculate the distance between $x(t) = t$ and $y(t) = t^2$.

Q6. (a) Prove that a subset G in a metric space (X, d) is open iff it is the union of all open balls contained in G.

(b) In a discrete metric space, prove that any subset G of X is open.

Q7. Let F be a subset of the metric space (X, d). Prove that F' is a closed subset of (X, d)

Q8 Find interior and set of limit points of the following sets:

(a) $A = (0,1) \cup [2,3]$ (b) $F = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{n}, \ldots\}$

(c) $A = \mathbb{Z}$ (d) $A = \mathbb{Q}$

Assume the usual metric on $\mathbb{R}$.

Q9. Let (X, d) be a metric space. Prove that $x \in \bar{F} \iff \exists$ a sequence $\{x_n\}$ of Points in F such that $x_n \to x$.

Q10. Let A be a subset of metric space (X, d). Prove that $\text{int}(A)$ or A° is the largest open set contained in A.

Q11. Let (X, d) be metric space and $F \subseteq X$. Prove that if x_0 is a limit point of F, then every open ball $S(x_0, r)$, $r > 0$ contains an infinite number of points in F.

Q12. Let (X, d) be a metric space and $F \subseteq X$. Prove that F is closed in X iff F^c is open in X.

29.11.2023

Test (Metric Spaces)

Time: 1:15 mins

B.Sc. (H) Mathematics II Year 23-24

Max Marks: 30

Attempt any six questions

Q1. Let (X, d_X) and (Y, d_Y) be metric spaces and $A \subseteq X$. Prove that a function $f: A \to Y$ is continuous at $a \in A$ iff whenever a sequence $\{x_n\}$ in A converges to a, the sequence $\{f(x_n)\}$ converges to $f(a)$.

OR

Prove that a mapping $f: X \to Y$ is continuous on X iff $f^{-1}(G)$ is open in X for all open subsets G of Y.

Q2. Let (X, d) be a metric space and suppose that $f_1: X \to \mathbb{R}$ and $f_2: X \to \mathbb{R}$ are cts fns. Prove that $f: X \to \mathbb{R}^2$ defined as $f(x) = (f_1(x), f_2(x))$ is also continuous.

Q3. Let A be a subset of the metric space (X, d). Define

$$f(x) = d(x, A) = \inf\{d(x, y) : y \in A\}, \quad x \in X$$

Prove that f is uniformly cts on X.

OR

Prove that $f: \mathbb{R} \to (-1, 1)$ defined by $f(x) = \dfrac{x}{1+|x|}$ is uniformly continuous.

Q4. Let (X, d) be a metric space and let $x \in X$ and $A \subseteq X$ be nonempty. Prove that $x \in \bar{A}$ iff $d(x, A) = 0$

Q5. Let A and B be disjoint closed subsets of a metric space (X, d). Prove that there is a continuous real-valued function f on X such that $f(x) = 0 \ \forall x \in A$, $f(x) = 1 \ \forall x \in B$ and $0 \le f(x) \le 1 \ \forall x \in X$.

Q6. Given an example of the following:

(a) Image of a Cauchy sequence under a continuous fn not be a Cauchy sequence.

(b) Image of a complete metric space under a one-one uniformly cts mapping need not be complete.

Q7. (a) Prove that two metrics d_1 and d_2 on a nonempty set X are equivalent iff identity maps $id: (X, d_1) \to (X, d_2)$ is a homeomorphism.

(b) Consider the metrics d and e defined on $C[0,1]$ by

$$d(f,g) = \sup \{ |f(x) - g(x)| : x \in C[0,1] \}$$

$$\cancel{d}\ e(f,g) = \int_0^1 |f(x) - g(x)|\, dx$$

Prove that d and e are ~~equivalent~~ equivalent.

Q8. (a) State and prove Contraction Mapping Principle.

(b) Give an example to show that condition $0 < \alpha < 1$ in contraction mapping principle can't be relaxed.

Q9 Let X be any set and $T: X \to X$ a map such that T^n has a unique fixed point $x_0 \in X$. ~~From~~ Prove that T has 'x_0' as its unique fixed point.

~~Q10 Let f be~~

Q10. Let f be an isometry between metric spaces (X_1, d_1) and (X_2, d_2). Show that f is a homeomorphism between X_1 and X_2. Is the converse true? Justify your answer.

Q11 Let f be a mapping of (X_1, d_1) into (X_2, d_2). Prove that f is continuous on X iff

(a) For every subset $A \subseteq X_2$, $(f^{-1}(A))^{\circ} \supseteq f^{-1}(A^{\circ})$.

(b) For every subset $A \subseteq X_1$, $f(\bar{A}) \subseteq \overline{f(A)}$.

Topic of Assignment

Mathematical Modelling and Graph Theory

Sr. No.	Name	Roll No	Section		
1	Ankit	21/17065	A	Study of plastic decomposition and tracing price acc. to it.	
2	Ayan	21/17141	A	Father of Actuarial science & his contribution in Mathematics	
3	Raghav Anand Nath	21/17007	B	RSA Algorithm and History of Cryptography	
4	Vishal	21/17081	B	RSA Algorithm and History of Cryptography	
5	Aman katewa	21/17093	A	Laplace Transformation, Commagraph Theory	
6	Rakhi	21/17037	B	Graph Theory & its applications.	
7	Ankit	21/17068	A	Unsolved Millennium Problems in Mathematics.	
8	Ruchi Patel	21/17043	B	Graph theory and its applications	
9	Dhruv Arora	21/17139	A	Brief history of Cryptography & RSA Algorithm	
10	Shipra Mangal	21/17073	B	Graph theory & its applications.	
11	Kalpak Khobragade	21/17125	A	Graph of polynomials & History	
12	Srishti Singh	21/17138	B	Irrational Numbers	
13	Sneha Gupta	21/17096	B	Hidden mathematics behind Video Games	
14	Shashank Dandriyal	21/17061	B	RSA algorithm and history of cryptography	
15	Deepanshu	21/17026	A	Mathematics in Sports & Games	
16	Vishwesh Tiwari	21/17146	B	Hidden mathematics behind mathematics.	
17	Jishan	21/17028	A	History of Cryptograph and RSA	
18	Ishita Mishra	21/17031	A	$\sqrt{2}$ is irrational - History.	
19	Gunjan Manral	21/17014	A	Cryptography & RSA Algorithm.	
20	Bhawna Dabas	21/17135	A	$\sqrt{2}$ is irrational - History.	
21	Chinmay Singh	21/17175	A	Contribution of Leonhard Euler to Maths	
22	Aryan Dev	21/17112	A	Cryptography & RSA Algorithm	
23	Aarti Bafila	21/17012	A	Brief history of cryptography & RSA algorithm	
24	Ravi Kumar Thakur	21/17128	B	~~RSA Algorithm and Cryptography~~ / Graph theory Navigating the network	
25	Anjali Upadhyay	21/17167	A	Study of plastic decomposition and tracing price acc. to it	
26	Tarul	21/17103	B	Scientist that got field medal	
27	Palak Jangir	21/17105	B	Irrational No.	
28	Archit Chandra	21/17042	A	Two Field Medalists (Most recent)	
29	Abhay Singh	2117164	A	Egyptian Mathematics	

Mridula

Sr. No.	Name	Roll No	Section		
30	Arnav Choudhary	21/17136	A	Fermat's Last Theorm	
31	Aman	21/17070	A	Study of Plastic Decomposition and tracing	rates of plastic using it
32	Aditya Kumar Jewliya	21/17123	A	Unsolved Millennium	Problems in Mathematics
~~33~~	~~PRABHAT KUMAR~~	~~21/17121~~	~~B~~		
34	Prabhat Kumar	21/17121	B	Graph Theory	& it's applications.
35	Harshit Gaur	21/17075	A	Graphs of polynomials	& History
36	Tamanna	21/17113	B	Application	of Graph Theory
37	Ashish Sangwan	21/17006	A	Field	Medalist.
38	Deepanshu	21/17019	A	Field Medalist	in Mathematics
39	Simran Dahiya	21/17020	B	Graph Theory	& it's application.
40	Kanika Gupta	21/17078	A	History of	Cryptography & RSA
41	Aman Malik	21/17166	A	Father of Actuarial Science and his Contribution in Mathematics	
42	Harsh Vashisht	21/17085	A	Unsolved Milennium	Problems in Mathematics.
43	Sanvi Rathore	21/17124	B	Use of graph Theory	in search engine & social media.
44	Thangvanlalmuon Gangte	21/17134	B	Graph Theory	
45	Kamal kumar chaudhary	21/17095	A	Field medalist	
46	Rebekah Rane Philip	21/17133	B	The Basis of	Group Theory
	Bhaskar	21/17063	A	History of Cryptography & RSA	
	Jayant	21/17163		History of cryptography & RSA	

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Mathematically Modelling & Graph Theory

Theory and Practical Test (07|12|23)

Time Allowed : 1 Hour.

Write Legendre Differential Equation of order α. (T-1)

Is $x=0$ an ordinary point or singular ~~pt~~. point (T-1)

of the above Differential Equation.

Solve Legendre Differential Equation of order $\alpha = 1$

in powers of x, on paper and in Mathematica. (T-6, P-6)

What is $P_1(x)$? (T-2)

~~[illegible]~~. Graph $P_1(x)$. (P-4)

Mathematical Modelling & Graph Theory

Practical Test + Theory Test

(30.10.23)

Max. Marks: 10 | Time Allowed: 40 minutes

1) Use Monte Carlo method to find area under the curve $y = x^2$ between $x = 0$ and $x = 3$. (5)

2) Solve the LPP:

Maximize $3x_1 + x_2$

subject to $3x_1 + 2x_2 \leq 18$,

$x_1 \leq 4$,

$x_2 \leq 6$,

$x_1, x_2 \geq 0$ (5)

Initials.

Mathematical Modelling & Graph Theory

Max. Marks: 10 Test - I (04/10/23) Time Allowed: 40 minutes

① Compare and contrast "Middle - Square Method" and "Linear Congruence Method" for generating random numbers. Give suitable examples. (6)

② Draw Q_3.
Is Q_3 Eulerian? Justify
Is Q_3 semi-Eulerian? Justify (4)

Mridula

Teacher's Name: Suresh kr tedar

B. Sc. (Hons) Mathematics Sem - III, Sec - A

class test Paper: Discrete mathematics

Attempt any four questions each question carry equal marks

Q.1 Define following terms (a) Partially ordered set (poset) (b) modular lattice (c) complete lattice

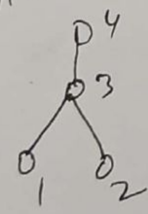
Q.2 Draw the Hass diagram of power set of set A with $\subseteq$ as relation where $A = \{a, b, c, d\}$

Q.3 Show/Check that the set (B, gcd, lcm) is a Boolean Algebra or not where B is set of all positive divisor of 45

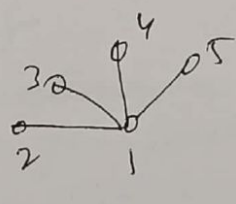
Q.4 Show that $\{\{1, 2, 3, 6, 9, 18\}, \text{gcd}, \text{lcm}\}$ does not form Boolean Algebra for the set of positive divisor of 18

Q.5 write/Describe the ordered pair in the rel^n whose Hasse diagrame in

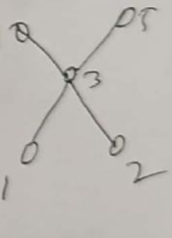
(a)



(b)



(c)



B.Sc. (Hons) Mathematics Sem - III, Sec - B

class - Test Paper

Attempt any four questions each carry equal marks

Q.1 Define (a) Lattice (b) Boolean Lattice (c) modular lattice

Q.2. Draw the Hass diagram of the relⁿ on set $A = \{1,2,3,4\}$

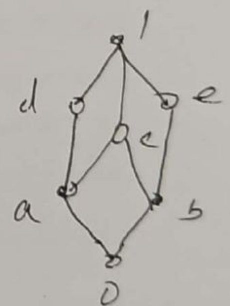
(a) $R_1 = \{(1,1)\ (1,2), (2,2), (2,4), (1,3)\ (3,3), (3,4), (1,4)\ (4,4)\}$

(b) $R_2 = \{(1,1)\ (2,2), (3,3)\ (1,3), (3,4), (3,5)\ (1,4), (4,4)\ (1,5)\ (2,3)\ (2,4)\ (2,5), (5,5)\}$

Q.3 Show that (B, gcd, lcm) is a Boolean Algebra if B is set of all positive divisor of 110

Q.4. Show that the set of B of all positive divisor of 72 is not a Boolean Algebra

Q.5 Consider lattice L in Fig Determine following



(a) which non-zero elements are join-irreducible

(b) which non-zero elements are meet irreducible

(c) which are atoms

(d) is L complete lattice ?

Addition Test Date: 11/12/2023

* Attempt any Four questions each questions carry equal marks.

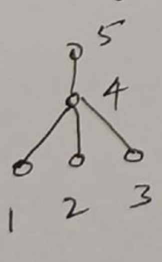
Q.1 Check wether the set (B, gcd, lcm) is Boolean Algebra, if B is set of all positive divisor of 108.

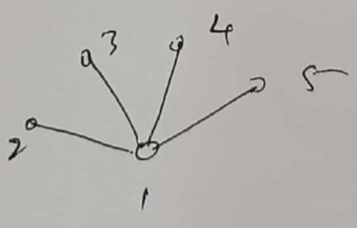
Q.2 Define following (a) Lattice (b) Boolean Lattice (c) complete lattice

Q.3. Draw the Hasse diagramme of power set of set $A = \{a, b, c, d\}$

Q.4 Draw the Hasse diagramme of the relation on set $A = \{1, 2, 3, 4, 5\}$ given by
$R = \{(1,1), (2,2), (3,3), (1,3), (3,4), (3,5), (1,4)$
$(4,4), (1,5), (2,3) (2,4) (2,5), (5,5)\}$

Q.5. Describe the ordered pair in the relation determined by the Hasse diagramme on set $A = \{1, 2, 3, 4, 5\}$





CLASS TEST (SET A)

SET - A

SHIVAJI COLLEGE

UNIVERSITY OF DELHI

Name of the Paper : Differential Equation

Name of the Course : Applied Physical Science with CS

Semester : III

Duration : 1 Hours

Maximum marks : 12

Attempt all Questions each Queastion carrying of 2 marks.

Question 1 Find the general solution of the differential equation 2 marks

$(2x^2 + y)dx + (x^2y - x)dy = 0$

Question 2 Find the general solution of the differential equation by using variation of parameter

$y''(x) + 6y'(x) + 9y(x) = e^{-3x}/x^3$ 2 marks

Question 3 Solve the system of equation 2 marks

$$x'(t) + y'(t) - x(t) - 2y(t) = 2e^t$$
$$x'(t) + y'(t) - 3x(t) - 4y(t) = e^{2t}$$

Question 4 The population x of a certain city satisfies the logistic law

$$x'(t) = (x/100) - x^2/(10)^8$$ 2 marks

where time t mneasured in years . Given that the population of the city is 100000 in 1980, determine the population as a function of time for t > 1980. In particular answer the follownig questions :

(a) What will be the population in 2000 ?

(b) In what year does the 1980 population double ?

Question 5 Find the general solution of the equation 2 marks

$$u_{xx} + 2u_{xy} + u_{yy} = 0$$

Question 6 Find the solution of the equation 2 marks

$$u(x + y)u_x + u(x - y)u_y = x^2+y^2$$

with the cauchy data u = 0 on y = 2x

सुनील कुमार मीना

CLASS TEST (SET-B)

SET - B

SHIVAJI COLLEGE

UNIVERSITY OF DELHI

Name of the Paper : **Differential Equation**

Name of the Course : **Applied Physical Science with CS**

Semester : **III**

Duration : **1 Hours**

Maximum marks : **12**

Attempt all Questions each Queastion carrying of 2 marks.

Question 1 Find the set of orthogonal trajectories of the family of curve $x - y = cx^2$ 2 marks

Question 2 Find the general solution of the differential equation by using variation of parameter 2 marks

$y''(x) + 3y'(x) + 2y(x) = 1/(1+e^{2x})$

2 marks

Question 3 Solve the system of equation

$$2x'(t) + y'(t) - 3x(t) - y(t) = t$$

$$x'(t) + y'(t) - 4x(t) - y(t) = e^t$$

Question 4 The human population of a certain island satisfies the logistic law with k=0.03, $\lambda = 3(10)^{-8}$, 2 marks
and time t measured in years.

(a) If the population in 1980 is 200,000, find a formula for the population in future years.

(b) Acoording to the formula of part (a), what will be the population in thr year 2000 ?

2 marks

Question 5 Find the general solution of the equation

$$u_{xx} + 2u_{xy} + 5u_{yy} + u_x = 0$$

2 marks

Question 6 Determine the integral surface of the equation

$x(y^2 + u)u_x - y(x^2 + u)u_y = (x^2 - y^2)u$

with data $x + y = 0$, $u = 1$

सुनील कुमार मीना

Assignment

Name of the Paper : GE – III Differential Equations

Name of the Course : Applied physical science with computer science

Semester : III

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Solve the following differential equations :

(i) $\frac{dy}{dx} = y\tan x + x^2\cos x$

(ii) $(x^2 - ay)dx + (y^2 - ax)dy = 0.$

P.T.O.

सुनील कुमार मीना

24/165

(b) Solve the Initial Value problem .

$$x\frac{dy}{dx}-3y=x^5y^{1/3}, \quad y(0)=1 .$$

Or

Solve the following differential equations :

$$\sin y\frac{dy}{dx}-2\cos x\cos y=-\cos x\sin^2 x .$$

(c) Find a family of oblique trajectories that intersect the family of parabolas $y^2 = cx$ at angle 60 degrees.

2. (a) Consider the differential equation :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0 .$$

Show that x^2 and $1/x^2$ are linearly independent solutions of this equation on the interval $0 < x < \infty$. Write the general solution of the given equation. Hence find the particular solution satisfying the initial conditions $y(2) = 3$, $\frac{dy}{dx}(2)=-1$.

सुनील कुमार मीना

(b) Find the suitable integrating factor for the differential equation

$(x^2 + y^2 + x)dx + xydy = 0$ and hence solve it.

(c) Given that $y = e^{2x}$ is a solution of the differential equation

$$(2x-1)\frac{d^2y}{dx^2} - 4(x+1)\frac{dy}{dx} + 4y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution also.

3. (a) Solve the Initial value Problem

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0, \quad y(0) = 1, \quad \frac{dy}{dx}(0) = 5.$$

(b) Find the general solution of the differential equation using method of Undetermined Coefficients

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 10y = 8\left(xe^{-2x}\right)$$

सुनील कुमार मीना

(c) Find the general solution of the differential equation using method of Variation of Parameters

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\frac{1}{1+e^x}.$$

4. (a) Given that $e^x \sin 2x$ is a solution of the differential equation

$$\frac{d^4y}{dx^4}+3\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+13\frac{dy}{dx}+30y=0$$

find the general solution.

(b) Find the general solution of the differential equation by assuming $x > 0$

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=4\sin(\log x)$$

(c) Find the general solution of the given linear system

$$\frac{dx}{dt}=2x+5y, \quad \frac{dy}{dt}=5x+12.5y$$

सुनील कुमार मीना

5. (a) Find the solution to the linear partial differential equation

$$x\,u_x + y\,u_y = u + 1,\ u(x,y) = x^2 \text{ on } y = x^2.$$

(b) Find the solution of the following partial differential equation by the method of separation of variables

$$u_x + 2u_y = 0,\quad u(0,y) = 3\exp(-2y).$$

(c) Reduce the equation

$$u_x - y\,u_y - u = 1,$$

into canonical form and obtain the general solution.

6. (a) Find the general solution of the differential equation by reducing it into the canonical form

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2.$$

(b) Reduce the equation

$$u_{xx} + 4\,u_{xy} + 4\,u_{yy} = 0$$

into canonical form and hence find its general solution.

P.T.O.

सुनील कुमार मीना

(c) (i) Find the partial differential equation by eliminating the arbitrary function f and g from the following equation

$$z = f(x + ay) + g(x - ay).$$

(ii) Find the partial differential equation by eliminating the arbitrary function f from the following equation

$$yz + zx + xy = f\left(\frac{z}{x + y}\right)$$

सुनील कुमार मीना

TEST FOR INTERNAL ASSESSMENT

B.Sc. (H) Mathematics Academic Session: 2023-2[4] (odd)

Semester I Section B

Paper : DSC-3 : Probability & Statistics

UPC ; 2352011103 Max. Marks : 20

Date : 07.11.2023 Time : 1 hr.

Instruction : Attempt all question. All questions carry equal marks.

Q1. Draw the Stem-leaf plot for the following data:

5.9 7.2 7.3 6.3 8.1 6.8 7.0 7.6 6.8 6.5
7.0 6.3 7.9 9.0 8.2 8.7 7.8 9.7 7.4 7.7
9.7 7.8 7.7 11.6 11.3 11.8 10.7

(a) Is their any outlier?

(b) What proportion of the data exceeds 10?

Q2 Find the mean, median & Sample Variance of the following data

154 142 137 133 122 126 135 135 108
120 127 134 122

Q3 State axiomatic defination of probability
Prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for any two events A & B.

Q4 (a) A certain shop repairs both audio & video components. Let A denote the event that the next component bought for repair is an audio component, & let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose $P(A) = 0.6$ & $P(B) = 0.5$. Find $P(B/A)$.

(b) State Baye's theorem.

Kumari Priyanka
07/11/2023

TEST FOR INTERNAL ASSESSMENT

B.Sc. (H) Mathematics Academic Session: 2023-24 (ODD)

Semester - I Section B

Paper: DSC-3 Probability & Statistics

UPC: 2352011103

Max. Marks: 20

Date: 11.12.2023

Time: 1 hr

Instructions: Attempt all questions. All questions carry equal marks.

Q1 State Central limit theorem. Let X = the number of different people sent text messages during a particular day by a randomly selected student at a large University. Suppose the mean value of X is 7 & the standard deviation is 6. Among 100 randomly selected such students, how likely is it that the sample mean number of different people texed exceeds 5? [Use $\Phi(-3.33) = 0.9996$]

Q2. Define Binomial distribution. Derive the formula for its mean and variance.

Q3. Define cumulative distribution function. The probability distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is a continuous random variable X with pdf

$$f(x) = \begin{cases} \frac{3}{2}(1-x^2), & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find its cdf.

Q4 Draw the stem-and-leaf plot for the following data

2.0 2.4 2.5 2.6 2.6 2.7 2.7 2.8 3.0 3.1 3.2
3.3 3.3 3.4 3.4 3.6 3.6 3.6 3.6 3.7 4.4 4.6
4.7 4.8 5.3 10.1

Khushi Priyanka
11/12/2023

— o —

Academic Session: 2023 – 2024 (Odd Semester)

Test for Internal Assessment Date: 21.11.2023

Name of Course	: **B.Sc. (Prog) with Computer Science**
Name of Paper	: **Differential Equations (Practical)**
Unique Paper Code	: 42357501
Semester	: V
Duration	: **1 hour**
Maximum Marks	: **20**

Instructions: *Attempt any two questions.*

Q1. Solve the system of differential equations

$$\frac{dx}{dt} = -0.0544\, z, \frac{dz}{dt} = -0.0106\, x$$

with initial condition $x(0) = 66, z(0) = 18$ and hence plot the solutions.

Q2. Solve $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 25\frac{dy}{dx} + 21y = 0$ and plot it's any four solutions.

Q3. Plot the characteristics of the partial differential equation

$$(u_x + y)u_x - y(x + y\, u)u_y = x^2 + y^2$$

Kumari Priyanka
21/11/2023

Academic Session: 2023 – 2024 (Odd Semester)

Date: 08.12.2023

Test for Assessment

Name of Course	: **B.Sc. (H) Mathematics**
Name of Paper	: **DSC-3: Probability and Statistics (Practical)**
Unique Paper Code	: **2352011103**
Semester	: **I**
Duration	: **1.5 hours**
Maximum Marks	: **10**

Instruction: *Attempt any two questions.*

Q1. For the following data find descriptive statistics, construct frequency table, stem-and-leaf plot, dot plot and histogram:

x	36	36	45	45	45	55	64	37	57	67	73	83	12	12

Q2. Fit the Binomial Distribution to the given data and hence find

(a) Probability at $x = 4$ (b) Probability for $x < 5$ (c) Probability for $x > 5$

x	0	1	2	3	4	5	6
f	5	12	19	28	21	14	2

where, x and f denote the random variable and the corresponding frequency.

Q3. The invasive diatom species Didymosphenia geminate has the potential to inflict substantial ecological and economic damage in rivers. An article described an investigation of colonization behaviour in that one aspect of particular interest was whether y = colony density was related to x = rock surface area. Here is representative data:

x	50	71	55	50	33	58	79	26	69	44	37	70	20	45	49
y	152	1929	48	22	2	5	35	7	269	38	171	13	43	185	25

Fit the simple regression model to this data. Calculate and interpret the slope of the regression line.

Kumari Priyanka
8/12/2023

B.Sc.(H) Mathematics Academic Session: 2023-2024 (ODD)
Semester-I (13)

Assignment for Internal Assessment Date: 14.11.2023

Paper: Probability & Statistics.

1. Define fourth spread. What is an outlier & when an outlier is extreme & when it is termed as mild.
For the following data calculate the median, upper fourth (third quartile) & lower fourth (first quartile) & hence the fourth spread.
9.69 13.16 17.09 18.12 23.70 24.07 24.29
26.43 30.75 31.54

2. (a) Define conditional probability.
(b) State Baye's Theorem.
(c) An individual has three different email accounts. 70% of her messages come into account A_1, whereas 20% come into account A_2 & the remaining 10% into account A_3. Of the messages into account A_1, only 1% are spam, whereas the corresponding percentages for accounts A_2 and A_3 are 2% & 5% respectively. A randomly selected mail is found to be a spam. What is the probability that it came in account A_2?

Kumar Prakash
14.11.2023

3. Let the random variable be defined as $X=1$ if a randomly selected vehicle passes an emission test & $X=0$ otherwise.
Assume that the probability mass function (pmf) of this variable is $p(1)=p$ and $p(0)=1-p$.
(i) Name the random variable & its type.
(ii) Compute $E(X^2)$
(iii) Show that $V(X)=p(1-p)$

4. For any random variable X, let $E(X)=5$ and $E[X(X-1)]=27.5$. Compute
(i) $E(X^2)$
(ii) $V(X)$
(iii) $V(2X+3)$

5. ~~Do~~ Explain the following
(a) Binomial Distribution
(b) Poisson Probability Distribution
(c) Negative Binomial Distribution
(d) Geometric Distribution
(e) Hypergeometric distribution
(f) Normal Distribution
(g) Exponential Distribution

Khumi Prajapati
14.11.2023

23/11/2023

B.Sc (H) Maths (Ist Sem) (Set-A)

(Test of Algebra)

Maxm Marks-12

Attempt any three questions-

Q1 (i) Find the geometric image for the Complex number z such that-

$$|z+1+i|<3 \text{ and } 0<\arg z<\frac{\pi}{6}.$$

(ii) Solve the equation. $x^4-7x^3+17x^2-x-26=0$. given that one root is $3+2i$.

Q2 (i) Suppose a, b and c are three non-zero integers with "a" and "c" relatively prime. Show that $\gcd(a,bc)=\gcd(a,b)$.

(ii) Solve the following Congruence if possible, If no solution exists, explain why not-

$$4x \equiv 2 \pmod{6}.$$

Q3 Find $|z|$ and $\arg z$ for $z=\frac{(2\sqrt{3}+2i)^8}{(1-i)^6}+\frac{(1+i)^6}{(2\sqrt{3}-2i)^8}$.

Q4 Prove that one root of $x^3+px^2+qx+r=0$ is negative of another root iff $r=pq$.

Ankush Kumar

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3/11/2023

B.Sc (H) Maths (Ist Sem) (Set-B)

(Test of Algebra) Max Marks-12

Attempt any three questions-

Q1 (i) Solve $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, whose roots are in arithmetical progression.

(ii) Find all the rational roots-

$2x^3 - [illegible]x^2 - 22x - 24 = 0$.

Q2 Solve the equations-

$z^6 + iz^3 + i - 2 = 0$.

Q3 (i) Using Euclidean algorithm find g.c.d(1004, -24) and express it as integral linear combination of the given integers.

(ii) Let a, b and c be three natural numbers such that gcd (a, c) = 1 and b divides c. Prove that gcd (a, b) = 1.

Q4 Solve the following pair of congruences:

$2x + 3y \equiv 1 \pmod 6$

$x + 3y \equiv 5 \pmod 6$

Anilkumar

1/2023

B.Sc (H) Maths (Ist Sem) Set-I

Test of Algebra Max Marks-12

Attempt any three questions-

Q1 (i) Find all the integral roots of-

$$x^7+2x^5+4x^4-8x^2-32=0$$

(ii) $x^4+15x^3+70x^2+120x+64=0$ given that the product of two of its roots is equal to the other two.

Q2 Find all complex numbers Z, such that $|Z|=1$ and $\left|\frac{Z}{\bar{Z}}+\frac{\bar{Z}}{Z}\right|=1$

Q3 (i) Solve the congruence $7x \equiv 8 \pmod{11}$.

(ii) Prove that $\gcd(n, n+1)=1$ for every natural number n. Find integers x and y such that $n.x+(n+1)y=1$.

Q4 Find the fourth roots of unity and represent them in the complex plane. Show that they forms a square inscribed in the unit ~~ci~~ circle.

B.Sc(H) Maths (Ist Sem) Set-1 06/12/23

Test of Algebra

Maxm Marks-12

Attempt any ~~two~~ questions -

Q1 Let G be the set of all 2×2 real matrices with non-zero determinant. Show that G is a group under the operation of matrix multiplication. Further show that it is not an Abelian Group.

Q2 Show that the set $G = \{1, 5, 7, 11\}$ is a group under multiplication modulo 12 with the help of the Cayley table.

Q3 Show that for any integer n, the set $H_n = \{n \cdot x \mid x \in \mathbb{Z}\}$ is a subgroup of the group $\mathbb{Z}$ of integers under the operation of addition. Further show that $H_2 \cup H_3$ is not a subgroup of $\mathbb{Z}$.

Q4 Let $n > 1$ be a fixed natural number. Let a, b, c be three integers such that $ac \equiv bc \pmod{n}$ and $\gcd(c, n) = 1$. Prove that $a \equiv b \pmod{n}$.

Q5 Find the polar representation of the complex number -

$$z = \sin a + i(1 + \cos a), \quad a \in [0, 2\pi).$$

Anilkumar

B.Sc(H). Maths (Ist Sem) Set-2

Test of Algebra

Maxm Marks-12

06/12/23

Attempt any two questions-

Q1 Let G be a group. Show that $|aba^{-1}| = |b|$ for all a and b in G. ($|x|$ denotes the order of an element x in G).

Q2 Show that the group $Z_n = \{0, 1, 2, \ldots, (n-1)\}$ is cyclic under the operation of addition modulo n. How many generators Z_n have? Further, describe all the subgroups of Z_{40}.

Q3 Show that $U(10)$ is an abelian group under multiplication modulo 10.

Anilesh kumar

Assignment - B.Sc (H) Maths Ist Sem

Max Marks - 12

...pt all the questions -

1. Solve the eqⁿ - (i) $z^2 + (2i-3)z + 5 - i = 0$.

   (ii) $z^5 + 1 = 0$

Q2 Find the seven seventh root of unity and from that for any positive integer n, the sum of their n^{th} powers vanishes, unless n is a multiple of 7 in which case the sum is 7.

Q3 If n is a positive integer, show that

$$(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos\frac{n\pi}{6}$$

Q4 Solve the following linear congruences.

(i) $9x \equiv 4 \pmod{6}$

(ii) $3x \equiv 2 \pmod{5}$

Q5 Compute (i) $z^n + \frac{1}{z^n}$ if $z + \frac{1}{z} = \sqrt{3}$

(ii) $$z = \frac{\left[(1-i)^{10}(\sqrt{3}+i)^5\right]}{(-1-i\sqrt{3})^{10}}$$

Q6 (i) Find the gcd (1800, 756)

(ii) If $ac \equiv bc \pmod{m}$ and $\gcd(c, m) = 1$ then $a \equiv b \pmod{m}$

Q7 For $a, b \in \mathbb{Z}$ define $a \sim b$ iff $3a + b$ is a multiple of 4.

(i) Prove that $\sim$ defines an equivalence relation

(ii) Find the equivalence class of 0 and 2.

Ankush kumar

Show that the set $\{5, 15, 25, 35\}$ is a group under multiplication modulo 40. What is the identity element of this group? Can you see any relationship between this group and $U(8)$?

Q9 (i) Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group but that $\{1, 2, 3, 4\}$ under multiplication modulo 5 is a group.

(ii) Show that the group $GL(2, R)$ is non-abelian by exhibiting a pair of matrices A and B in $GL(2, R)$ such that $AB \neq BA$.

Q10 (i) Prove that an abelian group with two elements of order 2 must have a subgroup of order 4.

(ii) Suppose that H is a proper subgroup of Z under addition and H contains 18, 30 and 40. Determine H.

(iii) Let G be a group and let $a \in G$. Prove that $C(a) = C(a^{-1})$.

Anlush kumari

Internal Exam

Vedic Mathematic -II

Max Marks: 20

Time 1 Hour

Q1: (i) Solve the following equation by Paravartya sutra

$$\frac{3}{x+1} + \frac{4}{x+2} = \frac{7}{x+4}.$$

(ii) Solve the system of equation by using Paravartya Yojayet method

$2x + 3y = 7$ and $4x + 6y = 8$.

Q2: (i) Solve the following by using "Samuccaya sutra".

(i) $\frac{3x+4}{6x+7} + \frac{4x+5}{2x+4} = \frac{5x+6}{2x+3} + \frac{2x+3}{6x+8}$,

(ii) $\frac{1}{x-8} + \frac{1}{x-9} = \frac{1}{x-5} + \frac{1}{x+2}$,

Q3: Define 'Anurupye Sunyamanyat' sutra and solve the following

(i) $5x + 8y = 40$ and $10x + 11y = 80$

(ii) $6x + 12y = 24$ and $48x + 96y = 192$

Test

Vedic Mathematics - I

Date :- 11/12/23 M.M. - 20

- Do any 5 questions
- All questions carry equal marks.

Solve the following questions.

1. $(108)^3$
2. Multiply 6543 by 9999
3. $\sqrt{4225}$
4. $\sqrt{3249}$
5. $\sqrt{20736}$
6. $(8989)^2$
7. Divide 7238761 by 524

Raman.

B.Sc (H) Mathematics

Assignment

I^{st} Semester

Elementary Real Analysis M.M.-12

Due Date:- 5/12/2023

- Do the following questions from the book "Introduction to Real Analysis" By Robert G. Bartle and Donald R. Sherbert (4th Edition)

Sec 3.1 :- Q^n 1, 3, 4, 5, 6, 11, 12, 13

Sec 3.2 :- Q^n 5, 6, 14, 19

Sec 3.3 :- Q^n 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 15

Sec 3.4 :- Q^n 4, 7, 8

Sec 3.5 :- Q^n 1, 2, 3, 10, 11

Poonam.

Test

B.Sc (H) Mathematics

I-B

Elementary Real Analysis M.M.-12

Date:- 3/11/2023

- Do any three questions
- All question carry equal marks.

Q1 Show that $\lim n^{1/n} = 1$

Q2 find the Infimum and Supremum (if exists) of the set $A = \{x \in \mathbb{R} \mid x > \frac{1}{x}\}$.

Q3 Show by definition that $\lim \frac{3n+1}{2n+5} = \frac{3}{2}$.

Q4 State Completeness property and show that every bounded below subset of $\mathbb{R}$ has infimum in $\mathbb{R}$.

Test

B.Sc. (H) Mathematics

I- A

Elementary Real Analysis

M.M.-12.

Date:- 3/11/2023

- Do any three questions
- All questions carry equal marks.

Q1 Show by definition that $\lim \frac{2n+5}{n+1} = 2$.

Q2 Show that $\lim n^{1/n} = 1$

Q3 Let S be non-empty subset of $\mathbb{R}$ that is bounded above. Show that

$$\sup(S) = -\inf\{-s : s \in S\}$$

Q4 Let X be non-empty set and let $f : X \to \mathbb{R}$ have bounded range in $\mathbb{R}$. Show that

$$\sup\{a + f(x) : x \in X\} = a + \sup\{f(x) : x \in X\}$$

where a is a constant.

Neeman.

Internal-II

Time 1 Hour.

Differential equations

Attempts any two questions.

Q1 (i) Find the general solution of the differential equation using undetermined coefficient methods.

$$y''' + 2y'' - 3y' - 10y = 8xe^{-2x}.$$

(ii) Find the general solution of the differential equation using method of variation of parameters.

$$y'' + 3y' + 2y = \frac{1}{1+e^{x}}$$

Q2: (i) Find the general solution of the differential equation

$$x^2y'' + xy' + y = 4\sin(\log x)$$

(ii) $y'' + 4y = 4\sec^2 2x$

Q3: (i) Reduce the equation $u_x - y u_y - u = 1$. into Canonical form.

(ii) Find the Solution of the linear partial differential equation

$$xu_x + yu_y = u+1, \quad u(x,y) = x^2 \text{ on } y = x^2$$

Differential equation

Max mark: 10

Attempts any two questions.

Q1: (i) Solve the following differential equation

$$\frac{dy}{dx} = y \tan x + x^2 \cos x$$

(ii) $$\frac{dy}{dx} - 2\cos x \cos y = -\cos x \sin^2 x$$

Q2: (i) Find the suitable integrating factor for the differential equation

$$(x^2+y^2+x)\,dx + xy\,dy = 0$$ and hence solve it.

(ii) Solve the initial value problem

$$x\frac{dy}{dx} - 3y = x^5 y^{1/3}, \quad y(0) = 1$$

Q3: (i) Solve $(x^2-ay)\,dx + (y^2-ax)\,dy = 0$

(ii) $x^2 y'' + xy' - 4y = 0$

Assignment I+II

Differential Equation

Q1: Solve the following differential equations

(i) $y'' - 6y' + 8y = 0, \quad y(0) = 1, y(0) = 6.$

(ii) $y'' - 6y' + 9y = 0, \quad y(0) = 2, y(0) = 8.$

(iii) $(D^3 - 2D^2 - 5D + 6)y = 2e^x + 4e^{3x} + 7e^{-2x} - 2x + 8e^{2x} + 15.$

(iv) $(D^2 - p^2)y = \sinh px.$

(v) $(D^2 - 4D - 5)y = e^{2x} + 3\cos(4x + 3).$

(vi) $(D^3 + D^2 + D + 1)y = \sin 2x + \cos 3x.$

(vii) $(D^3 - 1)y = x^5 + 3x^4 - 2x^3.$

Q2: Solve the following differential equation using variation of parameters method.

(i) $(D^2 + 2D + 1)y = e^{-x}\ln x.$

(ii) $(D^2 + 4)y = 4\sec^2 2x.$

(iii) $(D^2 - 3D + 2)y = xe^x + 2x$

(iv) $(D^2 - 6D + 9)y = e^{3x}$

(v) $(D^2 + a^2)y = x\cos ax$

Q3: Solve the following differential equation using undetermined coefficients method.

(i) $(D - 3)^4(D + 4)y = x^3e^{3x} + 6x^2$

(ii) $y'' + 6y' + 5y = 2e^x + 10e^{5x}.$

(iii) $y'' + 2y' + 4y = 13\cos(4x - 2).$

(iv) $y'' - y = e^x \sin 2x.$

(v) $y''' + y = 2x^2 + e^{2x} + 4\sin x.$

Assgnment III

~~Assignment III~~

Differential Equation

Q1: Form the PDE

(i) $z = xf(ax + by) + g(ax + by)$.

(ii) $(x - a)^2 + (y - b)^2 = z^2 \cot^2 \alpha$.

(iii) $F(x^2 + y^2 + z^2,\ z^2 - 2xy) = 0$.

(iv) $z = (x + y)\,\varphi\,(x^2 - y^2)$.

Q1: solve the following PDE

(i) $yzp - xzq = xy$.

(ii) $z(z^2 + xy)(px - qy) = x^4$.

(iii) $(y + zx)p - (x + yz)q = x^2 - y^2$.

(iv) $(y^2 + z^2)p - xyq + zx = 0$.

Assignment

Vedic Mathematic -II

Q1: (i) Write the short notes on main contribution of "BRAHMAGUPTA" in Vedic Mathematics.

(ii) Write the short notes on main contribution of "Srinivasa Ramanujan" in Vedic Mathematics

Q2 (i): Solve the following by using 'Paravartya Yojayet' sutra

$$\frac{3}{x+1} + \frac{4}{x+2} = \frac{7}{x+4}$$

(ii): Solve the following by using "Samuccaya sutra".

$$\frac{1}{x-8} + \frac{1}{x-9} = \frac{1}{x-5} + \frac{1}{x+2}.$$

(iii): Define 'Anurupye Sunyamanyat' sutra and solve the following

$$6x + 12y = 24 \text{ and } 48x + 96y = 192.$$

Q3 (i): Find the inverse of the matrix by using Urdhav Triyak sutra

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

(ii): The sum of three numbers is 2. If twice the second number is added to the sum of first and third, the sum is 2. By adding second and third number to five times the first number, we get 6. Find the three numbers (Using inverse of matrix by high- speed matrix algebra).

(iii): The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method (Using inverse of matrix by high- speed matrix algebra).

Q4 (i): Find the equation of a line parallel to $3x + 5y = 17$, passing through $(2, 1)$

(ii): In what ratio does the y-axis divide the line of $(7, 3)$ and $(5, 12)$.

(iii): Find the equation of a line passing through $(-5, 11)$ and parallel to $2x + y = 7$

Bhupi

06/12/23

Class Test

BSc (H) Mathematics Semester III, DSC - 8 Riemann Integration

Time: 1 hr

M.M: 10

Do any 5 questions.

Q1. Find the upper and lower Darboux integrals for $f(x) = x^2$ on $[0, b]$. Is f integrable?

Q2. Prove that every monotone increasing function f on $[a, b]$ is integrable.

Q3. Use Comparison Test to determine whether the following improper integral converge

i) $\int_1^\infty \frac{dx}{x\sqrt{x+1}}$ (ii) $\int_{-1}^\infty \frac{dx}{x^2+4x+6}$

Q4. Determine the convergence or divergence of following improper integrals

i) $\int_1^3 \frac{x}{\sqrt{x^2-1}}\,dx$ ii) $\int_0^1 \frac{1}{x\ln x}\,dx$

Q5 Find the volume of the solid generated when the region under the curve $y = x^2$ over the interval $[0, 2]$ is rotated about the line $y = -1$.

6. Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$ and x - axis is revolved about y axis.

7. Check for the convergence of $\int_0^\infty e^{-t} t^{s-1}\,dt$. Define Γ function.

Class Test

Name of the course : BSc (H) Mathematics 7/11/23

Semester : III

UPC : 2352012302

Name of Paper : DSC-8 : Riemann Integration.

M.M. 10 Time : 1hr

(Attempt any 5 questions).

Q.1. Find the upper and lower Darboux integrals for $f(x) = x^3$ on the interval $[0, b]$. Is f integrable.

Q2. Prove that every continuous fn f on $[a, b]$ is integrable.

Q3. Let f be a continuous function on $\mathbb{R}$ and define

$$G(x) = \int_0^{\sin x} f(t)\,dt, \quad x \in \mathbb{R}$$

Show that G is differentiable on $\mathbb{R}$ and compute G'.

Q4. Find the volume of the solid that results when the region enclosed by $y = \sqrt{x}$, $y = 0$ and $x = 9$ is revolved about the line $x = 9$.

Q5. Find the volume of the solid whose base is the region enclosed between the curve $x = 1 - y^2$ and the y-axis and whose cross sections taken perpendicular to the y-axis are squares.

Q6. Use cylindrical shells to find the volume of the solid generated when the region enclosed by

$y = \sqrt{x}$, $x = 4$, $x = 9$, $y = 0$

is revolved about y axis.