

Time - 40 minutes

Attempt all questions.

Max marks - 12

Q1 a) Is the following function continuous and differentiable?

$$f(x) = \frac{|x|}{x}$$

b) Evaluate $\lim_{x \rightarrow 2^+} \frac{x + |x-2|}{x-1}$. (4)

Q2 Prove that the equation $f(x) = x$ has at least one solution in $[a, b]$. (3)

Q3 a) Does the sequence $s_n = \frac{n^2-1}{n^2-n}$ converge? Why?

b) Find the linear approximation of the function $f(x) = \sqrt{1-x}$ around $x=0$ and use it to obtain an estimate of $\sqrt{0.95}$. Also, find an upper limit for the error of approximation. (2+3)