

1) Given the Cobb-Douglas function:

$$f(x, y) = x^a y^b$$

where $x, y > 0$ and $a, b \in (0, 1)$

- Show whether $f(x, y)$ is quasiconcave or quasiconvex.
- Carefully state all conditions Use Border-Hovewen method.
- Prove every statement e.g. Range of certain expression is strictly greater than zero or less than zero
- Sketch the level curves. (7 marks)

2) Given $g(x, y) = 3x^3 - 5y^2 - 225x + 70y + 23$

- Find all critical points & determine if max, min, saddlepoint, or inflection point.
- Explain each step / statement (6 marks)

3) Use the Lagrangian method to optimize

$$f(x, y) = 4x^2 - 2xy + 6y^2 \text{ s.t. } g(x, y) = x + y = 72$$

- Find f^*, x^*, y^* (optimal values)
- Estimate change in $f(x, y)$ due to a small change in the constant of the constraint function $g(x, y)$.

(7 marks)