Question 1:- (6 marks)

A double pendulum consists of a pendulum of mass m_1 and length l_1 to which a second pendulum of mass m_2 and length l_2 is suspended as shown in figure(1). The motion is considered in a plane so that the system has two degrees of freedom. Find T matrix and V matrix for this system and also find normal frequencies when $m_1 >> m_2$.

OR

Two equal masses (m) are connected to each other with the help of a spring with force constant k and then the upper mass is connected to a rigid support by an identical spring as shown in figure(2). The system is allowed to oscillate in the vertical direction. Show that the normal frequencies are $w^2 = (3 \pm \sqrt{5}) k/2m$.

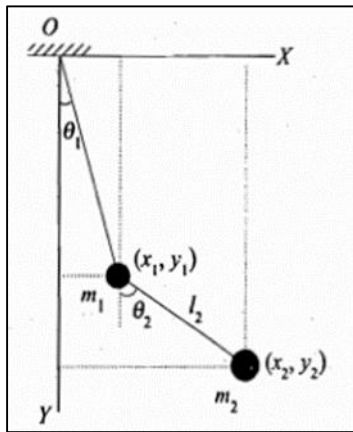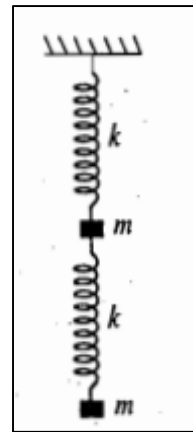Figure (1)

Figure (2)

Question 2:- (5 marks)

1) A beam of positive ions (each having mass m and charge q) passes through uniform crossed electric and magnetic fields:-

$$\vec{E} = E_0 \hat{y} \text{ and } \vec{B} = B_0 \hat{z}$$

Find the speed of those ions which pass undeviated.

2) If electric field is switched off then the undeviated ions (in part 1) move in a circular path of radius R in the presence of magnetic field alone. Find R and hence show that:-

$$\frac{q}{m} = \frac{E_0}{R B_0^2}$$

Question 3:- (6 marks)

A particle of mass m moves inside a bowl under gravity. If the surface of the bowl is given by the equation $z = a(x^2+y^2) / 2$, where a is a constant.

    (A) Write down Lagrangian of the system in cylindrical coordinate system.
    (B)   Find cyclic coordinate and its corresponding conjugate momentum.
    (C) Write down Hamiltonian of the system in cylindrical coordinate system.

Question 4:-   **(3 marks)**

Consider a particle of mass m moving in one dimension under a force with potential $U(x) = k(2x^3 - 5x^2 + 4x)$ where $k > 0$

(a) Find the stable and unstable equilibrium point.

(b) If particle oscillate about stable equilibrium point then natural frequency of oscillation is given by?