BSc (H) Mathematics, Sem IV (A) No.

Sequence of series of functions (26th April '24)

Time 45 min (Set A) MM : 15

All questions carry equal marks!

Q1 Show that the power series $\sum_{n=1}^{\infty} \sin \frac{x}{n^2}$ is only pointwise convergent for $|x| > a$ $(a > 0)$ and uniformly convergent on $[-a, a]$

Q2 If R is Radius of convergence of power series $\sum a_n x^n$ then prove that series is absolutely convergent for $|x| < R$ and divergent for $|x| > R$.

or

Suppose that $f(x) = \sum a_n x^n$ has radius of convergence $R > 0$ then show that

$$\int_0^x f(t)\,dt = \sum_{n=0}^{\infty} \frac{a_n x^{n+1}}{n+1} \quad \text{for } |x| < R$$

Q3 Find the exact interval of convergence of $\sum_{n=1}^{\infty} \frac{n^{\alpha} x^n}{n!}$

(Sushlir Madan)

Submitted by : Smm
(Prof Surbhi Madan)

$\boxed{25^{th}\ \text{April}\ 2024}$

BSc (H) Mathematics Sem IV, Sec B
Internal Exam (Test) 2024

Time 45 min | Seq. & Series of functions | MM : 15 | Set B

Each question carries equal marks.

Q1/ Show that the power series $\sum_{n=1}^{\infty} \frac{1}{n^2 x^2}$ is only pointwise convergent on $\mathbb{R} \sim \{0\}$ and uniformly convergent for $|x| \geq a$, $a > 0$

Q2/ Suppose that $f(x) = \sum a_n x^n$ has Radius of convergence $R > 0$ then show that

$$\int_0^x f(t)\,dt = \sum \frac{a_n}{n+1} x^{n+1} \quad \text{for } |x| < R$$

or

Let $\sum a_n x^n$ be power series with radius of convergence $R > 0$ (possibly $R = \infty$) if $0 < R_1 < R$. Then show that power series converges uniformly on $[-R_1, R_1]$ to a continuous function

Q3/ find the exact interval of convergence of the power series $\sum \frac{x^n}{n^n}$.