

Shiraji College, University of Delhi
Department of Mathematics
Internal Test
(Academic year 2023-2024)

Name of the Course:- B.Sc(H) Mathematics

Semester:- IV, Section B

Paper Name:- Sequence and Series of functions

Max. Marks:- 12

Faculty Name:- Dr. Vandana

Last Date of Submission:- 22nd April, 2024

Date of Conduct:- 22nd April, 2024

① State and prove Weierstrass M-test.

② If f_n is Continuous function on $D \subseteq \mathbb{R}$ to $\mathbb{R}$ for each $n \in \mathbb{N}$, and if $\sum f_n$ converges to f Uniformly on D then f is continuous on D .

③ Discuss the convergence and Uniform convergence of the series $\sum \sin\left(\frac{x}{n}\right)$, $x \in \mathbb{R}$, $n \in \mathbb{N}$ or

$$\sum \frac{1}{x^n + 1}, x \neq 0.$$

④ Show that the series $\sum \frac{(-1)^n \sin(nx)}{n^3}$ Converges Uniformly.

Vandana

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- ① Show that if $0 \leq x \leq a$ and $n \in \mathbb{N}$ then
- $$1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} \leq e^x \leq 1 + \frac{x}{1} + \dots + \frac{x^{n-1}}{(n-1)!} + \frac{e^a x^n}{n!}$$
- ② Show that if $n \geq 2$, then
- $$0 < e^n - \left(1 + 1 + \frac{1}{2!} + \dots + \frac{1}{n!}\right)^n < \frac{e}{n+1} < 1$$
- ③ If $x \geq 0$ and $n \in \mathbb{N}$, show that
- $$\frac{1}{x+1} = 1 - x + x^2 - x^3 + \dots + (-1)^{n-1} x^{n-1} + \frac{(-1)^n x^n}{1+x}$$
- Use this to show that $\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1} \frac{x^n}{n} + \int_0^x \frac{(-1)^n t^n}{1+t} dt$
- and that
- $$\left| \ln(x+1) - \left(x - \frac{x^2}{2} + \dots + (-1)^{n-1} \frac{x^n}{n} \right) \right| \leq \frac{x^{n+1}}{n+1}$$

Vandana