Class Test

Ordinary Differential Equations

Maniesh Kr Meena

B.Sc. (H) Maths II A

Set A

Maximum Marks : 12

Maximum Time : 1 Hour

Note: Attempt any four questions. Each question carry three marks.

Q. 1. What is the difference between IVP and BVP. Explain with examples.

Q. 2. If an archaeologist uncovers a seashell which contains 60% of the 14C of a living shell, how old do you estimate that shell, and thus that site, to be? (You may assume the half-life of 14c to be 5700 years.)

Q. 3. Consider the population model $\frac{dN}{dt} = aN - bN^2$,
where a and b are positive constants. Here bN^2 represents a death term due to overcrowding (i.e., proportional to N due to interactions of the population with itself).
(a) Find all the equilibrium points. Are there any conditions on the parameters a and b for the equilibrium population to remain positive?
(b) Determine the stability of each of the equilibrium points.
(c) It is claimed that this model is exactly the same as the logistic growth model. If this claim is true, then express the constants a and b in terms of the intrinsic growth rate r and carrying capacity K. If it is not true, explain why.

Q. 4. Give statement of Existence and Uniqueness theorem. Give an example of IVP that does not have unique solution.

Q.5. Write down differential equations for a model of a three species interaction with two predators Y and Z that compete for a single prey food-source X.
Also Find all possible equilibrium populations. Is it possible for all three populations to coexist in equilibrium?

Q.6. Consider the system

$$\frac{dX}{dt} = \beta_1 X\left(1 - \frac{X}{K}\right) - c_1 XY, \quad \frac{dY}{dt} = c_2 XY - \alpha_2 Y$$

for the dynamics of a predator-prey model, with density-dependent growth of the prey, and all parameters positive constants. Find all the equilibrium points. How do they differ from those of the standard Lotka–Volterra System.

Q. 7. Solve the initial value problem

$$(x^2 + 1)\frac{dy}{dx} + 3xy = 6x, \quad y(0) = 2.$$

Q. 8. Find general solution of following differential equation using method of variation of parameters.

$$y'' + y = \sin x$$

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Manish Kr Meena

Class Test

Ordinary Differential Equations

B.Sc. (H) Maths II B

Set B

Maximum Marks : 12 Maximum Time : 1 Hour

Note: Attempt any four questions. Each question carry three marks.

Q. 1. In a certain culture of bacteria, the number of bacteria increased sixfold in 1 h. How long did it take for the population to triple?

Q.2. Write down differential equations for a model of a three species interaction with one predator Z and two preys X and Y.
Also Find all possible equilibrium populations. Is it possible for all three populations to coexist in equilibrium?

Q. 3. Solve the following system of differential equations

$$\frac{dx}{dt} = 2x + 4y + 3e^{t}\,; \quad \frac{dy}{dt} = 5x - y - t^{2}.$$

Q. 4. Find general solution of $y''' + y' = \sin x + x^2 e^{2x}$.

Q. 5. Find general solution of following differential equation using method of variation of parameters.

$$y'' - y = \sin x + x\cos x.$$

Q. 6. Consider the system

$$\frac{dX}{dt} = \beta_1 X\left(1 - \frac{X}{K}\right) - c_1 XY, \quad \frac{dY}{dt} = c_2 XY - \alpha_2 Y$$

for the dynamics of a predator-prey model, with density-dependent growth of the prey, and all parameters positive constants. Find all the equilibrium points. How do they differ from those of the standard Lotka–Volterra System.