SHIVAJI COLLEGE, UNIVERSITY OF DELHI
Department of Mathematics
Internal Test (Academic Year 2023-24)

Name of Course: GE-IV — **Semester:** IV
Name of the paper: Elements of Real analysis — **Faculty Name:** Dr. S.D. Ram
Duration: one hour — **Maximum marks:** 12
Date of Test: 04 May, 2024 — **Set-A**

[1] State the integral test. Use the integral test check the convergence of $\sum_{n=2}^{\infty}\frac{1}{n\log n}$

[2] Check the convergence and absolute convergence of the following series:

a) $\sum_{n=1}^{\infty}(-1)^n\frac{Sinn\alpha}{n^3}, \alpha\in\Re$, b) $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n+2}$

[3] Define convergence of infinite series. Show that $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent for $p>1$ and divergent for $p<1$.

[4] Show that the sequence $\left(1+\frac{1}{n}\right)^n$ is convergent. Also find its limit.

[5] Define Cauchy's sequence. Show that every Cauchy's sequence is bounded.

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Internal Test (Academic Year 2023-24)

Name of Course: GE-IV — **Semester:** IV
Name of the paper: Elements of Real analysis — **Faculty Name:** Dr. S. D. Ram
Duration: one hour — **Maximum marks:** 12
Date of Test: 04 May, 2024 — **Set-B**

[1] State the limit comparison test for positive term infinite series hence test the convergence of the following series:

a) $\sum_{n=1}^{\infty}\frac{1}{n^2-n+1}$, b) $\sum_{n=2}^{\infty}\frac{1}{\sqrt{n+1}+\sqrt{n+2}}$

[2] State and prove Cauchy's criterion for the infinite series.

[3] Define a convergence sequence, Show that every convergence sequence is Cauchy's sequence.

[4] Show that the sequence $\{x_n\}$ defined by $x_1=1$ $x_{n+1}=\frac{3+2x_n}{2+x_n}$ $n\geq 2$. is convergence. Also find the limit.

[5] State Leibnitz test for alternating series. Check the convergence and absolutely convergence of the series: $\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{7}}+\ldots\ldots$.

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Department of Mathematics

Internal Test (Academic Year 2023-24)

Name of Course: GE-IV **Semester:** IV

Name of the paper: Elements of Real analysis **Faculty Name:** Dr. Subedar Ram

Duration: one hour **Maximum marks:** 12

Date of Test: 03 April, 2024 **Set-A**

[1] Prove that $\lim_{x\to\infty}\left(\frac{3n^2-4n}{n^2+5}\right)=3.$

[2] Prove that every convergent sequence is bounded.

[3] Show that $\sqrt[n]{n}=1.$

[4] For any ordered field F the following properties hold for $x, y \in F$

(a) $x<y$ iff $x\pm z<y\pm z$.

(b) If $z>0$ then $x<y \Rightarrow xz<yz$.

(c) If $z<0$ then $x<y \Rightarrow xz>yz$.

[5] Let $a<b$ in an ordered field F. then $a=\inf(a,b)$ and $b=\sup(a,b)$.

[6] If a sequence $\{a_n\}$ converges and $\forall \in \mathbb{N}$, $a_n \le K$ then $\lim_{x\to\infty} a_n \le K$.

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Internal Test (Academic Year 2023-24)

Name of Course: GE-IV **Semester:** IV

Name of the paper: Elements of Real analysis **Faculty Name:** Dr. Subedar Ram

Duration: one hour **Maximum marks:** 12

Date of Test: 03 April, 2024 **Set-B**

[1] Prove that $\lim_{x\to\infty}\left(\frac{2n+3}{3n-7}\right)=\frac{2}{3}.$

[2] Prove that a convergent sequence has unique limit.

[3] If $|a|<1$, then $\lim_{x\to\infty} a_n=0.$

[4] For any ordered field F the following properties hold for $x, y \in F$

(a) If $x<y$ and $u<v$ then $x+u<y+v$.

(b) If $0<x<y$ and $0<u<v$ then $xu<yv$ and $\frac{x}{v}<\frac{y}{u}$.

[5] If a sequence $\{a_n\}$ converges and $\forall \in \mathbb{N}$, $a_n \le K$ then $\lim_{x\to\infty} a_n \le K$.

[6] Let F be an Archimedean ordered field, $A \subseteq F$ and $u \in F$ then $u = Sup(A) \Leftrightarrow \forall \varepsilon > 0$.

(a) $\forall x \in A, x<u+\varepsilon$ and

(b) $\exists x \in A \ni x > u-\varepsilon$.

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Internal Test (Academic Year 2023-24)

Name of Course: B.Sc (H) Mathematics **Semester:** II
Name of the paper: Linear Algebra **Faculty Name:** Dr. Subedar Ram
Duration: one hour **Maximum marks:** 12
Date of Test: 02 April, 2024 **Set-A**

[1] Prove that for x, y vectors in $\mathbb{R}^n$, $||x+y||^2 = ||x||^2 + ||y||^2 \Leftrightarrow x.y = 0$

[2] Prove that for any vectors x, y in $\mathbb{R}^n$, $|||x|| - ||y||| \leq ||x+y||$.

[3] Use the Gauss-Jordan method, to solve the system, $5x_1 + 20x_2 - 18x_3 = -11$, $3x_1 + 12x_2 - 14x_3 = 3, -4x_1 - 16x_2 + 13x_3 = 13$.

[4] Find the rank of the matrix $\begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & 1 \end{pmatrix}$.

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Department of Mathematics

Internal Test (Academic Year 2023-24)

Name of Course: B.Sc (H) Mathematics **Semester:** II
Name of the paper: Linear Algebra **Faculty Name:** Dr. Subedar Ram
Duration: one hour **Maximum marks:** 12
Date of Test: 02 April, 2024 **Set-B**

[1] Prove that if x, y and z mutually orthogonal vectors in $\mathbb{R}^n$ then $||x+y+z||^2 = ||x||^2 + ||y||^2 + ||z||^2$.

[2] Prove that if $(x+y).(x-y) = 0$ then $||x|| = ||y||$ for any vectors x, y in $\mathbb{R}^n$.

[3] Use the Gauss-Jordan method, to solve the system, $-2x_1 - 3x_2 + 2x_3 - 13x_4 = 0$, $-4x_1 - 7x_2 + 4x_3 - 29x_4 = 0, x_1 + 2x_2 - x_3 + 8x_4 = 0$.

[4] Find the rank of the matrix $\begin{pmatrix} 3 & 5 & 2 \\ 4 & 2 & 3 \\ -1 & 2 & 4 \end{pmatrix}$.

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Department of Mathematics

Internal Test (Academic Year 2023-24)

Name of Course: B.Sc (H) Mathematics **Semester:** II
Name of the paper: Linear Algebra **Faculty Name:** Dr. Subedar Ram
Duration: one hour **Maximum marks:** 12
Date of Test: 02 April, 2024 **Set-C**

[1] Prove that $x.y = \frac{1}{4}(||x+y||^2 - ||x-y||^2)$ if x and y vectors in $\mathbb{R}^n$.

[2] (a) If x is a non-zero vectors in $\mathbb{R}^n$ then $u = \frac{1}{||x||}x$ is a unit vector in the same direction as x.

(b) If $u = \{2,3,-1,1\}$ in $\mathbb{R}^4$ then find the an unit vector v in the same direction as u.

[3] Use the Gauss-Jordan method to find the minimum integer value for the variables that will balance the chemical equation: $aAgNO_3 + bH_2O \longrightarrow cAg + dO_2 + eHNO_3$

[4] Find the rank of the matrix $\begin{pmatrix} 3 & 2 & 7 \\ -4 & 1 & 6 \\ 2 & 5 & 4 \end{pmatrix}$.

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Internal Test (Academic Year 2023-24)

Name of Course: B.Sc (H) Mathematics
Semester: II

Name of the paper: Linear Algebra
Faculty Name: Dr. S. D. Ram

Duration: One hour
Maximum marks: 12

Date of Test: 29 April, 2024
Set-A

[1] Verify the Cayley Hamilton Theorem for the matrix A= $\begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$.

[2] Use diagonalization method to determine whether matrix A is diagonalizable. If so, specify the matrices D and P and verify that $D = P^{-1}AP$ $A = \begin{pmatrix} 19 & -48 \\ 8 & -21 \end{pmatrix}$

[3] Find the eigenvalue and eigenvector of the given matrix $A = \{\{5, -8, -12\}, \{-2, 3, 4\}, \{4, -6, -9\}\}$.

[4] Express the x as a linear combination of the other vectors, if possible
$x = \{7, 2, 3\}, a_1 = \{1, -2, 3\}, a_2 = \{5, -2, 6\}, a_3 = \{4, 0, 3\}$.

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Department of Mathematics

Internal Test (Academic Year 2023-24)

Name of Course: B.Sc (H) Mathematics
Semester: II

Name of the paper: Linear Algebra
Faculty Name: Dr. S. D. Ram

Duration: One hour
Maximum marks: 12

Date of Test: 29 April, 2024
Set-B

[1] Express the x as a linear combination of the other vectors, if possible
$x = \{2, 2, 3\}, a_1 = \{6, -2, 3\}, a_2 = \{0, -5, -1\}, a_3 = \{-2, 1, 2\}$.

[2] Find the eigenvalue and eigenvector of the given matrix $A = \{\{2, 0, 0\}, \{-3, 4, 1\}, \{3, -2, 1\}\}$

[3] Verify the Cayley Hamilton Theorem for the matrix A= $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

[4] Use diagonalization method to determine whether matrix A is diagonalizable. If so, specify the matrices D and P and verify that $= P^{-1}AP$ $A = \begin{pmatrix} -18 & 40 \\ -8 & 18 \end{pmatrix}$

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Department of Mathematics

Internal Test (Academic Year 2023-24)

Name of Course: B.Sc (H) Mathematics
Semester: II

Name of the paper: Linear Algebra
Faculty Name: Dr. S. D. Ram

Duration: One hour
Maximum marks: 12

Date of Test: 29 April, 2024
Set-C

[1] Express the x as a linear combination of the other vectors, if possible
$x = \{5, 9, 5\}, a_1 = \{2, 1, 4\}, a_2 = \{1, -1, 3\}, a_3 = \{3, 2, 5\}$.

[2] Use diagonalization method to determine whether matrix A is diagonalizable. If so, specify the matrices D and P and verify that $D = P^{-1}AP$ $A = \begin{pmatrix} 13 & -34 \\ 5 & -13 \end{pmatrix}$.

[3] Verify the Cayley Hamilton Theorem for the matrix A= $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.

[4] Find the eigenvalue and eigenvector of the given matrix $A = \{\{7, 1, -1\}, \{-11, -3, 2\}, \{18, 2, -4\}\}$.

SHIVAJI COLLEGE, UNIVERSITY OF DELHI

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Assignment (Academic Year 2023-24)

Name of Course: GE-IV

Semester: II

Name of the paper: Real Analysis

Faculty Name: Dr. S. D. Ram

Due date: 22 April, 2024

Maximum marks: 12

[1] Let $a \geq 0$ be fixed non-negative element in an ordered field F. then for all $x, y \in F$

$a)\ |x| < a \Leftrightarrow -a < x < a.$

$b)\ |x| > a \Leftrightarrow x > a\ or\ x < -a.$

$c)\ |x - y| < a \Leftrightarrow y - a < x < y + a.$

[2] In any complete ordered field , every non-empty set that has a lower bound in F has a greatest lower bound in F.

[3] Let F be an Archimedean ordered field , $A \subseteq F$ and u F. then

$u = inf(A) \Leftrightarrow \forall \epsilon > 0$

$a)\ \forall x \in A,\ x > u - \varepsilon\ and$

$b)\ \exists x \in A\ such\ that\ x < u + \varepsilon.$

[4] Prove that

$a):-\lim\limits_{n\to\infty}\dfrac{n^2-2}{n^2+n}=1,\quad b):-\lim\limits_{n\to\infty}\dfrac{8n^2+3}{5n^2-2n}=\dfrac{8}{5},$

$c):-\lim\limits_{n\to\infty}\dfrac{2n^2-n}{n^2-5n-7}=2,\quad d):-\lim\limits_{n\to\infty}\dfrac{3n+4}{7n-1}=\dfrac{3}{7}.$

[5] Use algebra of limits to prove that $\lim\limits_{n\to\infty}\left(\dfrac{2n+3}{3n-7}\right)=\dfrac{2}{3}.$

[6] Define eventually constant sequence and give two examples.

[7] Prove that

$a):-\lim\limits_{n\to\infty}\dfrac{1+2+3+4+......+n}{n^2}=\dfrac{1}{2},\quad b):-\lim\limits_{n\to\infty}\dfrac{1^2+2^2+3^2+4^2+......+n^2}{n^3}=\dfrac{1}{3}.$

[8] A monotone sequence convergence iff it is bounded.

[9] Consider the sequence $< a_n >$ defined inductively by $x_1 = 1$ and $\forall n \in \mathbb{N},\ x_{n+1} = \sqrt{4x_n + 5}$. Prove that $< a_n >$ convergent and find the it's limit.

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Assignment (Academic Year 2023-24)

Name of Course: B.Sc (H) Mathematics(Section B) **Semester:** II

Name of the paper: Linear Algebra **Faculty Name:** Dr. S. D. Ram

Due date: 22 April, 2024 **Maximum marks:** 12

[1] Use the Gauss Jordan Method to find the value of A B and C in give partial fraction

$$\frac{5x^2+23x-5}{(x-1)(x-3)(x+4)} = \frac{A}{(x-1)} + \frac{B}{(x-3)} + \frac{C}{(x+4)}.$$

[2] Show that any row of A is in the row space of B

$$A = \begin{pmatrix} 0 & 4 & 12 & 8 \\ 2 & 7 & 19 & 18 \\ 1 & 2 & 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 & -4 & -21 \\ -2 & -4 & -6 & 5 & 27 \\ 13 & 26 & 39 & 5 & 12 \\ 2 & 4 & 6 & -1 & -7 \end{pmatrix}$$

[3] Using gauss Jordan method, find the equation of the circle $x^2 + y^2 + ax + by = c$. That goes through the points $(6,8)$, $(8,4)$, and $(3,9)$.

[4] Use the diagonalization method to determine whether matrix is diagonalible. Verify that $D = P^{-1}AP$.

$$A = \begin{pmatrix} 2 & 0 & -0 \\ -3 & 4 & 1 \\ 3 & -2 & 1 \end{pmatrix}.$$

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Department of Mathematics
Assignment (Academic Year 2023-24)

Name of Course: B.Sc (H) Mathematics(Section A) **Semester:** II

Name of the paper: Linear Algebra **Faculty Name:** Dr. S. D. Ram

Due date: 22 April, 2024 **Maximum marks:** 12

[1] Using gauss Jordan method, find the cubic equation $y = ax^3 + bx^2 + cx + d$ that goes through the points $(1,2), (2,-12), (-2,56)$ and (3,-54).

[2] Express the vector $\{13,-23,60\}$ as a linear combination of the vector $q_1 = \{-1,-5,11\}, q_2 = \{-10, 3, -8\}. q_3 = \{7,12,30\}$.

[3] Let

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 1 \\ -2 & 1 & 5 \\ 3 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & -5 \\ 2 & 3 & 0 \\ 4 & 1 & 1 \end{bmatrix}$$

Compute $R(AB)$ and $R(A)B$ to verify that they are equal

a) $R: <3> \leftarrow 3(2) + (3)$

b) $R: <2> \leftrightarrow <4>$

[4] Use the diagonalization method to determine whether matrix is diagonalible. Verify that $D = P^{-1}AP$.

$$A = \begin{bmatrix} 5 & -8 & -12 \\ -2 & 3 & 4 \\ 4 & -6 & -9 \end{bmatrix}$$