SHIVAJI COLLEGE, UNIVERSITY OF DELHI

DEPARTMENT OF MATHEMATICS

INTERNAL TEST (Academic Year 2023-24)

Name of the Course	: B.Sc.(H) Mathematics	Semester	: IV (Section B)
Name of the Paper	: Numerical Analysis	Faculty Name	: Dr. Neetu Rani
Duration	: $1\frac{1}{2}$ hrs	Maximum Marks	: 24
Date of Test	: 18th April, 2024	Set	: A

Note: 1. Attempt any six questions.

2. All questions carry equal marks.

1. Show that the convergence of the sequence generated by the formula $x_{n+1} = \frac{x_n^3+3x_n a}{3x_n^2+a}$ toward $\sqrt{a}$ is third order. What is the asymptotic error constant?

2. Approximate $\sqrt[3]{13}$ to three decimal places by applying the bisection method to within an absolute tolerance of 5×10^{-5}.

3. Define rate of convergence and order of convergence of an iterative method for finding an approximation to the root of $g(x) = 0$. Find the order of convergence of Newton's iterative formula.

4. Solve the system $Ax = b$ using LU decomposition method, where $A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$, $b = \begin{bmatrix} -3 \\ -12 \\ 6 \end{bmatrix}$.

5. Use Jacobi method to solve the given linear system of equations by taking $\boldsymbol{x}^{(0)} = \mathbf{0}$ and terminating iteration when $\left\|\boldsymbol{x}^{(k+1)} - \boldsymbol{x}^{(k)}\right\|_\infty$ falls below 5×10^{-6},

$4x_1 - x_2 = 2\,,\ -x_1 + 4x_2 - x_3 = 4\,, -x_2 + 4x_3 = 10\,.$

6. Write out the Newton form of the interpolating polynomial for $f(x) = \ln x$ passing through the points $(1, \ln 1), (2, \ln 2)$, and $(3, \ln 3)$. Use the polynomial to estimate both $\ln 1.5$ and $\ln 2.5$. What is the error in each approximation?

7. Let $x_0, x_1, x_2, \dots, x_n$ be $n+1$ distinct points from the closed interval $[a, b]$. If f is continuous on $[a, b]$ and has n continuous derivatives on the open interval (a, b), then show that there exists a $\xi \in (a, b)$ such that $f[x_0, x_1, x_2, \dots, x_n] = \frac{f^{(n)}(\xi)}{n!}$.

8. Let g be a continuous function on the closed interval $[a, b]$ with $g: [a, b] \to [a, b]$ and suppose that g' is continuous on the open interval (a, b) with $|g'(x)| \leq k < 1$ for all $x \in (a, b)$. If $g'(p) \neq 0$, then show that for any $p_0 \in [a, b]$, the sequence $p_n = g(p_{n-1})$ converges only linearly to the fixed point p.

Faculty Signature: NR

SHIVAJI COLLEGE, UNIVERSITY OF DELHI

DEPARTMENT OF MATHEMATICS

INTERNAL TEST (Academic Year 2023-24)

Name of the Course	: B.Sc.(H) Mathematics	Semester	: IV (Section B)
Name of the Paper	: Numerical Analysis	Faculty Name	: Dr. Neetu Rani
Duration	: $1\frac{1}{2}$ hrs	Maximum Marks	: 24
Date of Test	: 18th April, 2024	Set	: B

Note: 1. Attempt any six questions.

2. All questions carry equal marks.

1. Let a be a non-zero real number. For any x_0 satisfying $0 < x_0 < 2/a$, the recursive sequence defined by $x_{n+1} = x_n(2 - ax_n)$ converges to $1/a$. What are the order of convergence and the asymptotic error constant?

2. The function $f(x) = x^3 + 2x^2 - 3x - 1$ has a zero on the interval (-1, 0). Approximate this zero to within an absolute tolerance of 5×10^{-5}.

3. Define rate of convergence and order of convergence of an iterative method for finding an approximation to the root of $g(x) = 0$. Find the order of convergence of secant method.

4. Solve the system $Ax = b$ using LU decomposition method, where $A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}, b = \begin{bmatrix} 14 \\ 36 \\ 7 \end{bmatrix}$

5. For the given coefficient matrix and right-hand side vector, write out the components of the Gauss-Seidel method iteration equation. Then, starting with the initial vector $\boldsymbol{x}^{(0)} = \mathbf{0}$, perform three iterations of the Gauss-Seidel method, $\begin{bmatrix} 4 & 2 & -1 \\ 2 & 4 & 1 \\ -1 & 1 & 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

6. Construct the Lagrange form of the interpolating polynomial for $f(x) = e^x$ passing through the points $(-1, e^{-1}), (0, e^0)$, and $(1, e^1)$. Use the polynomial to estimate both $\sqrt{e}$ and $e^{-1/3}$. What is the error in each approximation?

7. If $x_0, x_1, x_2, \ldots, x_n$ are $n + 1$ distinct points from the closed interval $[a, b]$ and f is continuous on $[a, b]$ and has $n + 1$ continuous derivatives on the open interval (a, b), then show that for each $x \in (a, b)$ there exists a $\xi(x) \in (a, b)$ such that $f(x) = P(x) + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)(x - x_1)(x - x_2) \ldots (x - x_n)$, where P is the interpolating polynomial.

8. Let g be continuous on the closed interval $[a, b]$ with $g: [a, b] \to [a, b]$. Then show that g has a fixed point $p \in [a, b]$. Furthermore, if g is differentiable on the open interval (a, b) and there exists a positive constant $k < 1$ such that $|g'(x)| \leq k < 1$ for all $x \in (a, b)$, then show that the fixed point in $[a, b]$ is unique.

Faculty Signature: N.R.

Shivaji College, University of Delhi
Department of Mathematics
Internal Test - 2 (Academic Year 2023-24)

Name of the Course: B.Sc (H) Maths, Semester : IV (Section B)
Name of the Paper: Numerical Analysis, Faculty Name: Dr. Neetu Rani
Duration : 01 Hrs, Maximum Marks : 24
Date of Test : 7th May, 2024
Note : 1. Attempt any four questions.
2. All questions carry equal marks.

Q1.: Verify that the second-order central difference approximation for the second derivative provides the exact value of the second derivative, regardless of the value of h, for the functions $f(x)=1$, $f(x)=x$, $f(x)=x^2$ and $f(x)=x^3$, but not for the function $f(x)=x^4$.

Q2: Apply Runge-Kutta method of order 4 to approximate the solution of the given initial value problem over the indicated interval in t using the indicated number of time steps:

$x' = tx^3 - x \quad (0 \le t \le 1), \; x(0)=1, \; N=4.$

Q3: Write out the Newton form of the interpolating polynomial for $f(x)=e^x$ that passes through the points $(-1, e^{-1})$, $(0, e^0)$, and $(1, e^1)$.

Q4: Solve the given system by Gauss-Seidel method taking $x^{(0)}=0$ and terminate iterations when $\|x^{(k+1)} - x^{(k)}\|_\infty$ falls below 5×10^{-6}:

$4x_1 - x_2 = 2, \; -x_1 + 4x_2 - x_3 = 4, \; -x_2 + 4x_3 = 10.$

Q5: Write an algorithm for secant method and find out a simple zero of the function $f(x)=x^3+2x^2-3x-1$ on the interval $(-1,0)$ within an absolute tolerance of 5×10^{-5}.

Q6: Define the order of convergence and determine the order of convergence for the secant method.

Q7: Compute each of the following limits and determine the corresponding rate of convergence:

(i) $\lim_{n\to\infty} (\sqrt{n+1} - \sqrt{n})$ (ii) $\lim_{n\to 0} \frac{\cos x - 1 + \frac{x^2}{2} - \frac{x^4}{24}}{x^6}$

N.R.

SHIVAJI COLLEGE, UNIVERSITY OF DELHI

DEPARTMENT OF MATHEMATICS

INTERNAL ASSIGNMENT

(Academic Year 2023-24)

Name of the Course:	B.Sc. (Hons) Mathematics
Semester:	IV
Name of the Paper:	DSC-12: Numerical Analysis
Maximum Marks:	12
Faculty Name:	Dr. Neetu Rani
Last Date of Submission:	Within one week of completion of topic assigned (Jan 24-April 24)

Reference Book: Bradie, Brian (2006), A Friendly Introduction to Numerical Analysis

S. No.	Roll No.	Name	Exercise	Questions	
1	22/17048	Naman Maan	2.2	1, 2, 4-11	
2	22/17089	Nishant Soni	1.2	1-13, 16-21	
3	22/17108	Nitesh Kumar	3.5	1-10	
4	22/17050	Param Kumar	2.3	1-3, 5-16	
5	22/17051	Parul	7.4	1-4, 7, 8, 12, 15-17	
6	22/17052	Peeyush Garg	2.5	1-18	
7	22/17053	Pratik Raj	2.4	1-14	
8	22/17098	Pratyush Kumar	2.2	1, 2, 4-11	
9	22/17054	Prayag Singh	3.8	1-10, 13, 14, 16	
10	22/17055	Prince Kumar	7.4	1-4, 7, 8, 12, 15-17	
11	22/17056	Prince Kumar	5.5	1-12	
12	22/17057	Prince Kumar Jha	5.3	2-19	
13	22/17058	Rahul Kumar	6.2	1-16	
14	22/17097	Rajat Kumar	6.4	1-15, 17	
15	22/17088	Rakshit Pathak	7.2	1-15	
16	22/17059	Reena Kumari	7.2	16-24, 26	
17	22/17060	Sachin Kumar	1.2	1-13, 16-21	
18	22/17061	Sahil	2.1	1-5, 10-12, 14-16	
19	22/17096	Saksham Tyagi	2.1	1-5, 10-12, 14-16	
20	22/17062	Sakshi	2.2	1, 2, 4-11	
21	22/17063	Sakshi Swaraj	6.4	1-15, 17	
22	22/17064	Saloni Singh	6.4	1-15, 17	
23	22/17065	Saloni Yadav	2.4	1-14	
24	22/17103	Sanvi Batra	7.4	7, 8, 12, 15-17, 19, 21, 22	
25	22/17066	Satyam Kumar	5.5	1-12	
26	21/17090	Seema	5.1	1-17	

N'R

S. No.	Roll No.	Name	Exercise	Questions	
27	22/17067	Shashvat Kumar Mishra	7.4	7, 8, 12, 15-17, 19, 21, 22	
28	22/17068	Shivam Pandey	5.1	1-17	
29	22/17069	Shivam Sarang	7.2	16-24, 26	
30	22/17070	Shivanshu Kumar Chaurashiya	3.5	3-16	
31	22/17071	Shravan Kumar	2.3	1-3, 5-16	
32	22/17072	Shushant Gupta	3.5	3-16	
33	22/17073	Shweta Kumari	7.2	16-24, 26	
34	22/17074	Sneha Dudeja	7.4	7, 8, 12, 15-17, 19, 21, 22	
35	22/17075	Sumit Nayak	3.8	1-10, 13, 14, 16	
36	22/17091	Sunith Kumar Pandey	5.1	1-17	
37	22/17076	Tanuj Kumar	6.2	1-16	
38	22/17077	Tisha Ahuja	2.5	1-18	
39	22/17078	Tushar	5.3	2-19	
40	22/17079	Vimlendra Mohan Pathak	2.5	1-18	
41	22/17080	Vishal Singh	7.2	1-15	
42	22/17095	Vishwas	5.1	1-17	
43	22/17081	Vivek Kumar Rawat	7.4	1-4, 7, 8, 12, 15-17	
44	22/17082	Yash	2.5	1-18	
45	22/17083	Yash Gupta	7.4	1-4, 7, 8, 12, 15-17	
46	22/17084	Yash Pal	7.4	7, 8, 12, 15-17, 19, 21, 22	
47	22/17085	Yashdev	7.4	1-4, 7, 8, 12, 15-17	
48	22/17086	Yogesh	7.4	7, 8, 12, 15-17, 19, 21, 22	

Note: 1. Use only A-4 size paper for solving the assignment questions.

2. Submit your assignment within one week of completion of topic assigned.

Faculty Name: Dr. Neetu Rani

N. R.