Date: 10.5.24

B.Sc. (H) Mathematics II Year (Section B)

Test: Complex Analysis (Including Practicals)

Max. Marks: 20

Do any four questions

Q1. Write the two Laurent Series in powers of z for the function $f(z) = \frac{1}{z(1+z^2)}$.

in certain domains and specify those domains

Q2. Find the singular points of the function

$$f(z) = \frac{1}{\sin(\pi/z)}$$

Is any singular point isolated also? Justify.

Q3. Evaluate $\int_C \frac{5z-2}{(z)(z-1)}\,dz$ $\quad C: |z| = 2$.

Q4. Evaluate $\int_C \frac{3z^3+2}{(z-1)(z^2+9)}$ $\quad C: |z| = 4$

Q5. Show that when $0 < |z| < 4$

$$\frac{1}{4z-z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}$$

Dated: 19.04.2024

Test 2

Complex Analysis

B.Sc.(H) Mathematics. III Year (2023-24)

Time: 1 hr

Max. Marks: 30

Attempt any six Questions

Q1(a) Is Mean value theorem for derivatives of complex-valued fns true? Justify your answer.

Q1.(b) Show that if m and n are integers,

$$\int_0^{2\pi} e^{im\theta} e^{-in\theta} d\theta = \begin{cases} 0 & \text{when } m \neq n \\ 2\pi & \text{when } m = n \end{cases}$$

Q2. Let $f(z) = \pi \exp(\pi \bar{z})$ and C is the boundary of the square with vertices at the points $0, 1, 1+i$ and i, the orientation of C being in the counter clockwise direction.

Q3. $f(z)$ is defined by means of the equations

$$f(z) = \begin{cases} 1 & \text{when } y < 0 \\ 4y & \text{when } y > 0 \end{cases}$$

and C is the arc from $z = -1-i$ to $z = 1+i$ along the curve $y = x^3$.

Q4. (a) State and prove ML Theorem.

(b) Show that if C is the boundary of the triangle with vertices at the points 0, $3i$ and -4 oriented in the counterclockwise direction, then

$$\left| \int_C (e^z - \bar{z}) dz \right| \leq 60.$$

Q5 Let C_R denote the upper half of the circle $|z| = R$ $(R > 2)$ taken in the counterclockwise. Show that

$$\left| \int_{C_R} \frac{2z^2-1}{z^4+5z^2+4}\, dz \right| \leq \frac{\pi R\,(2R^2+1)}{(R^2-1)(R^2-4)}.$$

Further show that the value of the integral tends to zero as $R \to \infty$.

Q 6 (a) Evaluate $\displaystyle\int_C \frac{1}{z^2+2z+2}\, dz$ $C: |z|=1$

(b) Evaluate $\displaystyle\int_C \frac{\cos z}{z(z^2+8)}$ C is boundary of square whose sides lie along $x = \pm 2,\ y = \pm 2$.

Q 7 (a) Evaluate $\displaystyle\int_C \frac{dz}{(z^2+4)^2}$ $C: |z-i| = 2$

(b) Let C be simple closed contour in the sense.

Let $\displaystyle g(z) = \int_C \frac{s^3+2s}{(s-z)^3}\, ds$

Show that $g(z) = 6\pi i z$ when z is inside C.

and $g(z) = 0$ when z is outside C.

Q 8. State and prove Liouville's Theorem.

Q 9. State and prove Cauchy Integral Formula.

Q 10. Prove that if a function f is analytic at a given point, then its derivatives of all orders are analytic there too.

Q 11. Let f be continuous on a domain D. If

$$\int_C f(z)\, dz = 0$$

for every closed contour C in D, then prove that f is analytic throughout D.

Max. Marks: 40

B.Sc (H) Mathematics II Year (2023-24)

Time. 90 mins

Test : Complex Analysis

Dated : 18.03.2024

Do any seven questions

Q1. Find and sketch, showing corresponding orientations, the images of the hyperbola

$$x^2 - y^2 = c_1 \; (c_1 < 0)$$

under the transformation $w = z^2$.

Q2. (a) Prove that $\lim_{z \to \infty} f(z) = w_0$ iff $\lim_{z \to 0} f\left(\frac{1}{z}\right) = w_0$

(b) Find $\lim_{z \to 0} \left(\frac{z}{\bar{z}}\right)^2$.

Q3 (a) Let f denote the function whose values are

$$f(z) = \begin{cases} \bar{z}^2/z & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases}$$

Determine whether $f'(0)$ exists or not? If so, find $f'(0)$.

(b) Let $f(z) = \operatorname{Re} z$. Show that $f'(0)$ doesn't exist at any point.

Q4. (a) Let $f(z) = \cos x \cosh y - i \sin x \sinh y$. Prove that $f''(z) + f(z) = 0$ by showing the existence of $f'(z)$ and $f''(z)$.

(b) Let $f(z) = e^x e^{-iy}$. Is f differentiable at any point? Justify.

Q5. Prove that if $f'(z) = 0$ everywhere in domain D, then $f(z)$ must be constant throughout D.

OR

Suppose that $f(z)$ and $\overline{f(z)}$ are both analytic in domain D. Show that $f(z)$ must be constant throughout.

Q6. (a) Verify that $f(z) = \sin x \cosh y + i \cos x \sinh y$ is entire.

(b) Show that $f(z) = 2xy + i(x^2 - y^2)$ is nowhere analytic.

Q7. Solve $e^z = 1 + \sqrt{3}i$

Q8. Show that $\overline{\exp(iz)} = \exp(i\bar{z})$ iff $z = n\pi$ $(n = 0, \pm 1, \pm 2 \ldots)$.

Q9 (a) Find the value of $\log(-1 + \sqrt{3}i)$

(b) Show that $\text{Log}(-1+i)^2 \neq 2\,\text{Log}(-1+i)$

Q10. Show that $\log(i^2) \neq 2 \log i$ when $\log z = \ln r + i\theta$, $(r > 0, \frac{3\pi}{4} < \theta < \frac{11\pi}{4})$

OR

Show that $\log(i^{1/2}) = \frac{1}{2} \log i$.

Q11. Show that $\overline{\sin(iz)} = \sin(i\bar{z})$ iff $z = n\pi i$, $n = 0, \pm 1, \pm 2, \ldots$

OR

Show that $\sin \bar{z}$ is not an analytic function.

Q12. Find all the roots of the equation $\sin z = \cosh 4$.

Q13. Prove that $|\sin z|^2 = \sin^2 x + \sinh^2 y$. Hence prove that $\sin z = 0$ iff $z = n\pi$ $(n = 0, \pm 1, \pm 2, \ldots)$

Q14. (a) Show that $f(z) = 2x^2 + y + i(y^2 - x)$ is not analytic at any point.

(b) Find the points where $f(z) = (x^3 + 3xy^2 - x) + i(y^3 + 3x^2y - y)$ is differentiable. Is f also analytic at ~~the~~ any of these points?

Dated : 04.05.24

Assignment (Complex Analysis)

B.Sc.(H) Mathematics III Year (2023-24)

Q1. (a) Find the Maclaurin series expansion of the function

$$f(z) = \frac{z}{z^4+9}.$$

(b) Show that if $f(z) = \sin z$, then

$$f^{(2n)}(0) = 0 \quad \text{and} \quad f^{(2n+1)}(0) = (-1)^n \quad (n = 0, 1, 2, \ldots)$$

Hence find the Maclaurin series ~~of~~ for $\sin z$.

(c) Derive Taylor series representation

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}} \quad (|z-i| < \sqrt{2})$$

(d) Derive the expression

$$\frac{\sinh z}{z^2} = \frac{1}{z} + \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+3)!} \quad (0 < |z| < \infty)$$

(e) Show that when $0 < |z| < 4$

$$\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}$$

Q2 (a) Write the two Laurent series in powers of z that represent the function

$$f(z) = \frac{1}{z(1+z^2)}$$

(b) Give two Laurent series expansions in powers of z for the function

$$f(z) = \frac{1}{z^2(1-z)}$$

and specify the regions ~~where~~ in which those expansions are valid.

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Q.3 Let $C: |z|=1$, taken counter clockwise direction. Show that

$$\int_C \exp\left(z+\frac{1}{z}\right) dz = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!\,(n+1)!}$$

Q 4. Use the theorem in sec 71, involving a single residue, to evaluate the following functions around $|z|=2$ in the +ve sense:

(a) $\dfrac{z^5}{1-z^3}$ (b) $\dfrac{1}{1+z^2}$ (c) $\dfrac{1}{z}$.

Q5. Suppose that a function is analytic throughout the finite plane except for a finite number of singular points $z_1, z_2, --, z_n$. Show that

$$\operatorname*{Res}_{z=z_1} f(z) + \operatorname*{Res}_{z=z_2} f(z) + -- + \operatorname*{Res}_{z=z_n} f(z) + \operatorname*{Res}_{z=\infty} f(z) = 0$$

Q6. Evaluate $$\int_C \frac{3z^3+2}{(z-1)(z^2+9)} dz$$

(a) $C: |z-2|=2$ (b) $|z|=4$.

Q7 Evaluate $$\int_C \frac{(3z+2)^2}{(z)(z-1)(2z+5)} dz \qquad C: |z|=3$$

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