

SHIVAJI COLLEGE, UNIVERSITY OF DELHI

DEPARTMENT OF Mathematics

INTERNAL TEST (Academic Year 2023-24)

Name of the Course : BSc (H) Mathematics

Semester : IV

Name of the Paper : Multivariate Calculus

Faculty Name : Dr. BABITA GUPTA

Duration : 1 hr

Maximum Marks: 12

Date of Test : 29/04/24

Attempt any 5 questions.

1. Let  $f$  be the function given by

$$f(x, y) = \begin{cases} \frac{xy^2}{x^2 + y^4} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

Is  $f$  continuous at  $(0, 0)$ ? explain.

2. In what direction is the function defined by

$$f(x, y) = xe^{2y-x}$$

increasing most rapidly at the pt  $(2, 1)$  and what is the maximum rate of increase.

3. Find the absolute extrema of  $f(x, y) = xy - 2x - 5y$  on triangular region with vertices  $(0, 0)$ ,  $(7, 0)$  and  $(7, 7)$ .

4. Sketch the region and compute the double integral

$$\iint_D (x+y) dA ; \text{ where } D \text{ is triangle with vertices } (0, 0), (0, 1) \text{ and } (1, 1).$$

5. Evaluate the given integral by converting to polar coord.

$$\int_0^3 \int_0^{\sqrt{9-x^2}} x dy dx$$

6. Find the volume of the solid bounded by  $z = 2$  and  $2z = x^2 + y^2$ .

Faculty Signature:

Bhupla

**SHIVAJI COLLEGE, UNIVERSITY OF DELHI**  
**DEPARTMENT OF MATHEMATICS**  
**INTERNAL TEST (Academic Year 2023-24)**

Name of the Course : BSc(H) Mathematics

Semester : Even

Name of the Paper : Multivariate Calculus

Faculty Name : Babita Gupta

Duration : 1 Hour

Maximum Marks: 12

Date of Test : 11/03/24

Attempt any 5 questions

1. Show that  $\lim_{n \rightarrow \infty} \frac{x^2 - y^2}{x^2 + y^2}$  does not exist.
2. For  $f(x, y) = x^2y + y^3x^2 + \cos(x^2)$  determine  $f_x, f_y$  and  $f_{xy}$ .
3. Write the equation of tangent plane to the surface  $Z = 10 - x^2 - y^2$  at  $(2, 2, 2)$
4. If  $Z = f(u - v, v - u)$  prove that

$$\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = 0$$

5. The directional derivative of  $f(x, y, z)$  at the point  $P_0$  is greatest in the direction of  $v = i + j - k$  and has value  $5\sqrt{3}$  in this direction. What is the directional derivative of  $f$  in the direction of  $w = i + j$ .
6. For  $f(x, y) = -x^3 + 9x - 4y^2$  find the critical points and identify as relative maximum, relative minimum or saddle point.
7. Determine the total differential for  $f(x, y) = \cos(x^2y)$ .



Faculty Signature: